THE INITIAL TREATMENT OF THE AREA MEASUREMENT IN THE SELECTED ELEMENTARY MATHEMATICS TEXTBOOKS FROM US AND KOREA

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This study compared area lessons from Korean textbooks and US standards-based textbooks to understand differences and similarities among these textbooks, as well as how these textbooks address known learning challenges in area measurement. Several well-known challenges have been identified in previous studies, such as covering, array structure, and linking array structure to area formula. We were interested in knowing if textbooks addressed these issues in their treatments of area measurement, and in doing so provided students with opportunities to overcome or become familiar with known challenges. The results show that both countries textbooks demonstrated similar limitations, only few area and area related lessons are covered and three important learning challenges in area measurement are not covered well, which need to be informed to practicing teachers.

Keywords: Area, Textbooks, Learning Challenges

For the last few decades, mathematics education researchers have been interested in how students in different countries learn mathematics. Such interests are the result of studies such as the Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA), which indicate that students in East Asian countries perform consistently well. One important area we can examine among various opportunities to learn (OTL) is what textbooks offer to students for their learning, as textbooks play an important role in lesson enactment process, teachers use textbooks and other resources to select and modify tasks to prepare their lessons (Remillard & Heck, 2014). Among many mathematical topics, US students' performance in measurement is weaker than any other content area on TIMSS (Mullis, Martin, Foy, & Hooper, 2016). While researchers conducting international comparative studies of textbooks have examined various mathematical topics (Cady, Hodges, & Collins, 2015; Son & Hu, 2016), area lessons have not been examined and compared often. The purpose of this study was to examine and compare area lessons in US Common Core-aligned textbooks and Korean textbooks, and explore how textbooks address well-known learning challenges in area measurement.

Related Literature

Textbooks in the curriculum enactment process

Although not all contents in textbooks will automatically be transformed to mathematics lessons directly (teachers will likely modify textbook contents), examining the treatment of mathematical topics in textbooks can tell us how much attention is given to that specific topic. In the curriculum enactment process, teachers select and possibly modify mathematical tasks and activities from textbooks and other curriculum materials (Remillard & Heck, 2014). Textbook content and how teachers enact their lessons jointly influence what students experience in their

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classrooms (Smith, Males, Dietiker, Lee, & Mosier, 2013). For example, if textbooks do not address well-known challenges, we can interpret that as a possible reason for students' struggle because it is possible that those challenges are not well reflected in the teacher's lesson plans and decrease students' OTL to learn and get familiar with those challenges (Smith, Males, & Gonulates, 2016).We cannot say textbooks are the only reason for students' struggle in learning area measurement. However, limited coverage of topics will limit opportunities to learn for students and is a possible explanation for their performances.

How Students Learn Area Measurement

Foundational concepts for area include covering a region without gaps or overlaps, counting unit measures, iterating, understanding array structure and linking the number of squares to length and width (Battista, 2004). Being able to use same – sized units repeatedly to cover a region and iterating are fundamental skills to understand measurement in general (Smith et al., 2016). However, studies have shown that it is challenging for students to develop a good conceptual understanding of area. Students are not able to cover a two dimensional region with equal-sized units. Instead, they often use unequal unit or leave gaps or overlaps (Battista, 2004; Outhred & Mitchelmore, 2000). Without having conceptual understanding, students often use the area formula *length width* without understanding why and for the wrong figures (Zacharos, 2006). However, when students learn using the conceptual approaches of partitioning and covering (partitioning, filling a given space and seeing array structure), they are more likely to develop a better understanding of area measurement (Huang, 2017; Na, 2012; Outhred & Mitchelmore, 2000). Here are research questions that we attempt to answer.

- How do US and Korean textbooks distribute attention to area and area-related lessons?
- In what order do the curricula present concepts related to measuring area, and do the sequences differ significantly between textbooks?
- How well do the curricula address well-known students' challenges in learning area measurement?

Methods

Data Sources

Three textbooks series - *enVisionMath*, *Go Math*, and *MyMath* - are Common Core-aligned textbooks, which were not examined in recent study (Smith et al., 2016). A total of 9 US textbooks were examined, three textbooks from grade 1 through grade 3 from each publisher. Two recent studies examined length and area lessons in other non - Common Core American textbooks and also several other studies examined textbooks that were developed before the introduction of the Common Core State Standards (CCSS) (Hong & Choi, 2014, 2018; Smith et al., 2013; Smith et al., 2016; Son & Senk, 2010). Therefore, examining these common core aligned textbooks series may expand our understanding of how more current American textbooks treat volume measurement. In all, 431 (*enVision Math*), 441 (*My Math*) and 550 (*Go Math*) items were analyzed.

For Korea, the textbooks examined in this study, Elementary School Mathematics, were published by the government and the only mathematics textbooks used in elementary schools in Korea. For Korean textbooks, we examined 129 items from grades 1, 3 and 5. Table 1 describes the number of pages and lessons examined for this study.

Table 1: Textbooks used in the study

Textbook Series	Publisher	Publication Date	Pages	Items	Lessons
envisionMath series	Pearson	2015	86	431	18
Go Math series	Houghton Mifflin	2015	133	550	15
MyMath series	McGraw-Hill	2014	69	441	12
Korean Textbooks	The Ministry of Education in Korea	2014, 2015	31	129	6

Framework for Analyzing Textbooks

When textbooks are analyzed, exposition (e.g. introductory paragraphs, text boxes with definitions, formulas, or theorems), worked examples (problems presented together with an explained solution) and exercise problems (mathematical items students are expected to solve) should be examined because they can provide potentially different OTL for students.

We also searched for studies that examined measurement lessons in textbooks. Smith and his colleagues (2013, 2016) examined length and area measurement in US textbooks. Their framework specifically targets how textbooks address challenges in learning length and area. We adopted and modified Smith and colleagues' framework to analyze area measurement lessons. Table 2 describes our framework for this study.

Table 2: Analysis framework of content and problems

Area of Focus

• Number of area and area related lessons
• Timing and topic sequence
• Procedural and conceptual knowledge
• Known challenges in learning area measurement
    o Covering with equal-sized units
    o Row and column array structure
    o Area formula and definition
• Response type

Coding Procedures and Examples

Each exposition, worked example, and exercise problem in textbooks has its own instructional purpose (potentially different OTL). Each exposition introduces mathematics content including definitions, formulas, and procedures; each worked example demonstrates how certain problems are solved; and each exercise problem gives students opportunities to engage in problem solving. Thus, when we discussed our unit of analysis, we first considered each worked example, exercise problem, and exposition as one unit of analysis as each item provides OTL to teachers and students. Figure 1 shows examples of how we coded each item.

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Translation:

- This is the floor plan for Chul Soo's home. Find out the area of each room.
  Parents' Room
  Chul Soo's Room
  Brother's Room

Figure 1. Coding examples from a Korean textbook (The Ministry of Education in Korea, 2015, p. 139)

First, we decided there are three items (three exercise problems or three units) to be coded. Students are asked to use the area formula to compute the area of each room. In terms of topic, these items are coded as area formula because students just need to multiply numbers to get the correct answers. In terms of procedural and conceptual knowledge types, these were coded as procedural (only multiplying two numbers is required). Finally for response type, these were coded as short response (only numbers are required). For all other textbook pages, we used the same method to identify expositions, worked examples, and exercise problems to count the number of units to be coded from each page.

Reliability

Each textbook included exposition, worked examples and exercise problems. After discussing the established codes, two authors coded approximately 20% of the textbook items to check inter-rater reliability. After comparing codes for sample items and finding an acceptable high inter-rater reliability, the authors coded all textbook items jointly to produce a final set of tables for analysis, resolving coding differences of individual items when they arose. To determine reliability, we applied a generalizability theory D study (Alkharusi, 2012). This technique produced a reliable coefficient of 0.964.

Results

Area Measurement Lessons in Textbooks

Table 3: Number and Percentage of Area and Area Related Lessons to the Total Lessons

	Grade	Area and Area-Related Lessons	Total
enVision Math	1	5 (4.5%)	110
	2	8 (6.9%)	116
	3	5 (4.2%)	119
Go Math	1	3 (3.0%)	101
	2	4 (3.6%)	110
	3	8 (7.6%)	105
MyMath	1	3 (3.2%)	95
	2	2 (2.2%)	92
	3	7 (6.2%)	113
Korean Textbooks	1	1 (1.4%)	70
	3	1 (1.5%)	68
	5	4 (5.7%)	70

Table 3 indicated that little curricula attention was given to area and area-related topics in these textbook series, supporting earlier findings by Smith et al. (2016). These percentages were less than the 1% to 12% range found by Smith et al. (2016).

Time and Sequence

With regards to the sequencing of topics, Korean and US textbooks differed. The Korean textbook series first introduced area in grade 1, with a lesson titled "Comparing Area". This was a short lesson that asked students to compare and make visual judgments between two objects. These items were not found in US textbooks. After a brief introduction to area in grade 1, Korean textbooks had one lesson about partitioning shapes into equal parts in grade 3. Lessons on area then began in grade 5, where the textbook introduced unit squares, area of rectangles and the area formula. In contrast, all three US textbook series included several lessons in grades 1 and 2 about partitioning regions, including rectangles and circles. Then, the area formula was introduced in grade 3 US textbook series. In terms of timing, it appeared that Korean textbooks introduced area first in grade 1, but ideas of partitioning, understanding and finding area of rectangles were found earlier in US textbook series.

Procedural, Conceptual and Conventional Knowledge

The majority of items (83 to 100 % in all textbooks) in both countries' textbooks were procedural. This finding supports previous work examining American textbooks, where procedural items accounted for more than 87% of items in US textbooks (Smith et al., 2016). Such findings imply that the focus of area lessons in both American and Korean textbooks are more about procedures than concepts. Again, this can be one way to lead both countries' students to a more procedural understanding of area.

Knowledge Needed in Understanding Area Measurement

Covering. Covering may be introduced with drawing, using tiles or iterating. Only limited number of covering items were included in textbooks (less than 17 % area items in each grade). Compared to US textbooks, Korean textbooks included covering items much later, only introducing them in the fifth grade. Such a lack of inclusion is problematic, as previous studies have shown that it was challenging to second graders to cover a region completely without gaps or overlaps (Battista, Clements, Arnoff, Battista, & Caroline Van Auken, 1998; Lee, 2010). We also noticed that when textbooks include covering items, the terms "gaps" and "overlaps" are used only few times: less than 10 times in each American textbook series and never in the Korean textbooks. Since students often struggle with covering a region without gaps and overlaps (Outhred & Mitchelmore, 2000; Sarama & Clements, 2009), careful attention to covering and explicit remarks about why gaps and overlaps are important will lead students to a more conceptual understanding of what it means to measure area.

Array structure. Items in this group included drawing, tiling or partitioning a region into rows or columns (the terms "rows" or "columns" needed to be included, or students needed to have opportunities to show array structure) and then counting them or using partial array structure. In US textbook series, partitioning a rectangle into rows and columns appeared first in grade 2, and grade 3 but coverage of the topic was brief and limited (less than 7 % area items in each grade). In Korean textbooks, the topic of array structures appeared in grade 5 but limited number of items were included (about 11 % of its area-related items).

Area definition and formula. Items were coded as area formula if they showed that multiplying two numbers gives the area, or if they used the *length width* formula to compute the area. Both countries' textbooks used array structure to introduce the area formula. Korean

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textbook lets students derive the formula after working on tasks counting unit squares in grade 5 textbook. US textbook series also used an array structure in grade 3. As previously mentioned, with limited opportunities to explore how array structure and the area formula are related, students are likely to resort to more procedural approaches to area. Such a tendency was reinforced by every textbook, as they all moved quickly to items, where students were only required to use a procedure, multiplying length and width.

Other area topics

One of the frequent topics in both countries' textbooks is counting unit squares in a shape without focusing on array structure (ranging from 23% to 41% of items in grade 3 textbooks and about 26 % in grade 5 Korean textbook). With such a heavy focus on counting squares, paired with limited experience with array structures in previous years, it will be challenging for students to connect array structure to the area formula. Prior studies have shown that even with drawn lines and squares, the connection between counting unit squares and area is not apparent to elementary students (Battista, 2004; Battista et al., 1998). Students may count unit squares procedurally without seeing array structure or understanding the purpose of not having gaps or overlaps. With limited opportunities to experience covering and array structure in grades 1 and 2, it will be challenging for students to see array structure when they are trying to count unit squares.

Summary and Conclusion

This study compared area lessons from Korean textbooks and US textbooks to understand differences and similarities among these textbooks, as well as how these textbooks address known learning challenges in area measurement. Our results indicated that textbooks from both countries paid modest or limited attention to area measurement lessons. In terms of timing, US textbook series introduced area related topics, partitioning, covering, array structure and area formula much earlier than Korean textbooks. In terms of sequence, US textbooks progressed through partitioning, tiling and then presenting the area definition and formula. Studies showed, 2nd and 3rd graders often struggled with covering and array structure (Battista, 2004; Sarama & Clements, 2009). However, textbooks from both countries introduced such topics either later or not at all. Such findings indicate that both timing and sequencing were an issue for both countries' textbook.

Both countries textbooks placed strong focus on procedures rather than concepts. Also, the most frequent items were partitioning regions without array structure. Compounded with issues of sequencing and timing, a procedural focus, and limited coverage of important conceptual area ideas are highly like to lead to students' challenges in learning area measurement. What may we conclude from our findings? As we mentioned previously, although textbooks do not provide mathematics lessons directly (content will likely be modified by teachers), they are one of the main resources teachers use when planning lessons. With the issues identified in these textbook series thus far, it is possible that limitations in textbooks can lead to area lesson plans that do not reflect challenges and important concepts of area measurement. In turn, limited coverage may lead to limiting elementary students' learning opportunities and they may be inclined to adopt a more procedural understanding of area, without attaining a conceptual understanding. With our findings, it will be important to provide teachers with additional supports so that they can attempt to modify tasks in these textbooks to properly address students' learning challenges in area measurement.

In terms of international comparative studies, we cannot say that the learning opportunities these textbooks offered are directly related to US students' performances in TIMSS. However,

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our results showed that how these textbooks treat area measurement could be one of the reasons for US students' TIMSS results. Despite Korean students' high performances in measurement, we did find that Korean textbooks shared many of the same conceptual limitations as US textbooks. As Smith and colleagues (2016) noted, further studies are required to examine the link between curriculum use and students' performances in assessments in order to make more distinct claims about influence of textbooks on students' performances.

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