SUPPORTING SECONDARY STUDENTS' PERSEVERANCE FOR SOLVING CHALLENGING MATHEMATICS TASKS

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Perseverance, or initiating and sustaining productive struggle in the face of obstacles, promotes making sense of mathematics. Yet, engaging in struggle can be grueling and is avoided for some students. I investigate the effect of scaffolding mathematics tasks on student perseverance. The results show how prompting secondary students to conceptualize a mathematical situation prior to problem-solving can encourage re-initiating and re-sustaining mathematically productive effort upon reaching an impasse. For learning mathematics with understanding, these findings suggest specific methods by which student perseverance in problem-solving can be supported.

Keywords: Instructional Activities and Practices, Problem-Solving, High School Education

Perseverance, or initiating and sustaining in-the-moment productive struggle in the face of one or more mathematical obstacles, setbacks, or discouragements, is a productive process by which understandings are developed. The idea of withstanding uncertainty and persevering past obstacles has long been recognized as a key process by which meanings can be formed (Dewey, 1910; Polya, 1971). These ideas have been echoed for learning mathematics because students develop their understandings through productive struggle, or as they grapple with mathematical ideas that are within reach, but not yet well formed (Hiebert & Grouws, 2007). Additionally, reconciling moments of impasse, or times of significant uncertainty or discouragement, is critical for mathematics learning; the processes of struggle to approach, reach, and make continued progress despite a perceived impasse puts forth cognitive demands upon the learner that are conducive for development of conceptual ideas (VenLehn et al., 2003). As such, encouraging student perseverance has been made explicit as a mathematical and pedagogical demand for reform to help improve teaching and learning in mathematics education (NCTM, 2014).

Supporting Student Perseverance with Mathematics Tasks

Despite widespread educational support around the notion of struggling with challenging mathematics, both students and teachers can be reluctant to engage in and offer opportunities for perseverance (DiNapoli & Marzocchi, 2017). Consequently, several recent research efforts have sought to make explicit classroom practices that support or constrain student perseverance with challenging mathematics. Kapur (2009) found that providing consistent opportunities for students to persevere with unfamiliar mathematical tasks encouraged more variability in problem-solving strategies and greater learning gains, compared to providing consistent opportunities to engage with more procedural mathematics. All students, however, were unlikely to make additional attempts at solving a problem after reaching an impasse – an area of concern. Bass and Ball (2015) explored the nature of perseverance by implementing classroom tasks with familiar entry points yet a complex structure (i.e., low-floor/high-ceiling tasks). The researchers observed children leveraging these opportunities to persevere in their efforts despite challenge and seemingly make mathematical progress. The authors urge for future research to document the student perspective around such perseverance and if it was indeed productive. Warshauer's (2014) exploration of productive struggle found that teachers used probing questions and encouragement as effective methods to nurture perseverance at times of immense student

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struggle, but had great difficultly providing consistent support for all classroom students for logistical reasons. Noteworthy outcomes of these studies are ideas for how perseverance can be operationalized, and suggestions for future research to carefully study perseverance from the student point of view to better understand how it can be supported.

Additionally, in a recent journal series on nurturing perseverant problem solvers, several teaching practices were identified and described as advantageous. These included encouraging independent student thinking by restricting teacher reassurance feedback during problem-solving (Bieda & Huhn, 2017), scaffolding students' experiences with tasks through assessing questions, advancing questions, and judicious telling (Freeburn & Arbaugh, 2017), and scaffolding student engagement with mathematics through establishing a culture of guiding, exploratory self-questioning (Kress, 2017). These studies offer insight into effective teacher moves for supporting student perseverance with mathematical tasks, but also call attention to logistical concerns about teachers providing targeted support for each and every student (Warshauer, 2014).

One method to bypass these logistical concerns is to embed similar scaffold supports into the mathematical tasks themselves to better insure each student has an opportunity to engage with them. Anghileri (2006) explains different levels of scaffolds that can be applied to tasks to help students problem-solve. Most relevant to supporting student perseverance are embedded conceptual thinking scaffolds, which provide opportunities for students to conceptualize the situation by making connections from their prior mathematical knowledge to the task at hand and mapping out their own strategies for problem-solving. Such conceptualization scaffolds provide a structure for thinking and acting that can organize a problem-solving plan coming entirely from a student's own ideas. Moreover, these scaffolds align with Polya's (1971) stages of problem-solving through which learners approach a task. While stages 3 and 4 help describe the actions of perseverance, it is stages 1 and 2 that can theoretically support those actions by encouraging students to conceptualize the mathematical situation at hand (see Figure 1).

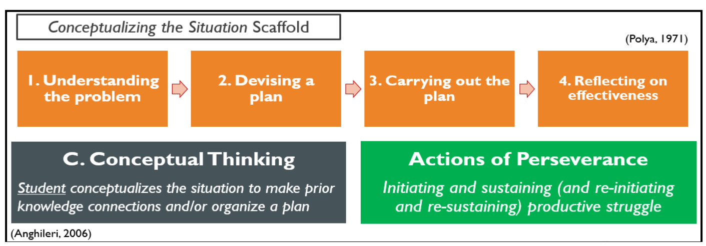

Figure 1. Scaffolding Perseverance in Problem-Solving

There is ample work that shows how conceptualization scaffolds encourage initial engagement with challenging mathematical tasks: when students have opportunities to make connections to what they already know and record all of their ideas and plans prior to the actual execution of problem-solving strategies, they are better suited to initiate and sustain their engagement with the task (e.g., Hmelo-Silver & Barrows, 2006). However, it is less clear how conceptualization scaffolds affect student perseverance upon reaching a perceived impasse – a moment that could be leveraged for key conceptual learning gains (VenLehn et al., 2003). If scaffolds are to support student perseverance during problem-solving, they must not only support initiating and sustaining productive struggle at the outset, but also support re-initiating and re-sustaining productive struggle after a student encounters a significant setback and is uncertain about how to continue. Therefore, via the student point of view, this study addressed how initially engaging with conceptualization scaffolds embedded at the start of low-floor/high-ceiling mathematics tasks supported perseverance, especially after a perceived impasse.

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Methods

The participants for this study were 10 ninth-grade students from one suburban-area high school algebra class in a Mid-Atlantic state. These participants were purposely chosen to have demonstrated, via pretest, the prerequisite knowledge necessary to initially engage with each mathematical task included in the study.

To collect data, each participant was observed engaging with five mathematical tasks, one per week. These tasks were rated as analogous, expert-level tasks by the Mathematics Assessment Project (MAP) because of their low-floor/high-ceiling structure, two objectives, and required generalization a mathematical situation. I also solicited independent mathematics education experts to rate the difficulty of these tasks; no differences were reported. Additionally, each participant was given an opportunity to reflect on task difficulty after their participation and reported no differences. Three tasks were randomly chosen to be scaffolded, and two tasks were randomly chosen to be non-scaffolded. The conceptualization scaffold embedded into the scaffolded tasks was "*Before you start, what mathematical ideas or steps do you think might be important for solving this problem? Write down your ideas in detail*." Each participant worked on these set of five tasks in a random order. For context and to help follow the results in this paper, Cross Totals (a scaffolded task) asked students to generalize rules about how to arrange the integers 1-9 in a symmetric cross such that equal horizontal and vertical sums would be possible or not possible. Triangular Frameworks (a non-scaffolded task) asked students to generalize rules about how to build different triangles using the triangle inequality theorem if the longest side was even or odd in length.

This study prioritized the student point of view of their perseverance, so I incorporated many opportunities for participants to make explicit their in-the-moment perspectives during problem-solving. For each task and participant, I conducted think-aloud interviews while they worked on a task and video-reflection interviews immediately after they finished working. Additionally, once a participant had engaged with all five tasks (and thus all five think-aloud interviews and video-reflection interviews), I conducted an exit interview to give each participant an opportunity to comment on their overall experience working on the five tasks. In all, I conducted 11 interviews with each participant, or 110 interviews in total for this study.

To analyze the data, I developed the Three-Phase Perseverance Framework (3PP) (see Table 1) that I used to operationalize the construct in this context. It was designed to reflect perspectives of concept, problem-solving actions, self-regulation, and making and recognizing mathematical progress. The 3PP considered first if the task at hand warranted perseverance for a participant (the Entrance Phase), considered next the ways in which a participant initiated and sustained productive struggle (the Initial Attempt Phase), and considered last the ways in which a participants re-initiated and re-sustained productive struggle, if they reached an impasse as a result of their initial attempt (the Additional Attempt Phase). A participant was determined to have reached a perceived impasse if they affirmed they were substantially stuck and unsure how to continue (VenLehn et al., 2003). Mathematical productivity was determined based on the extent to which the participant perceived themselves as better understanding the mathematical situation as a result of their efforts. To substantially capture the student point of view during engagement with tasks, coding decisions – or whether or not certain engagement constituted evidence of perseverance – depended on student cues from all interviews.

Table 1: Three-Phase Perseverance Framework (3PP)

Entrance Phase	
Clarity	Objectives were understood

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Initial Obstacle	Solution pathway not immediately apparent
Initial Attempt Phase	
Initial Effort	Engaged with task
Sustained Effort	Used problem-solving heuristics to explore task
Outcome of Effort	Made mathematical progress
Additional Attempt Phase (after perceived impasse)	
Initial Effort	Engaged with task
Sustained Effort	Used problem-solving heuristics to explore task
Outcome of Effort	Made mathematical progress

I used a point-based analysis with the 3PP to help inform deeper investigation of the ways in which participants persevered on scaffolded and non-scaffolded tasks. Each participants' experiences with each task were analyzed using the framework, and each component in the Initial Attempt and Additional Attempt Phases were coded as 1 or 0, as affirming evidence or otherwise, respectively. Coding decisions were based off of interview quotes. For instance, participants earned a 1 for an Outcome of Effort component only if they affirmed perceiving mathematical progress; such decisions were not based on my own perceptions. Since each task had two objectives and six components per objective, there were 12 framework components to consider, per participant, per task. Thus, 3PP scores ranged from 12 to 0, depicting optimal to minimal demonstrated perseverance in this context, respectively. Participants did not need to completely solve the task to earn a 12, they just had to exhibit perseverance in all 3PP components. Once 3PP scores were determined for all participants' experiences with all tasks, I conducted matched-paired *t*-tests to compare means of 3PP scores between work on scaffolded and non-scaffolded tasks. I also inductively coded interviews to uncover from the participant perspective why they persevered differently on scaffolded tasks compared to on non-scaffolded tasks. For trustworthiness, I enlisted help from two independent coders to analyze participant perseverance and their reasons for doing so. Our inter-rater reliability was 93%.

Results

Participants demonstrated higher quality perseverance when working on scaffolded tasks compared to on non-scaffolded tasks, in general, as evidenced by significantly greater mean total perseverance scores ($M_S = 8.73$, $SD_S = 2.07$, $M_{NS} = 6.00$, $SD_{NS} = 2.60$; $t(9) = 6.816$, $p < .001$) (see Table 2), as well as by participant reports. Differences in participants' perseverance in the Additional Attempt Phase of the 3PP drove this general finding. Participants demonstrated higher quality perseverance after encountering a perceived impasse while working on scaffolded tasks compared to on non-scaffolded tasks. This was evident by significantly greater mean Additional Attempt Phase perseverance scores ($M_S = 3.67$, $SD_S = 2.50$, $M_{NS} = 1.60$, $SD_{NS} = 2.58$; $t(9) = 4.083$, $p = .003$) (see Table 2), as well as by participant reports. All participants affirmed in the Entrance Phase that they understood the objectives, but did not know how to solve each task. Also, all participants reported a perceived impasse as a result of their engagement with each task.

Table 2: Three-Phase Perseverance Scores

Total Perseverance Scores (Maximum of 12 Points)		
Task Type	**Mean**	***n***
Scaffold	8.73	30 (3 tasks per 10 participants)
Non-Scaffold	6.00	20 (2 tasks per 10 participants)
Difference	2.73*** ($p < 0.001$)	

Additional Attempt Perseverance Scores (Maximum of 6 Points)		
Task Type	**Mean**	***n***
Scaffold	3.67	30 (3 tasks per 10 participants)
Non-Scaffold	1.60	20 (2 tasks per 10 participants)
Difference	2.07** ($p < 0.01$)	

The most prevalent difference between participants' perseverance on scaffolded and non-scaffolded tasks was whether and how they re-initiated and re-sustained a productive additional attempt at solving the problem. This means that while working on scaffolded tasks, participants often continued to productively struggle toward a solution after reaching a perceived impasse. This was not often the case after reaching an impasse while working on non-scaffolded tasks. When considering the specific Additional Attempt Phase components of re-initiating, re-sustaining, and outcome of effort, participants working on scaffolded tasks demonstrated more aspects of perseverance compared to their work on non-scaffolded tasks (see Table 3).

Table 3: Perseverance Frequencies in Additional Attempt Phase

Component	On Scaffolded Tasks	On Non-Scaffolded Tasks
Re-Initiated Effort	80% (24 out of 30)	35% (7 out of 20)
Re-Sustained Effort	70% (21 out of 30)	35% (7 out of 20)
Productive Outcome of Effort	57% (17 out of 30)	20% (4 out of 20)

While talking about their engagement with the five problem-solving tasks in the study, all participants reported a general positive effect of their preliminary conceptualizing work prompted by the scaffolds on their mathematical engagement. Related, none of the participants engaged in any noticeable preliminary conceptualizing work on tasks that did not prompt it (i.e., the non-scaffolded tasks). Most of the participants explicitly mentioned in their interviews that the reason they did not engage in such preliminary work on non-scaffolded tasks was because the task did not specifically ask them to do so, even though they recognized such work was helpful to them. Several more specific themes emerged from the analysis of all interviews that helped explain why participants found it easier to persevere on scaffolded tasks compared to on non-scaffolded tasks, especially after reaching an impasse (e.g., the scaffolds helped participants re-visit a different idea, re-conceptualize the situation after a mistake, stay organized, and establish momentum to stay interested and engaged).

Illustrative Case: James's Experiences with a Scaffolded and Non-Scaffolded Task

To illustrate how perseverance was better supported in scaffolded tasks compared to non-scaffolded tasks, especially by helping participants make a quality additional attempt, consider participant James's experiences with the Cross Totals task and Triangular Frameworks task. For James, Cross Totals was his fourth overall task and his third and final scaffolded task. Triangular Frameworks was his fifth and final task, the second of two non-scaffolded tasks. James passed through the 3PP Entrance Phase on both tasks by affirming he understood the objectives, but that had "no idea what to do." James earned 3PP scores of 6 in his Initial and Additional Attempt Phases, resulting in a maximum overall score of 12 for his work on Cross Totals. On Triangular Frameworks, James earned 3PP scores of 6 and 0 in his Initial and Additional Attempt Phases, respectively, resulting in an overall score of 6, with the most notable difference being no additional attempt at solving after reaching an impasse on Triangular Frameworks.

On Cross Totals, James began his work with the scaffold prompt, brainstorming about the mathematics present, including ideas about the parameters of possible and impossible cross totals and that the middle number would be included in both horizontal and vertical sums. After recording his conceptualization ideas, James started his initial attempt at solving the problem stating he would try to "find some possible ones first, and those might connect to impossible ones." He initiated and sustained his effort by guessing and checking different arrangements of integers along two lines representative of the lines of the cross, with one integer in the middle of both lines, and finding their sums. James noticed he was not having success with his plan, and affirmed he was at an impasse when he said "I'm stuck. This is harder than I thought." During his video-reflection, James revealed "I felt stressful here because I never learned this and there was not any way I would know how to do it. I definitely panicked." He did, however, believe he made some mathematical progress toward both objectives by figuring out "that you couldn't just throw numbers in [the cross], you had to think about big numbers and small numbers."

After James revealed he was at an impasse during his think-aloud, he paused and admitted "I don't know what to do now." Frustrated, he started looking around at his papers in front of him and eventually pointed to his list of mathematical ideas under the scaffold prompt and said, "Well I haven't used [the middle number] yet really. Something might be special about the middle number." During his video-reflection, he explained his point of view during these moments, "I had kind of forgotten that I wrote all this stuff down. So I saw again that middle number idea. It was kind of like my life-preserver…it kept me from giving up."

When James revisited his original conceptualization of this problem, prompted by the embedded scaffold, he was propelled forward into making an additional attempt at solving. He thought-aloud about his plan to re-initiate his effort by studying the middle number in possible and impossible cross totals. He began re-sustaining his effort by changing his point of view, a different problem-solving heuristic, and examined the given example. He noticed that "this one has 9 in the middle to get a 27, they are balancing the big and small numbers... something about the evens and odds." James wrote down his observations around the provided example and went on to explore his own examples (see Figure 2) and solve the problem by concluding and defending that "all solutions are odd, like the middle can't be even to do a cross total."

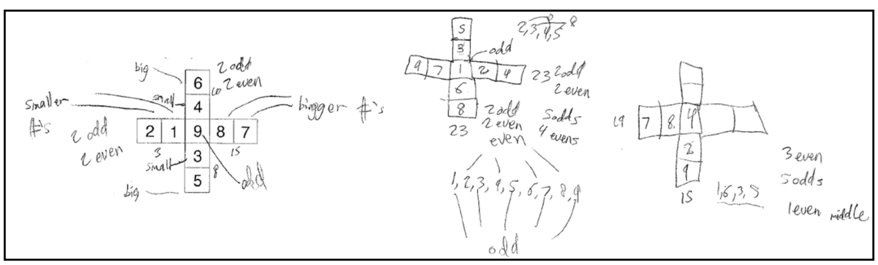

Figure 2. James's Work in Additional Attempt Phase on Cross Totals

On Triangular Frameworks, a non-scaffolded task, James did not record his initial conceptualization of the mathematics in the task. He started his initial attempt toward both objectives by saying "I'll try to find some other evens and odds that work out." He initiated and sustained his effort by guessing and checking different triangles, the same initial heuristic he used in Cross Totals the week before. James examined the given example of six frameworks made with a longest side of 7m, and made mathematical progress when he built four frameworks with a longest side of 6m, writing "6-5-4, 6-5-3, 6-5-2, 6-4-3" on his paper. James concluded, "It looks like it's one less for odd or two less for even, so the rules might be that." Before James wrote his rules, he said, "Wait, let me look at something." Clearly troubled, James went on to write “5-4-3” and “5-4-2” on his paper. Then, seemingly ignoring what he had just done, he wrote his rules about the situation (rules that were incorrect), that if the longest side, *c*, is odd he can make *c*-1 frameworks, and if *c* is even he can make *c*-2 frameworks. Finally, he said “I’m done” and stopped working without making an additional attempt.

While reviewing the video of the aforementioned moments during his first attempt at solving Triangular Frameworks, James admitted that he was, in fact, at an impasse during the latter stages of his first attempt. He said, “I was looking at if 5 was the longest side and my rule didn’t work! There should have been four of them but there were only two, 5-4-3 and 5-4-2…so I panicked and pretended like it worked.” When asked why he pretended, he said, “I panicked when it didn’t work. I didn’t even know where to start to fix it. So I really wanted to stop.” James’ experiences with Cross Totals and Triangular Frameworks are illustrative of the ways in which participants persevered while working on scaffolded tasks compared to on non-scaffolded tasks. Similarly to his engagement on Cross Totals, James leveraged his initial conceptualization work while working on the two other scaffolded tasks to help make a quality additional attempt at solving despite encountering frustrating impasses. On the other hand, similar to his work on Triangular Frameworks, James did not make an additional attempt at solving the other non-scaffolded task; he cited overwhelming stress and feeling disorganized after encountering a setback as the primary reason he did not persevere. During his exit interview, James shared his perspective on how initially attending to conceptualizing the mathematical situation had a positive effect on his engagement for all scaffolded tasks:

> You have to have a plan of what you can try to do…Like the ones that made me write down what I thought first. That was really good. I used that a lot because sometimes you forget where you’re going in a problem, it’s like chaos – like on [Triangular Frameworks]. But on other ones, like [Cross Totals] I got stuck but had a way out of it. It seems easier with that because you don’t have to think of a way out when you’re mad or stuck or something.

For James, and for almost all participants in this study, responding first to the conceptualization scaffold served as a “life-preserver” of sorts later, when participants were “panicked” and most tempted to give up. In these moments, the conceptual thinking recorded after engaging with the scaffold prompt acted as an organizational toolbox from which to draw a fresh mathematical idea, or a new connection between ideas, to use to help re-engage with the task upon impasse and to continue to productively struggle to make sense of the mathematical situation. Participants were persevering in problem-solving cyclically, with each additional attempt as a new opportunity to productively struggle supported by their own conceptual ideas (see Figure 3). Without recording their conceptual thinking on non-scaffolded tasks, participants felt lost and frustrated after a setback and often gave up without making an additional attempt at solving.

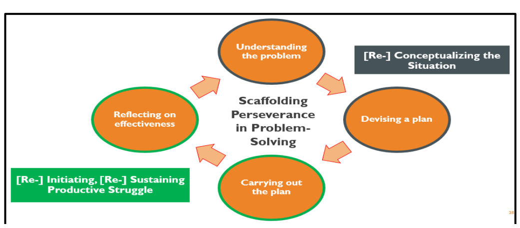

Figure 3. Rethinking Scaffolding Perseverance in Problem-Solving

Discussion and Conclusion

Prior research on supporting student perseverance with mathematical tasks was limited to a

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focus on the low-floor/high-ceiling structure of the task itself (e.g., Bass & Ball, 2015), teacher moves that nurtured independent student thinking (e.g., Freeburn & Arbaugh, 2017), and using conceptualization scaffolds to encourage initial engagement (e.g., Hmelo-Silver & Barrows, 2006), but did not examine in detail the student perspective around moments when perseverance is necessary, especially upon perceived moments of impasse. Addressing classroom logistical concerns by embedding scaffolds directly into low-floor/high-ceiling tasks (Anghileri, 2006), this study extends past research by investigating from the student point of view how conceptualization scaffolds help students persevere re-initiate and re-sustain productive struggle after reaching an impasse. In sum, the results showed positive effects of prompting students to record their initial conceptualization of a mathematical situation prior to problem-solving, as evidenced by more and higher-quality perseverance on scaffolded tasks compared to on non-scaffolded tasks. Arguably the most notable effect occurred when students reached an impasse while problem-solving, because often they were able to leverage their initial conceptualization to overcome that obstacle and continue to make progress. This implies such scaffolding can help students persevere past impasses and continue to make progress, if provided the opportunity to explore mathematics as a discipline of activity (Schoenfeld & Sloane, 2016). Further, greater efforts are needed to establish attending to conceptual thinking as a normative and welcomed school practice during all problem-solving, not just when prompted (Warshauer, 2014).

Future work should seek to further validate the impact of conceptualization scaffolds on student perseverance after impasse. Especially important is collecting evidence to refute the alternative explanation that differences in task difficultly could explain these data reported here. Although I made several efforts to ensure each task was similarly difficult (vetted by MAP, analyzed by experts, rated by participants), a follow-up study could randomize the assignment of scaffolds to tasks for each participant to better control for this potential bias.

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