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Impact and Prevalence of Diagrammatic Supports in Mathematics Classrooms

Bryan J. Matlen[1], Lindsey E. Richland[2], Ellen C. Klostermann[2], and Emily Lyons[2]

[1] WestEd, Redwood City CA 94063, USA
[2] University of Chicago, Chicago IL, 60637, USA
bmatlen@wested.org

Abstract. Mathematical problem solving typically involves manipulating visual symbols (e.g., equations), and prior research suggests that those symbols serve as diagrammatic representations (e.g., Landy & Goldstone, 2010). The present work examines the ways that instructional design of student engagement with these diagrammatic representations may impact student learning. We report on two studies. The first describes systematic cross-cultural differences in the ways that teachers use mathematical representations as diagrammatic supports during middle school mathematics lessons, finding that teachers in two higher achieving regions, Hong Kong, and Japan, more frequently provided multiple layers of support for engaging with these diagrams (e.g. making them visible for a longer period, using linking gestures, and drawing on familiarity in those representations), than teachers in the U.S., a lower achieving region. In Study 2, we experimentally manipulated the amount of diagrammatic support for visually presented problems in a video-based fifth-grade lesson on proportional reasoning to determine whether these multiple layers of support impact learning. Results suggest that learning was optimized when supports were used in combination. Taken together, these studies suggest that providing visual, temporal, and familiarity cues as supports for learning from a diagrammatic representation is likely to improve mathematics learning, but that administering these supports non-systematically is likely to be overall less effective.

Keywords: Mathematics Learning, Comparison, Diagrammatic Representations, Analogy, Cognitive Supports

1 Introduction

Teaching students mathematics that is flexible, transferrable, and connected across topics is crucial to high quality instruction; however, despite decades of agreement to this pedagogical goal, many students struggle with such mathematical thinking (e.g., Polya, 1954; Bransford, Brown & Cocking, 2001; National Mathematics Panel, 2008). Given the difficulties students experience in learning mathematics, a key goal is to improve instruction.

Improving mathematics instruction requires an understanding of the cognitive processes involved in acquiring mathematics knowledge and skills. In this paper, we highlight one challenge to thinking mathematically: learning to *perceive* mathematical problems and symbolic equations as sets of relationships (arithmetic, proportional, inequality, etc.) (see Richland, Stigler & Holyoak, 2010). The ability to perceive the relational structure of mathematics allows the problem solver to more easily draw connections across problems or mathematical ideas, and to think more conceptually about mathematics.

Mathematical representations are diagrammatic if they convey structural properties through their spatial and perceptual attributes. As an illustration, in the equation $3 + (4 – 5)$, a learner must know to subtract 5 from the 4 before adding three because the parentheses convey priority in carrying out the operations. Thus, part of the students' task in working with mathematical representations is to perceive the relevant features and structural relations that are embedded in their perceptual instantiations. Effective mathematical instruction, therefore, requires attention to how perceptual representations are presented in order to support students' understanding of mathematical concepts (Kellmann & Massey, 2013; Richland & McDonough, 2010).

Considering mathematical equations and symbolic representations as diagrams allows one to formulate insights into how to best support learners in identifying the core relationships within these representations. Diagrams can use spatial cues and sparse representations to highlight relationships rather than simply depict iconic information (Ainsworth, 2006; Michal et al. 2016), however it is not the case that students learn from any and all experiences with a diagram (Rau, 2017). Rather, it is clear that not only must the diagram be informative and relevant, but also pedagogical practices for supporting students' thinking and must improve the likelihood that students notice and attend to the key relationships being depicted (e.g., Richland & McDonough, 2010). To improve students' attention to relationships, we can draw on strategies for ensuring that students learn from diagrams to inform mathematical pedagogy.

In the present paper, we begin by reviewing principles deriving from perceptual learning, mathematics, and reasoning literatures to highlight strategies for how to improve attention to relationships within diagrams: a) use visual representations, make them visible while discussing them and subsequent representations, b) use hand movements (linking gestures) to move between instructional diagrams, and c) draw on material that is familiar to learners (see below for more detail). We then describe two sets of data suggesting that combining these strategies systematically may be the most potent way to improve student learning from mathematical diagrams. First, we describe an analysis of cross-national data collected as part of the Third International Mathematics and Science Study (TIMSS, Hiebert et al, 2003) showing that teachers in two regions that outperform the United States, Hong Kong and Japan, used these pedagogical principles in combination more systematically than did U.S. teachers. These data suggest both that these strategies must be considered in combination, rather than as separable practices, but also that these may be correlated with student learning. In a second study, we report an experimental design in which we tested the efficacy of them being used together. The results support the correlational data identified in study one, together providing consistent indications that these are important pedagogical practices that can support students' attention during engagement with mathematical diagrams, and that doing so has consequences for student learning even when the mathematical diagram and the audio-stream of the lessons are identical.

1.1 Perceptual Learning in Mathematics

Though mathematics has traditionally been viewed as involving conceptual learning, there is evidence that mathematics learning is highly perceptual (e.g., Kellman & Massey, 2013). For instance, mathematical concepts are frequently represented in the form of symbolic notations, which themselves contain perceptual attributes that are connected to structural properties. As an illustration, Landy and Goldstone (2010) had subjects solve simple equations that were presented either spatially consistent with the order of operations (e.g., solving 3   +   (4 – 5)) or spatially inconsistent (e.g., solving 3 + (4   -   5)) – these authors found that the spatial distance between influenced problem solving: Subjects were less accurate at solving simple equations when they were spaced in ways that were inconsistent with the order of operations, suggesting that even adults represent simple equations as types of diagrams.

As another illustration of perceptual learning in mathematics, interventions that support perceptual learning processes have shown some promise for improving students learning outcomes. For example, Kellman and colleagues have developed visual matching exercises that engage learners in linking different representations of mathematical concepts. In these exercises, students do not formally solve problems or conduct calculations; rather, students learn to identify the attributes that connect different mathematical representations. Despite never formally solving problems, students who engage in these linking activities are more accurate and at later problem solving than students that do not engage in them (Kellman et al. 2008; Kellman, Massey, & Son, 2010). A related perceptual learning intervention that allows students to perform physical manipulations of equations that are consistent with the grammatical rules of algebra has shown promise for supporting students’ algebraic understanding (Ottmar, Landy, & Goldstone, 2012).

Because mathematics involves perceiving the relevant structure in representations, diagrammatic supports that highlight structure can be a powerful tool to promote mathematical understanding and fluency (Rau, Aleven, & Rummel, 2009; Rittle-Johnson, Star, & Durkin, 2009). At the same time, simply providing diagrams may not result in successful learning (Rau, 2017). Often domain leaners fail to notice relevant correspondences between representations unless highly supported in doing so (e.g., Alfieri, Nokes-Malach & Schunn, 2011; Gick & Holyoak, 1980, 1983; Richland & McDonough, 2010). Children and domain novices (both characteristics of k-12 school children) are most susceptible to missing key elements of comparisons and attending to irrelevant salient features that impede relational thinking, in part due to low cognitive processing resources (e.g., Richland, Morrison & Holyoak, 2006). We next review research on how to support reasoning with diagrams in mathematics.

1.2 Diagrammatic Supports

While diagrams can serve as effective cognitive supports for learning in mathematics, students need support in comprehending diagrammatic representations before they can benefit from them. Understanding of visualizations in mathematics can be supported by cognitive aids that highlight relevant structural properties in the representations. The science of learning has made advances in understanding of how students best learn with diagrammatic representations. These principles for supporting diagrammatic fluency are discussed, below.

Making Representations Visible Simultaneously. Research suggests that learning in general is facilitated by the use of simultaneous diagrammatic representations (Gadgil, Chi, & Nokes, 2013; Gentner, 2010; Matlen et al. 2011; Richland & McDonough, 2010). In the domain of mathematics, Rittle-Johnson and Star (2007) found that middle-school age students were more likely to improve in solving algebraic equations when students compared multiple worked out equations as compared to studying them in isolation. Simultaneous presentation prompted students to compare the two domains and highlighted the relevant structural attributes of the equations. Other mathematics studies have shown learning gains when two visual representations are displayed simultaneously versus sequentially, leading to gains in procedural knowledge, flexibility, and conceptual understanding (e.g., Richland & Begolli, 2016; Rittle-Johnson, Star & Durkin, 2009).

Use spatial organization to highlight key relations. Whenever two representations are compared there are many similarities and differences that could be attended to. Learning is enhanced when the spatial organization of the representations highlights the alignments. For example, Kurtz & Gentner (2013) found participants were faster and more accurate at detecting differences in skeletal structures when two skeletal images were presented in the same orientation relative to when they were presented in a symmetrical orientation. Further, Matlen, Gentner, and Franconeri, (2014, in prep) found that placing images in direct spatial alignment, such that a student need not move through one object to find alignments with another, optimized the speed and accuracy with which analogies were processed. In contrast, impeded alignments were slower and led to more errors.

Use linking gestures to move between spatial representations. Linking gestures are hand movements that move between two (or more) representations that are being compared, sometimes highlighting the specific alignments between these representations, and other times simply providing support for noticing the relevance of one representation to another (Alibali & Nathan, 2007, Alibali, Nathan & Fujimori, 2011; Richland, 2016). For instance, Richland and McDonough (2009) provided undergraduates with examples of permutation and combination problems that incorporated visual cueing, such as gesturing back and forth between problems and allowing the examples to remain in full view, versus comparisons that did not incorporate visual cueing. Students who studied the problems with visual cueing were more likely to succeed on difficult transfer problems. Linking gesture use is correlated with high mathematics learning in students (Richland, in 2016) and teacher gesture is well known to improve learning outcomes (see Goldin-Meadow, 2003).

Don't overload learners' cognitive resources. Reasoning with multiple representations requires adequate working memory (WM) and executive function (EF) resources, leading to reasoning failure and lower rates of learning when resources are overloaded or non-functioning (e.g., Richland, Morrison & Holyoak, 2006; Walz et al, 2000; Cho et al, 2007). When the contributions of working memory (WM) and inhibitory control (IC) were examined separately on children's successful learning and transfer from a classroom lesson based on an instructional analogy, we found that both explained distinct variance for predicting improvements in procedural knowledge, procedural flexibility, and conceptual knowledge after a 1-week delay (Begolli et al, in press). WM & IC were less predictive at immediate post-test, suggesting that these functions are not simply correlated with mathematics skill, but may be particularly important in the process of durable schema-formation (Begolli & Richland, 2015). To reduce cognitive load during comparison, visual representations from familiar examples or domains can be used when possible in order to help students understand unfamiliar examples or domains (Duit, 1991).

1.3 Implementing Supports in Practice

The above supports provide guidance for instructional decisions in classrooms. However, many mechanisms of learning operate simultaneously in everyday classrooms, and may augment or undermine each other, meaning that theories that explain learning in isolation may actually differ from those that explain learning in classrooms. Despite a large research base on what supports reasoning with diagrammatic representations, little research has explored the combinatorial use of supports, and this is particularly true in the context of authentic classrooms learning environments.

One reason why this may be important is that some supports may seemingly contradict one another. For instance, supports 1-3 described above are hypothesized to function because they reduce the cognitive processing load on reasoners to notice and draw inferences based on similarities between the representations; however, it is possible that adding simultaneous visual representations, spatial alignments, and gestures to process simultaneously could instead augment processing load. Thus, studying the integration of these principles is key to understanding how supports function in combination, resulting in more informed recommendations for how to best structure diagrammatic supports for classroom learning.

1.4 The Present Studies

The present studies examine the combinatorial use of diagrammatic both descriptively and experimentally. Study 1 consisted of a cross-cultural examination of the use of diagrammatic supports in middle school classrooms. The study builds on prior research by Richland, Zur, and Holyoak (2006) who coded the frequency of diagrammatic supports using a sample of eighth-grade mathematics lessons taught in the U.S., Hong Kong, and Japan from the Third International Mathematics and Science Video Study (TIMSS, Hiebert et al, 2005). These authors found that U.S. teachers regularly use comparison and contrasting cases in mathematics instruction, yet they do so without using diagrammatic supports as frequently as East Asian teachers. However, in this research, the frequency of co-occurrence of the supports was not examined. Thus, the present investigation re-analyzed lessons from the U.S., Hong Kong, and Japan to understand the combinatorial use of diagrammatic principles in these regions. In Study 2, we experimentally examine the impact of different combinations of diagrammatic supports on middle school students' mathematics learning.

2 Study 1

2.1 Method

Videodata were collected as part of the Third International Mathematics and Science Video Study (TIMSS, Hiebert et al, 2005) through a randomized probability sample of all eighth-grade mathematics lessons taught in the U.S. and seven higher achieving regions internationally. These data were analyzed and reported in a previous study (see Richland, Zur, & Holyoak, 2007). The current study involved a re-analysis of codes from a set of thirty lessons that were randomly selected from the U.S., Hong Kong, and Japan. Each lesson was taught by a different teacher, and all verbalized or visually presented comparisons were identified and then coded for their presence of principles for supporting student comparison efforts.

- sourceVisAvail = the source domain of the comparison was visually available
- gestureComp = use of linking gestures for comparison
- visualAlignment = problems were spatially aligned
- sourceUnfamiliar = whether the source of the comparison was unfamiliar

The data for the present study were re-analyzed to explore the extent to which diagrammatic supports were used in combination cross-culturally in western and eastern regions. Prior reports of this data indicated that teachers in Japan and Hong Kong were more likely to use diagrammatic supports than teachers in the U.S. (Richland, Zur, & Holyoak, 2007). Thus, in the present study we combined Japanese and Hong Kong lessons in the present analysis.

The data set consisted of 588 previously coded analogies in 30 lessons from each of the three regions listed above (*n* = 10 each). Codes for the diagrammatic supports for each analogy relative to the total number of analogies presented were averaged within each lesson.

2.2 Results

To determine the extent to which diagrammatic supports co-occurred with one another, Pearson correlations were conducted between the supports within each region. The results of this analysis for East Asian and U.S. lessons are presented in

Figures 1 and 2, respectively. As can be seen from Figure 1, in East Asian lessons, supports evidenced moderate to high positive correlations (ranging from .45 to .68), indicating that supports are used moderately often in combination. In addition, all correlations within East Asian lessons were statistically significant from a zero correlation (*p*s < .05). In contrast, correlations between supports in the U.S. were inconsistent in their direction (ranging from -.48 to .29; see Figure 2). Moreover, no correlations in U.S. lessons were statistically different from a zero correlation (all *p*s > .15). Though we view these findings as primarily descriptive, the results suggest that in addition to using diagrammatic supports less frequently in the U.S. than in East Asian countries (Richland, Zur, & Holyoak, 2007), U.S. teachers are also less likely to use supports in combination.

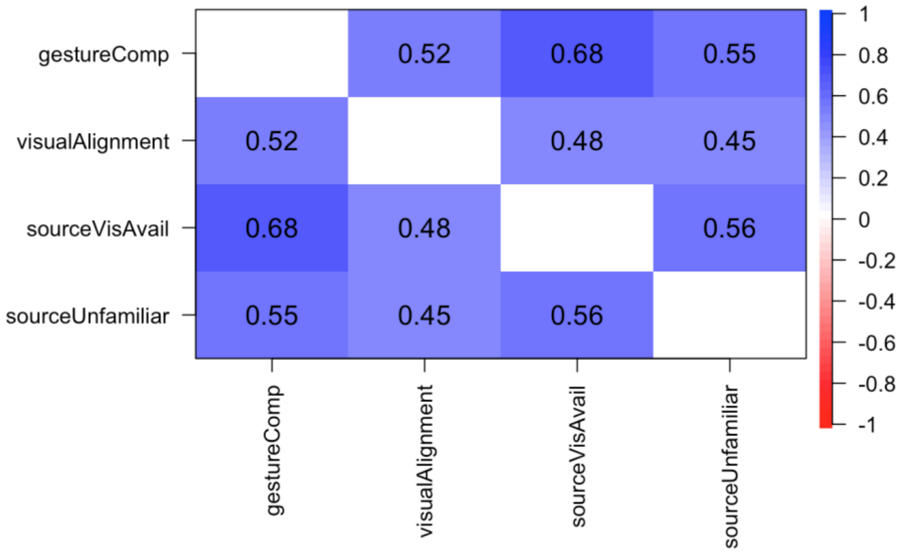

Fig. 1. Pearson correlations between diagrammatic supports in East Asian mathematics lessons (i.e., Hong Kong and Japan, *N* = 20). All correlations are statistically significant at alpha < .05.

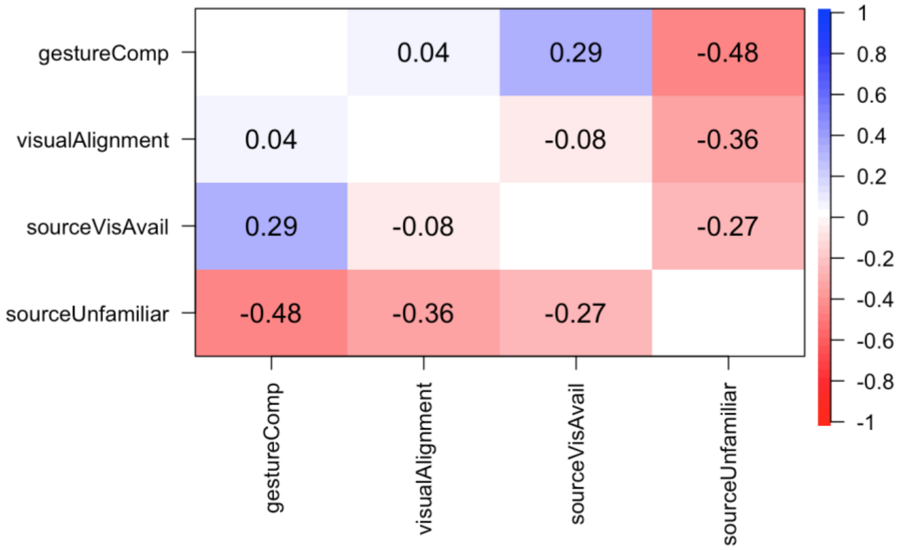

Fig. 2. Pearson correlations between diagrammatic supports in U.S. mathematics lessons (*N* = 10). No correlations are statistically significant (all *ps* > .15).

Prior research suggested that U.S. teachers regularly use comparison and contrasting cases in mathematics instruction, yet they do so without adequately supporting students in drawing these connections (Hiebert, 2003; Richland, Zur & Holyoak, 2007). Importantly, teachers' rates of supporting students in drawing connections between mathematical ideas or problems during problem solving was the single factor that differentiated all higher and lower achieving peer countries in the Trends in International Mathematics and Science 1999 Video Study (Hiebert, 2003). Though NCTM and disciplinary panels have long recommended helping students make mathematical connections (see National Mathematics Panel, 2008; Polya, 1954), this is still a serious challenge for teachers (Hiebert, Stigler, Jacobs et al, 2005). The present study suggests that U.S. teachers also use these supports less frequently in combination than in east Asian countries – this was particularly true when the source problem was unfamiliar. This finding contrasts to East Asian teachers, who were more likely to use supports when the source problem was unfamiliar.

3 Study 2

Study 1 revealed that teachers in two higher achieving regions, Hong Kong and Japan, more frequently provided multiple layers of support for engaging with these diagrams in systematic ways, such that if one support strategy were used another was often used (i.e. making representations visible for a longer period, spatial alignment between diagrams, using linking gestures, and drawing on familiarity in those representations).

This correlated use of support strategies mapped onto student achievement patterns, such that teachers in Hong Kong and Japan used these practices more than teachers in the U.S., which is a lower achieving region. This trend was suggestive of a relationship to achievement, but not conclusive. Thus in Study 2, we experimentally manipulated the amount of diagrammatic support provided for visually presented problems in a video-based fifth-grade lesson on proportional reasoning, to determine whether these multiple layers of support impact learning.

3.1 Method

The present experiment, we independently manipulated the familiarity of a source example problem with the amount of diagrammatic support provided to assess the influence of familiarity and visual supports together, separately, and in comparison to instruction without either of them. Specifically, the design was a 2 (familiarity condition: Unfamiliar or Familiar) x 2 (support condition: All Support or No Support) between subjects randomized trial.

Participants. Two hundred sixty-seven 5th grade students participated in this study. Forty-nine participants (18%) were excluded because they did not complete either the familiarity manipulation or one of the three math assessments. Of the remaining 218 participants, 61 were in the Familiar-All Support condition, 50 were in the Familiar-No Support condition, 56 were in the Unfamiliar-All Support condition, and 51 were in the Unfamiliar-No Support condition (See Table 1 for demographic representations of students). The study was run in nine total classrooms in five schools in the Chicago area. Four of these schools were public charter schools, while one was a Catholic school.

Table 1. Demographics of participating students in the analytic sample.

Demographic	Percent in Sample	Number in Sample Total *N* = 218
Females	58%	126
African-American	16%	34
Hispanic	57%	124
White	13%	29
Other Race(s)	14%	31

Procedure. Classrooms were visited after permission from the school's administration and teachers was granted. Each participating classroom was visited three times over a two-week period. Students were told that the goal of the study was to understand the best ways to teach kids math.

Visit 1. The first visit to the classroom lasted approximately 1 hour. Students completed two baseline measures, and then were randomly assigned to either a familiarity training condition or a no familiarity training condition. The baseline measures were the following::

1. *Patterns of Adaptive Learning Survey (PALS)*. This 24-item measured goal orientation, and consisted of three scales (Mastery Goal Orientation, Performance-Approach Goal Orientation, and Performance-Avoid Goal Orientation) (Midgley et al., 2000). One question about students' level of math anxiety ("Math makes me feel nervous") and two questions about students' long division abilities ("I've been taught long division before" and "I can do long division") were added to the end of this survey.
2. *Content Knowledge Assessment*. This assessment was a researcher-designed test consisting of 7 items that assessed students' baseline level of knowledge of rate and ratio concepts and long division abilities (see Begolli & Richland, 2016 for test properties).

On the content knowledge pre-test, students were instructed to attempt each problem and were asked to show all of their work, even if they weren't able to get a final answer. The pre-measures took approximately 45 minutes for students to complete.

After students completed the pre-measures, they were randomly assigned to one of the two familiarity conditions. Half of the students in each classroom were given long division instruction (Familiar condition) while the other half was given practice with long division problems (Unfamiliar condition). All students had been previously instructed in long division, these were simply opportunities to retrieve and strengthen the familiarity of these procedures. Students in both familiarity conditions were given a worksheet containing the same three long division problems. For students in the Familiarity condition, the first problem was worked out for them step-by-step, with instructions for each step. Students were asked to solve the second problem themselves, but were given those same step-by-step instructions with space next to each instruction for students to complete that step. Students were then asked to solve the third problem on their own. Students in the Control condition were given the 3 long division problems and were simply asked to solve each problem and show their work.

Visit 2. Researchers returned to each classroom 2-7 days later (mean= 4.3 days, median= 4.5 days), for a 90 minute session. Students were assigned to one of two video-based instruction conditions: *All Supports* or *No Supports*. Assignment was random but with the constraint that one problem on the baseline test was scored and used in order to minimize any differences between baseline performance across conditions.

All of the videos contained the identical audio stream of information, and the lessons were the same teacher and classroom, but one video included more visual access to the mathematical diagrams on the board and to linking gestures, while the other video did not have these pedagogical supports.

The lesson content involved a lesson about ratio, centered on a comparison between multiple ways that different students solved a word problem involving a set of ratios. . Both video lessons began with a teacher asking students to solve a ratio problem any way they would like (see Table 2). Two students in the video were then asked to share the method they used to solve the problem. The first student in the video indicated that he used the *Least Common Multiple* (LCM) method to solve the problem. The teacher then solved the problem on the board using the LCM method described by the student. A second student in the video told the class how he used division to solve the problem. The teacher solved the problem on the board using the division method described by the student. Finally, the teacher discussed the definitions of rate and ratio, summarized the lesson, and compared the two solution methods. Students in the study completed problems and answered teacher questions in a packet along with students who appeared in the video lesson.

The All Support and No Support videos differed in three ways (see Figure 3): 1) In the All Support video, the two methods of solving the problem (LCM and division) remained visible on the board throughout the lesson. In the No Support video, the LCM solution was not visible again once the discussion of the division method began. 2) In the All Support video, these two solution methods were presented on the board in a parallel structure so that comparisons between the two solution methods could be made more easily. In the No Support video, the two solution methods were not shown at the same time, so this way of organizing the board could not support students in making comparisons between the two solution methods. 3) In the All Support video, the teacher used linking gestures while comparing the two solution methods. Gestures were not used in the No Support video. After the video lesson, students in the study completed an immediate post-test to assess how much they learned from the video lesson. Finally, students completed a 10-item survey that tested their level of engagement with the lesson.

Table 2. Students were provided the following prompt (accompanied by the table, below) in each video lesson: *Ken and Yoko shot several free throws in their basketball game. The result of their shooting is shown in the table. Who is better at shooting free throws?*

Shooter	Shots Made	Shots Tried
Ken	12	20
Yoko	16	25

Fig. 3. Screen shots of identical points in the AS and NS videos during which the teacher compared two solution strategies. Parallel organization on the board, two visible solution strategies, and gesture were used in the All Support condition, but not in the No Support condition.

Visit 3. Researchers returned for a third visit 4-9 days later (mean = 6.7, median = 7). Students completed 3 tasks during this visit, which lasted 1 hour. As a group, students completed the d2 Test of Attention, a paper and pencil test that measures concentration and selective attention (Brickencamp & Zilmer, 1998). Next, students completed the delayed post-test to assess how much they remembered from the video lesson during our second visit. Finally, students completed a demographics questionnaire.

3.2 Results

Outcomes of interest in the present investigation concerned students' performance on the content knowledge assessment. Specifically, we were interested in increases in the correct strategy use at post-test vs. pre-test (students' use of either the LCM or division strategies), and decreases in the incorrect subtraction strategy from pre- to post-test. For this reason, we concentrated our analyses on the problems that required students to choose a strategy and solve the problem on their own (this analysis does not include student responses to multiple choice questions). Outcome scores represent an average across all non-multiple choice questions.

Correct strategy use. To explore the presence of correct strategy use across conditions, we conducted separate 2 (familiarity condition) x 2 (support condition) between subjects ANOVAs on students' gains from pre-test to post-test (Visit 1 vs. Visit 2) and from gains from pre-test to delayed post-test (Visit 1 vs. Visit 3), using the correct use of either the LCM or division strategy as the outcome variable. The analysis for the pre- to post-test ANOVA revealed significant main effects of support condition (F(1,214) = 11.52, *p* = .001) and familiarity condition (F(1,214) = 4.36, *p* = .04), and a marginally significant interaction between support and familiarity conditions (F(1,214) = 3.11, *p* = .08) on correct strategy use. Games-Howell post-hoc tests revealed that these effects were primarily driven by the performance of students in the Familiar-All Supports condition, who performed significantly better than students in the three other conditions (*ps* < .02).

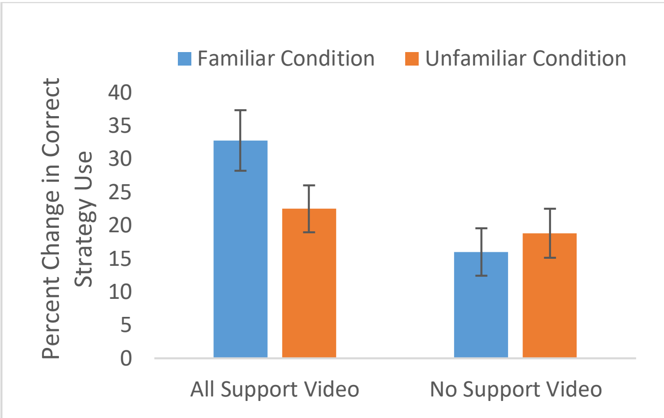

Fig. 4. Average percent change in correct strategy use from pre-test to delayed post-test by condition.

At delayed post, the ANOVA analysis revealed a significant main effect of support condition (F(1,214) = 6.73, *p* = .01), no main effect of familiarity condition (F(1,214) = .90, *p* = .35), and a marginally significant interaction between support and familiarity conditions (F(1,214) = 2.76, *p* = .10) on correct strategy use. We show delayed post performance in Figure 4, as this time-point represents learning that is sustained over time, and is arguably the strongest test of our hypotheses. To explore the interaction, we conducted Games-Howell post-hoc tests to make comparisons between conditions. This analysis revealed that the interaction was driven primarily by higher performance in the Familiar-All Support condition relative to the Familiar-No Support condition (*p* = .02) and the Unfamiliar–No Support condition (*p* = .09).

Decreases in use of the incorrect, subtraction strategy. A common incorrect strategy for comparing ratios involves the use of subtraction, where students subtract part of the whole (e.g., shots made from the total amount tried) (see Begolli & Richland, 2016). To explore the use of this strategy, we conducted a 2 (familiarity condition) x 2 (support condition) between subjects ANOVA on students' decreases in use of the subtraction strategy from pre to post-test and from pre- to delayed-post-test. The ANOVA on decreases from pre to post-test revealed a main effect of support (F(1,214) = 6.35, p= .01) but no effect of familiarity condition and no interaction between support and familiarity. Post-hoc Games-Howell tests revealed a marginally significant effect for students in the Familiarity-All Support condition to decrease their use of the misconception more often than students in the Unfamiliarity-No

Support condition (*p*= .06)

The ANOVA on decreases from pre to delayed post-test revealed only a significant main effect of the support condition (F(1,214) = 7.98, *p* = .005) (see Figure 5). Marginal differences in the decreased use of the subtraction strategy were found for pair-wise comparisons between the Familiar-All Support condition vs Unfamiliar-No Support condition (*p* = .10) and the Unfamiliar-All Support condition vs the Unfamiliar-No Support condition (p=.06).

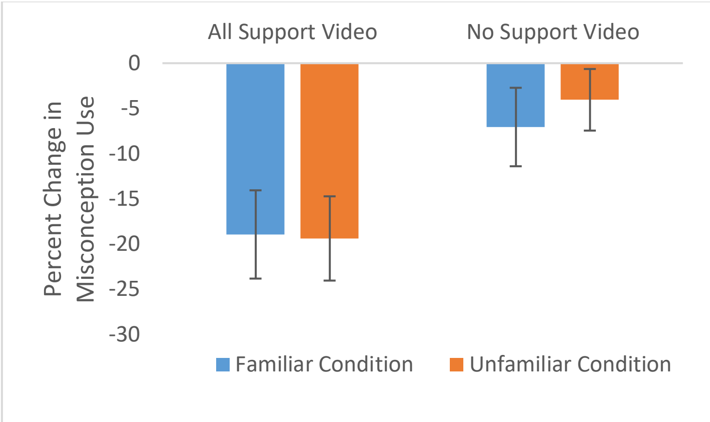

Fig. 5. Average percent change for the incorrect subtraction strategy from pre-test to delayed post-test by condition.

4 General Discussion

The studied presented here involved an observational and experimental exploration of the use of diagrammatic supports in combination in middle school mathematics classrooms. Study 1 involved a cross-cultural examination of teachers' use of diagrammatic supports and find that teachers in Hong Kong and Japan more frequently combine diagrammatic supports in mathematics lessons, whereas U.S. teachers combine supports less systematically in mathematics lessons. Study 2 manipulated the amount of diagrammatic support for visually presented problems in a fifth-grade lesson on proportional reasoning. Results suggest that learning about proportions is optimized when supports are used in combination. Though we did not explore whether combinations of supports aid learners more than single supports, these studies together suggest that providing visual, temporal, and familiarity cues as supports for learning from a diagrammatic representation is likely to improve mathematics learning.

The present research is an early attempt to explore how cognitive supports for diagrams interact in authentic environments, and is consistent with recent calls to explore instructional complexity in authentic contexts. For example, Koedinger, Booth, and Klahr (2013) estimate that there are on the order of trillions of instructional decisions that must be made during the course of classroom teaching, and suggest that more investigations are needed that explore how instructional principles interact with one another, as well as how they interact with the content to be learned. These authors suggest that educational technology environments can used to test a large number of permutations of instructional combinations to address this problem (e.g., Koedinger, Booth & Klahr, 2013). Similarly, we use video methodology to deeply situate this work in authentic student learning environments that are complex and that routinely combine multiple pedagogical principles, while maintaining internal validity of our experimental approach. In doing so, we attempt to understand how instructional combinations impact learning of mathematics in both an internally and ecologically valid way.

Though our approach directly examines the relationship between principle enactment and student learning, future work can more directly examine the issues that teachers confront during the course of enacting principles. For example, teachers must enact principles while they are attempting to hold both the content of the lesson and students' understanding of the content of the lesson in mind – presumably a high demand on cognitive resources – nevertheless, little work has addressed how enactment of principles influences teacher cognition. Explorations of this issue in future work might better inform theory on how principles can be optimally used in applied contexts. Moreover, our future reports will examine relationships between student characteristics known to correlate with mathematics learning, such as anxiety and executive function, to exposure to combinatorial support use and learning. This work will shed light on ways in which diagrammatic supports interact with other factors in applied contexts.

Author Note

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