MODELS AND MODELING WORKING GROUP

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The Models and Modeling Working Group at PME-NA has provided a forum for discussing and collaborating on projects fundamental to research on mathematical modeling since the first PME-NA conference in 1978. We propose to convene this Working Group at PME-NA 41 with a dual purpose: (1) to build on a theme begun in Greenville, holding a focused workshop to pursue innovations in activity design to connect modeling activities to mathematically rich, socially-engaged inquiry into questions from the world outside of school; and (2) to continue to invite newcomers to the Models and Modeling Perspective (MMP), giving them an introduction to this design research tradition.

Keywords: Modeling, Problem Solving, Design-Based Research, Design Experiments

The Models and Modeling Working Group was initiated in the inaugural year of the PME-NA conference in 1978, and it has met frequently since then. Over the 41 years of its existence, the working-group format has offered a vehicle for coordinating substantial research efforts and for fostering collaborations and mentoring relationships within the “Models and Modeling Perspective” (MMP). The MMP takes a pragmatic approach to fundamental questions in mathematics education, such as “What ... beyond having a mathematical idea ... enables students to *use it* in everyday problem-solving situations?” (Lesh, Landau, & Hamilton, 1983, qtd. in Lesh & Doerr, 2003, p. *i*., emphasis added). Such questions foster interdisciplinary collaborations connecting a broad range of theoretical perspectives.

A function of the Working Group has been to pursue innovations in design-based research (cf, Kelly, Lesh, & Baek, 2008) – to discuss and extend the ways in which a focus on models and modeling can be used both to support learning in STEM, and to study such learning processes in action. Indeed, calls for STEM integration (English, 2016; English & King, 2015) and for attending to Engineering perspectives (Diefes-Dux, Moore, Zawojewski, Imbrie, & Follman, 2008; Roehrig, Moore, Wang, & Park, 2014) have only increased the potential of this long-standing tradition to contribute to efforts to innovate in teaching and research.

This year, we propose to convene the working group to pursue an area of innovation in activity design for STEM integration. We aim to connect a signature *genre* of mathematical modeling activity created by the MMP, Model-Eliciting Activities (MEAs), with real-world modeling contexts that engage with important broader societal questions. In particular, we will workshop a particular MEA, refining it (and/or creating a cluster of thematically related MEAs), and connecting it with (a) related Citizen Science projects; (b) resources and information housed in centers for informal learning (e.g., zoos and museums); and (c) bearing on deep issues of social and ecological justice.

History of the Working Group

Early in its history, the Group focused heavily on the design and analysis of particular, self-contained activities that enabled groups of learners to engage realistic and deep forms of modeling and that produced an auditable trail of thinking, exposing their thought processes to teacher and researcher observers. Thus, this initial line of research pursued modeling activities as research instruments, analogous to group-level versions of Piaget's interview tasks. In this phase of the field's development, a primary effort involved elaborating design principles for these and documenting images of *idea development* that they promoted.

Quickly, these activities were recognized for their potential as powerful learning environments, yet this "turn" raised questions about how different student groups' work on open modeling problems could be "processed" by the whole class, bringing out common themes and connecting them to more conventional mathematical terminology, algorithms, and procedures. Gradually over time, researchers associated with the Group expanded their perspectives to consider implementations and curricular sequences that had longer time-duration, and that integrated models and modeling into the experience of learning mathematics in more extensive ways.

Several broad patterns in this more extensive and disseminated approach to modeling in the curriculum have emerged, and there is no sense that the MMP has yet exhausted the space of possibilities. These broader perspectives open both exciting opportunities and significant challenges. On the one hand, new questions can be researched, opening the way for new forms of contact and interaction with classroom practice and with learning outside of school; on the other, the approach raises new challenges at the level of methodology, data analysis, and forms of evidence that are convincing backings for claims about learner activity.

We propose convening the Group at PME-NA 41 to continue a style of work that has characterized the Group's collaborations over the past several years. In particular, based on our experiences in 2015-2018, we propose a work-session structure that can serve two dual purposes: (a) making substantive progress in work on a particular cluster of Model-Eliciting Activities, exploring how to use activities in the MEA genre to spark student connections with citizen science and ecological conservation; while also (b) integrating newcomers to Models and Modeling as a research area.

For this Working Group, these two goals are ***both*** essential: we propose to gather, not as a closed expert group, but as a broad group of educators and researchers. To welcome newcomers and new perspectives into the group, the structure we are proposing for the Working Group will provide initial introductions to the approaches and characteristics of the MMP and the activity designs the tradition has developed, but we also plan to engage both newcomers and returning participants in the work of continuing the refine the MEA structure in general and the Pelican MEA in particular to address issues of equity, sustainability, and environmental stewardship, which represent urgent opportunities and problems of research and practice in mathematics education.

In the following sections, we provide a very brief overview of the field of research represented by the Models and Modeling Perspective; we outline patterns in research efforts that have extended modeling activities over longer timescales; and we describe our plan of work in detail, illustrating how these goals are addressed as well as how we plan to productively integrate newcomers to the Group over the three working sessions offered in the Conference.

The Models and Modeling Perspective (MMP)

Since the 1970s, MMP researchers and educators have engaged in design research directed at understanding the development of mathematical ideas among groups of learners. A key principle behind this work has been that learners' ideas develop through, and in relation to conceptual entities called *models*. The core construct of a "model" and the activity of "modeling" are both central to the MMP; and they are also multidimensional, playing multiple roles in the MMP theory.

Models & Modeling Working Group founder Dick Lesh and Helen Doerr provide the following working definition of *models*:

> conceptual systems (consisting of elements, relations, operations, and rules governing interactions) that are expressed using external notation systems, and that are used to construct, describe, or explain the behaviors of other system(s)—perhaps so that the other system can be manipulated or predicted intelligently (Lesh & Doerr, 2003, p. 10)

In this spirit, "being a good modeler is in large part a matter of having a number of fruitful models in one's 'hip pocket.'" (Lesh, 1995, personal communication, qtd in Lehrer & Schauble, 2000).

But the term "model" not only applies to features of "target knowledge" that are built through learning: the interpretive systems that people *bring* to problems are also "models." When explicitly expressed through representational media, such models—personal interpretive systems—can provide illumination into how students, teachers, and researchers adapt, formulate, and apply relevant mathematical concepts in particular situations or contexts (Lesh, Doerr, Carmona, & Hjalmarson, 2003). In fact, "models" are the shape and vestment of most all knowledge—whether this knowledge appears as the patterns of perspectives and pre-conceptions that learners bring with them "in the door"; the shared ways of thinking that a group of learners build in solving a problem; or the systematic accounts of phenomena that represent the normative views of a scientific discipline at a given moment in its history.

An early finding of research in the Models and Modeling Perspective (MMP) was that, under appropriate conditions, groups of learners can be supported in producing external representations of the models they bring to a situation, and that when these groups put their initial models into conversation with one another and in contact with the real world, they can be supported to revise, and refine them in rapid and iterative cycles, building toward a more robust model that reflects their achievement of a shared way of thinking. In particular, when individuals and groups encounter problem situations with specifications that demand a model-rich response, their models can be observed to grow through such relatively rapid cycles of development toward solutions that satisfy these specifications.

While the elicitation of initial ways of thinking is valuable, MMP researchers' interest quickly turned to this process of model refinement—that is, to the dynamics of model***ing*** (as opposed to the statics of mod***els***), and to the features of activity environments that foster modeling and make it visible for teachers and researchers. The dynamics of modeling represent, for the MMP, an account of *idea development*, as observed in the discourse and other representations produced by groups of learners as they iteratively work to mathematize and formulate a solution that meets the needs of a concrete *client* in a realistic setting.

Thus, the MMP tradition became focused squarely on local conceptual development (Lesh & Harel, 2003): that is, on investigating the micro-evolution of ideas in groups of students (and teachers). The resources and tools its researchers produced were first and foremost designed to study idea development and the range of possibility for this mode of learning activity. The results of this work include a body of *Model-Eliciting Activities* (MEAs), in which students are presented with authentic, real-world situations where they repeatedly express, test, and refine or revise their current ways of thinking as they endeavor to generate a structurally significant product—that is, again, a *model*, comprising conceptual structures for solving the given problem. These activities differ markedly from some other environments dedicated to applications of specific mathematical concepts and procedures. In contrast, MEAs give students the opportunity to create and adapt mathematical models in order to interpret, explain, and/or predict the behavior of real-world systems (Zawojewski, 2013). An extensive body of MMP research has produced accounts of learning in these MEA environments (Lesh, Hoover, Hole, Kelly, & Post 2000; Lesh & Doerr, 2003), design principles to guide MEA development (Hjalmarson & Lesh, 2007; Doerr & English, 2006; Lesh, et. al., 2000; Lesh, Hoover, & Kelly, 1992) and accounts and reflections on the design process of MEAs (Zawojewski, Hjalmarson, Bowman, & Lesh, 2008).

As an example of the activity-type of Model-Eliciting Activities (MEAs), consider the Pelican Colony problem (Moore et al, 2015; Pompei, 2010) (below, and **Figure 1**). In this problem, the client is Alice Heart, a Wildlife Biologist from the U. S. Fish and Wildlife Service. The problem provokes students to grapple with notions of area, and for many (though by no means all) groups it prompts them to invent a measure analogous to "nest density" in the context of a pelican conservation effort to arrive at a procedure that the client can use.

An excerpt from the client letter, which includes the "call to action" for students, is given below:

> The U.S. Fish and Wildlife Service needs a procedure to estimate the number of nests at each pelican colony. Because pelicans are very sensitive to disturbances while they are incubating their eggs, we are not able to physically walk through every colony and count nests (this would also take too much time and cost too much!). We have hired pilots to fly our biologists over nesting colonies so they can take aerial photographs of the sites. As pelican colonies can be quite large (hundreds or thousands of nests), each photograph shows only a portion of the entire site. We have maps based on satellite images that are taken annually, which show us the shape and size of each colony site. We are enlisting your team's help to create a procedure that will allow us to estimate the number of nests in a pelican colony, based on the photograph that shows a sample of the colony, and a map that shows the size and shape of the entire site.

Students are provided with aerial photographs and colony outlines (Figure 1) for two sample pelican colonies. The client would like to be able to determine the size of a colony (in terms of the number of nests) from these pieces of data, and students are tasked with developing a procedure to 1) compare the sizes of different nests and 2) estimate the size of a nest based on an aerial photograph and an outline of the colony.

Student groups iteratively develop solutions to this problem in the time allotted—usually 50-60 minutes for this MEA. Afterwards, groups may present their work for a discussion of the modeling possibilities of these presentations (see Brady & Jung, 2019a; b, for suggestions of the value of such presentations); the students solutions can be used as a springboard for a class discussion following the five-practices structure described by Stein, Engle, Smith, and Hughes (2008); or the teacher may organize a "poster session" for the groups to share and learn from each others' solutions (Lesh, 2010). In one version of this activity structure, one member of each group hosts a presentation of the poster showing their results. The other students circulate to learn about other groups' solutions, using a Quality Assurance Guide to assess the results produced by others in the class. These instruments are submitted to the teacher and contribute to assessment in various ways, providing evidence for the achievements of both individuals and groups.

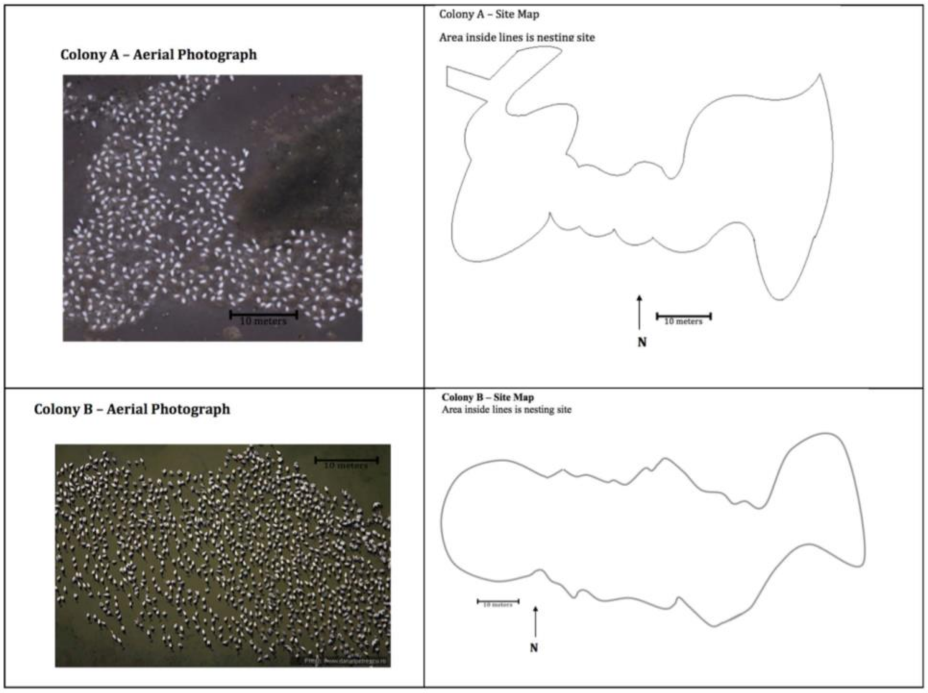

Figure 1: Two Pelican Colonies. Outlines, and Aerial Photographs.

MEAs like the Pelican problem present learners with situations where familiar procedures, ways-of-operating, and constructs may be *applicable,* but where they are also *insufficient*. In this sense, they support a learning environment that has a "low threshold" but a "high ceiling" (cf Papert, 1980). That is, on the one hand they are accessible to learners from a wide range of levels of ability, experiences, or knowledge (from upper elementary school through graduate school). On the other hand, learners encountering these problems find that they have no ready-made solution they can apply to address the client's needs. As a result, groups learners must engage in sense-making and solution-construction processes that position them as mathematical creators and also put them off balance in comparison to typical school-mathematics tasks. Indeed, this uncertainty is part of the design of MEAs, illuminating fundamental conceptual issues associated with the core mathematical structures involved.

MEA Design Principles

As individual MEAs emerged, an intense period of design research ensued to understand them as a genre of learning tasks that could (a) stimulate mathematical thinking representative of that which occurs in contexts outside of artificial school settings (Lesh, Caylor, & Gupta, 2007; Lesh & Caylor, 2007); (b) enable the growth of productive solutions through rapid modeling cycles; and (c) leave behind "auditable trails" - researchable traces of learners' ways of thinking during the process (Kelly & Lesh, 2000; English et al., 2008; Kelly, Lesh & Baek, 2008). The success of MEAs as an activity genre and as research tools was both enabled by and illustrated by the MMP's articulation of a set of six design principles (Lesh & Harel, 2003; Lesh et al., 2000; Hjalmarson & Lesh, 2007). These principles indicate essential elements of MEAs and their classroom implementations, enabling them to serve as rich contexts for student problem solving. Table 1, below, also indicates "touchstone" tests for whether each of these six principles has been realized in a given implementation setting.

Table 1: Six Design Principles for MEAs and Touchstones (see also Brady et al, 2017)

Principle	Touchstone Test for its Presence
Reality Principle	Students are able to make sense of the task and perceive it as meaningful, based on their own real-life experiences.
Model Construction Principle	To solve the problem, students must articulate an explicit and definite conceptual system (model).
Self-Evaluation Principle	Students are able to judge the adequacy of their in-process solution on their own, without recourse to the teacher or other "authority figure".
Model Generalizability Principle	Students' solutions are applicable to a whole range of problems, similar to the particular situation faced by the "client" in the MEA.
Model-Documentation Principle	Students generate external representations of their thinking during the problem-solving process.
Simplest Prototype Principle	The problem serves as a memorable representative of a kind of mathematical structure, which can be invoked by groups and by individuals in future problem solving.

Nested Levels of Modeling: Multi-Tiered Design Research

In parallel with learner-focused research using MEAs, researchers also have observed that *teachers'* efforts to understand their students' thinking involve yet another process of modeling: In this case, teachers engage in building models of student understanding. Although these teacher-level models are of a different category from student-level models, students' work while engaged in MEAs does provide a particularly rich context for teachers' modeling processes. Following this line of inquiry, the MMP community has also produced tools and frameworks that can be useful to teachers in making full use of MEAs in classroom settings, while also providing *researchers* with insights into *teachers'* thinking.

Finally, at a third level of inquiry, researchers' own understandings of the actions and interactions in *curricular activity systems* (Roschelle, Knudsen, & Hegedus, 2010) involving students, teachers, and other participants in the educational process can also be studied through the lens of model development. Multi-tier design experiments in the MMP tradition have done precisely this, involving researcher teams in self-reflection and iterative development as well (Lesh, 2002). Therefore, the MMP version of multi-tier design research can involve at least three levels of investigators— students, teachers, and researchers—all of whom are engaged in developing models that can be used to describe, explain, and evaluate their own situations, including real-life contexts, students' modeling activities, and teachers' and students' modeling behaviors, respectively. The situation can be further enriched by considering other educational stakeholders and learning settings, such as interactions between academic coaches and teachers (Baker & Galanti, 2017), and between schools and community organizations, and between students and parents.

Otten, S., Candela, A. G., de Araujo, Z., Haines, C., & Munter, C. (2019). *Proceedings of the forty-first annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. St Louis, MO: University of Missouri.

Constructing Curricular Materials to Support Modeling at Larger Timescales

Over the past 15 years, MMP researchers have continued this direction of work in their own teaching and in partnerships with K-12 classroom teachers. Within the domain of statistical thinking in particular, this effort has produced resources and tools sufficient to support *entire courses* in several versions and including accompanying materials related to learning and assessment aimed at both student and teacher levels.

In their work on MEAs, students have rich but idiosyncratic mathematical experiences that need to be *unpacked* and placed into relationship with each other and with more canonical concepts, practices, and procedures from the discipline. To investigate such matters, MMP researchers attend to learner activity beyond the scope of single MEAs, formulating tools and designs for Model Development Sequences, or MDSs (Arleback, Doerr, & O'Neil, 2013; Doerr and English, 2003; Hjalmarson, Diefes-Dux, and Moore 2008; Lesh, Cramer, Doerr, Post, & Zawojewski, 2003). Work here includes approaches for extending the modeling dynamics that MEAs foster and for unpacking and making explicit learners' ways of thinking, so that they are available to be reflected upon by the classroom group as a whole and systematized in relation to big ideas in the discipline (see also Brady, Eames, & Lesh, 2015 for tentative MDS design principles).

Because the courses supported by these materials were designed explicitly to be used as research settings, for investigating the interacting development of students' and teachers' ways of thinking, the materials were modularized so that important components could be easily modified or rearranged for a variety of purposes in different implementations. In particular, by selecting from and adapting *the same core collection of MEAs*, and surrounding them with MDS activities tailored to the learning goals and emergent ideas of different classrooms, parallel versions of the course have been developed for: (a) middle- or high-school students, (b) college-level elementary or secondary education students, and (c) workshops for in-service teachers. Existence-proof versions of these courses have produced impressive gains (see, e.g., Lesh, Carmona, & Moore, 2009).

As a result of the breadth of the MMP, other models for engaging with MEA-style modeling at larger timescales have also emerged. These larger structures reflect other professional and theoretical interests and concerns, such as a commitment to iterative design-based inquiry (Eames, et al, in press); a focus on socio-mathematical norms (Yackel & Cobb, 1996), or the dynamics of coaching (Baker & Galanti, 2017).

Connecting to Themes Supporting Sustained Inquiry

While Model Development Sequences and other MEA-based approaches (cf Eames et al, in press) explore ways to sustain authentic modeling experiences over longer timescales, we have recently become interested in ways to build upon the *thematic* interests that MEAs awake in learners. Entering the world of the MEA's client, learners become deeply engaged in a world-view, and they are positioned as assisting in a cause or concern that they adopt for the duration of the problem, often quite energetically. We are asking ourselves, how might teachers or educators at a larger scale (e.g., schools, districts) build upon student interest in MEA clients' problems to offer sustained inquiry into the issues and concerns raised there? And the reciprocal question, how might important social issues (e.g., questions of social or ecological justice), or movements (e.g., citizen science or citizen activism projects) be a fruitful source for MEA design, yielding not only opportunities for rich mathematical modeling but also entry points for engaging of the student population with these out-of-school themes, contexts, and groups.

Research and Discussion Themes to Guide the Working Group

In the past several, our previous PME-NA working group meetings in East Lansing, Tucson, Indianapolis, and Greenville have brought together over 40 participants from the US, Canada, and Mexico. Participants have described ongoing implementation and research across a wide range of grade levels and educational settings. In their work, attendees reported that they apply a variety of interpretive lenses and frameworks to modeling, and they situate their work in a variety of ways with respect to other current trends in mathematics education research. Moreover, in pursuing their practice, they developed definitions of core MMP constructs that were both broadly compatible (enabling productive discussion) and differently specialized and exemplified (enabling illuminating debate).

We have thus found Working Group meetings to offer unique opportunities to connect research voices and viewpoints, spurring conversations between research groups that have common inspiration and compatible interests, but very diverse local experiences and perspectives. The Working Group meetings have also consistently been a vehicle for connecting "old timers" with "newcomers." Some of the giants in the PME-NA community (e.g., Dick Lesh, Lyn English, Helen Doerr, Margret Hjalmarson, Jim Middleton, Tamara Moore, Eric Hamilton, and others) have also been leaders in the MMP, and in each of our sessions, we have invited participation from one or more of these leaders to offer perspectives on our thinking and on the field as a whole. We see these interactions as an important aspect of newcomers' (and old-timers') experience of the conference as a site for the exchange of wisdom, perspective, and enthusiasm among participants.

In our planning leading up to the Greenville we identified a compelling opportunity to experience and workshop an MEA that was still being designed. Thus we configured our work as a "researcher-level MEA" – where our "client" was the research team preparing the MEA for its first implementation, and where we experienced the problem as learners (working on the problem together during session 1) and as researchers (reflecting on our experiences and work-shopping possible revisions or additions to the activity).

As we learned more about the work of our "client" through workshopping the MEA, we became interested in the subject matter of the problem (which involved Box Turtles and conservation efforts surrounding them), and we identified connections between the way conservation efforts and issues entered into this problem and into work that members of our group had done in their own MEA designs. Our client's connections with a major ecological foundation were distinctive, but we felt that the idea of building an MEA that would awaken students' interest in ecological or social issues was not only compelling but possibly a repeatable and generalizable approach. This year, we aim to prove out that emergent conjecture. Specifically, we aim to explore how conservation and sustainability can serve as a context for authentic mathematical modeling—and how such modeling efforts can stimulate interest and concern for conservation and sustainability.

Session Outline: Advancing the Research Agenda while Building Community and Capacity

The working group will meet in three sessions over the course of the conference. As the organizers and facilitators do the preparatory work for the conference, these plans will be refined, but the broad outlines here reflect our current thinking.

As mentioned above, our experience of the working group over the past three PME-NA conferences has highlighted the value of these meetings for both (a) establishing and "hashing out" plans for innovative collaborative research, and (b) inviting interested newcomers to the

MMP, providing them opportunities to engage with its principles and practices and to interact with some of its founding members. Although it imposes an intense challenge for organizing and facilitating the working group, we aim to continue to support these two strands of activity. In part, we are committed to both because we recognize the importance to the MMP both to advance its agenda and rejuvenate its participant group. But in addition, we recognize that these two threads are in fact inseparable. Some of our most interesting theoretical discussions have come out of the friendly challenges from newcomers/outsiders, and we aim to cultivate and integrate rather than cordon off these voices and perspectives.

Key among the preparatory efforts for the facilitator group will be to select the focal MEA. One proposal is to return to the Box Turtle MEA workshopped last year, and use the session to explore adaptations and variations that could enable learners from diverse geographic settings to connect with local conservation efforts and/or citizen science projects. An alternative would be to pursue another of the design efforts emerging among others in the facilitator group, which also combine conservation, community engagement, and citizen science. In either case, the focus will be on creating MEA-style activities that foreground the mathematical modeling challenges inherent in pursuing such efforts. Below, we describe the session plan assuming the "Box Turtle" choice; changing the focal MEA will change the details but not the high-level purpose of each session or the "story arc" of the Working Group overall.

Session One

Our goal in Session One is to equip a diverse attendee group to participate in workshopping the focal MEA. With Box Turtles, this will involve a slightly time-compressed experience of the MEA as learners, followed by a report of the implementations since last PME-NA of this activity, and patterns in both students' modeling work and in the interests that students and teachers have expressed in conservation topics.

Sessions Two and Three

The goal of these sessions is to make substantive contributions to the effort to identify opportunities to connect the MEA with local conservation efforts, and to provide suggested structures for activating those connections. For instance, one of the emergent themes in our prior work with the Box Turtle MEA was the challenge of structuring observations (of turtle's features) and converting observed phenomena into measurable quantities (and into a *procedure* for measuring them). This is both an important theme in the history and philosophy of science (Daston and Lunbeck, 2011) and a theme recognized as appropriate and valuable for science learning, even at a young age (Eberbach & Crowley, 2009). Moreover, as Trumbull et al (2005) describe how identifying and stabilizing clear and repeatable procedures for observation is critical efforts like those central to organizing successful citizen science efforts.

A possible goal, then would be to refine the design of this MEA so that it (1) uncovers and delves into an important issue and practice involved in participating in citizen science projects, and (2) raises awareness and interest in the social and ecological themes that often *inspire* participation in citizen science projects. Together, these features of a refined MEA could both motivate and prepare students to engage in conservation projects, preparing them to experience citizen science through the lens of modeling, and modeling through the lens of engaged citizenship.

Session Three will be dedicated to making concrete research plans for continuing work after the Conference. This will include plans for multiple implementations of the revised focal MEA and other thematically-connected MEAs. We will foreground implementation opportunities, identify opportunities for cross-institution IRB proposals and additional collaborations to create new MEAs that exhibit similar connections with social issues and movements.

Acknowledgements

Proposing and facilitating this Working Group is a highly collaborative effort. Authors after the first contributed equally, and their authorship order has been determined at random.

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