Conceptions of experiments in teaching mathematics and teaching physics in Vietnam

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Abstract

This article presents a part of the Inter-Tetra project. This study firstly reviews the literature on conceptions of experiments in didactics of math and didactics of physics to design questionnaires to survey the conceptions of experiments in teaching physics and teaching math in Vietnam. Thereafter, the survey has been conducted among the math and physics pedagogical students in the Hanoi National University of Education (HNUE). The results of the survey have revealed the similarities and differences of the concepts of an experiment in teaching math and physics. In addition, this study also discusses the causes and consequence of the similar and dissimilar concepts of experiments.

1. Introduktion

Mathematics and Physics have an intimate relationship. Not only do they complement each other, but both share the similarities in the historical development (Krause & Witzke, 2015). The interaction of mathematics and physics lead educators to implement integrated teaching of math and science (Galili, 2018). This also facilitates the teaching activities to enhance students' competency. The aim of the Inter-TeTra (Interdisciplinary Teacher Training) project is to implement the fruitful discussions about the similarities and differences between mathematics and physics in teacher training (Krause et al., 2019). In this study, we built a questionnaire based on different notions of experiments to record data about the perception of math and physics teacher students in order to find out the similarities and differences in perception of pedagogical students in these two subjects on experiments and on the way of using experiments in teaching physics and math.

2. Experiments in research and teaching

2.1. Experiments in Physics and Mathematics

Galilei and the first physicists used empirical research methods to emphasize the aspect of arbitration between arguments, while mathematicians like Gauß often approached experiments as initial symbols or a way to illustrate abstract objects (Philipp et al., 2015).

Despite the similarities between these two subjects, the roles of experiments in physics and mathematics are still different due to the characteristics of each subject. The experiment in physics research plays a key role in the history of physics. The role of experiments in mathematics research has just been more apparent recently as the numerical methods and the simulation software appear.

Experiments in physics can be classified into 4 aspects: observation, measurement, the experiment itself and the epistemological role of the experiment –

the so-called experimental method (Schwarz, 2009). This is quite similar to the classification of (Etkina et al. 2002) in which they stated 3 types of experiments: observational experiments, testing theory model experiments, and application experiments.

In order to have a comparison on the role of experiments in mathematics and in physics, we base on two typical notions which are shown the table 1.

1. Observational exper- iment: The goal is to observe a new phenom- enon. Students later de- vise explanations for the observations.1. Ga intuiti 2. Dis terns a 3. Us place	ining insight and
 Testing experiment: The goal is to test whether the explana- tion devised for some observed phenomenon works. Students use ex- planations that they constructed for type 1 Experiments to make a prediction about the outcome of a new ex- periment. Application experi- ment: The goal is to ap- ply the explanation that has been tested in type 2 experiments to ex- plain new phenomena Testing principal (1997) Testing principal	on. covering new pat- and relationships. ing graphical dis- to suggest under- mathematical ples. esting and espe- falsifying conjec- ploring a possible to see if it is worth l proof. Suggesting ap- hes for formal eplacing lengthy derivations with uter-based deriva-

or to design technical	
devices.	

 Tab. 1: Conception on experiments in physics and mathematics

Borwein (2008) also notes that the above activities are quite similar to the role of laboratory experimentation in the physical and biological sciences, especially "computational experimentation" in physical science and engineering.

2.2. Role of experiments in physics teaching and mathematics teaching

In teaching physics, an experiment is an indispensable medium. This is not only reflected in textbooks but also in any literature on the didactics of physics. The experiment is always considered as a multi-purpose tool bringing high value for learning.

Besides, it is quite unusual to integrate experiments into teaching math. The number of articles applying experiments in teaching math is still limited. There are two ways to use experiments in teaching math. The first form is experimental mathematics, the numerical method and simulation software. The second form is using hands-on mathematics – like "Mathematik zum Anfassen" (Beutelspacher, 2018).

In the first approach from Jinyuan-Li (2003), it is recommended that the following steps can be used to apply experiments into math teaching:

• set out from some practical examples (including examples designed by students themselves)

- do experiments on the computer
- find out the rules among them
- make abstractions
- verify and prove your abstractions.

In the second approach, experiments in teaching maths are homologous with exploration experiments or hands-on experiment in physics, though Beutelspacher (2018) distinguishes mathematics from sciences such as physics or chemistry, where experiments are used to verify a theory or to falsify a wrong hypothesis. Also, mathematical experiments are not used to simply illustrate a definition or a theorem. The role of a mathematical experiment works "bottom-up": starting from experience, leading to insight. It is an impulse. If the experiment is good, this impulse is so strong that it enables the visitor to ask the right questions, to get the right conceptions and, finally to get by an "Aha-moment" the right insight.

In reality, teachers also take some experiments or retell some illustrative experiences as teaching students some math topics at elementary and junior high schools (Beutelspacher, 2018). Hamiltons' Icosian Game, the "Tower of Hanoi" and the Soma cube -Hein or tessellation (Ilucová, 2006), are examples of such types of experiments.

In some cases, math teachers also use experiments for testing the hypothesis or for theoretical reasoning. For example, to find the relationship between the height of a person and the length of footsteps when walking (Axel & Michael, 2015).

Thus, we see that the conception of experiments in mathematics and physics are quite similar (in a simulated experiment, illustrated experiment) but there are still many differences. In order to detect how these conceptions of experiments affect pedagogical students, the specific research questions are:

Is there a difference between conceptions about the experiment (definition, role, a necessity in teaching) of physics and math pedagogical students?

Is there a difference between conceptions about the experiment (definition, role, a necessity in teaching) of students before and after studying didactics of their subjects?

3. Method of study

3.1. Context

The HNUE is one of the biggest and oldest universities in Vietnam, training teachers and lecturers in educational research. Most of researchers in physics and math education in Vietnam before 2000 were considerately influenced by the Soviet education because almost all of them studied there. In recent years, the German education have also been conveyed with a source of staff studied in Germany. Despite the trend of globalization as well as the comfort in accessing information resources with the diversity of education perspective, the current researches of mathematics and physics in Vietnam have still influences and thereby have many similarities with the kind of teaching in Germany.

For that reason, the physics pedagogical students at the HNUE are fully trained and have similar views on the use of experiments in teaching physics. Besides, the use of experiment in teaching math is just quite a bit mentioned in the studies of math teaching (Trần, 2009). Now a few students have reached to hands-on math through open math days with active participation of several private STEM math or education centers (Pomath-center).

It is noticed that math has always been considered a "king subject" in Vietnam for a long time, the best students will choose math and the entrance exam to the mathematics department of HNUE is always the most demanding among the universities in Vietnam.

3.2. Sample selection

Because the curriculum in HNUE is in the parallel model in which students will study the subjects of fundamentals of teaching of math and science in the 2nd and 3rd year, we choose first-year students (before being taught the fundamentals of teaching) and the third-year student (after studying the fundamental of teaching) to attend the survey (tab. 2).

Gen- der	Male 75 (22%)	Female 261 (76,5)	Others 5 (1,5%)
Year	1st Year 134 (39,3%)	2nd year 90 (26,4%)	3rd year 107 (31,4 %)
Sub- jects	Math 103 (30,2%)	Physics (69,8%)	
Total	N = 341		

Tab. 2: Population of research

3.3. Questionnaire (see Appendix)

The questionnaire is designed based on two notions. The first one is the theory about conceptions about experiments in mathematics and in physics. The second one is the experiences of the authors including observations and answers of short interviews with teachers and students through the following questions:

• What is an experiment?

• What functions do experiments perform in teaching?

• Tell about an experiment which is most impressive to you as you a student.

Based on these, we discussed and clarified that there are six possible conceptions about experiments as follows.

- 1. Experiment as a part of a scientific method
- 2. Experiment as a way for proving a hypothesis

3. Experiment as a measurement for inducing a new law

- 4. Experiment as a simulation or demonstration tool
- 5. Experiment as a game
- 6. Experiment as a trial activity

Each of these perspectives was made explicit in the items in the questionnaire (see appendix). With the question number 3, the sentences corresponding to the types of conceptions are: item 1 for type 1; item 5 and item 9 for type 2; item 4 and item 7 for type 3; item 3, item 6 and item 8 for type 4; item 9 for type 5; item 2 for type 6.

With the question number 4, we coded by clarifying the students' conceptions given in the 6 types above. In the case that the students give a definition that coincides with two or more of the above conceptions, we take the most obvious conception or the conception first recorded by the student as a student's conception.

On the other hand, these conceptions have shown the reliability with open answers and tasks that require students to draw the most impressive experiment. From the students' answers, we classify students' perception of an experiment according to the 6 conceptions as above.

The questions 5 and 6 go into students' perspective about the importance and function of experiments in teaching physics and teaching mathematics.

4. Results and conclusion

4.1. Comparison of conceptions of experiment between math and physics students

4.1.1. Conceptions of experiment

About the conceptions of experiments, the result of the frequency of selection and data analysis shows that the conceptions of physics and math students are different on items 1, 4, 5, 8, and 10. Besides, we find out that students in the two faculties do not agree in the conception of considering an experiment as a part of the scientific method and as a simulation and demonstration tool. Physics students tend to treat an experiment as a part of the scientific method and a way of proving a hypothesis. Meanwhile, math students tend to consider experiments as the first step of the cognitive process or as just a trial and error method. Taking the data of question number 4, the frequency of the choice of the definition according to the types of experiments between the math and physics students is also distinctive with the results which are revealed in Fig. 1.



Fig. 1: Comparison of frequency distribution of selecting the type of experiment of math (green) and physics (blue) students.

4.1.2. Role of experiments in teaching

There is no difference between the first-year physics student and the first-year math student. Based on what is shown in question number 5, we draw a conclusion on the students' perspectives for the necessity of experiment in teaching.

Both math and physics students think that the use of experiments in physics is more crucial than in mathematics and there is no statistically significant difference in the results of this perspective of math and physics pedagogical students (tab. 3).

Group Statistics					
	Faculty	N	Mean	Std. De- viation	Std. Er- ror Mean
5_nes_math	Phy	238	3.63	.734	.048
	Math	103	3.84	.801	.079
5_nes_phys	Phy	238	4.76	.444	.029
	Math	103	4.76	.474	.047

Tab. 3: Mean score of question 5

4.1.3. Functional aspects of the experiment

The survey result of the 6th question is analyzed in the following table.

Roll of	Physics	students	Math S	tudents
experiment	Yes	No	Yes	No
Help	215	23	93	10
students to	(90,3%)	(9,7%)	(90,3%)	(9,7%)
understand				
the lesson				
Engage the	219 (92	19 (8%)	89	14
students	%)		(86,4%)	(13,6%)
more				
Illustrate	140	98	69	34
the	(58,8%)	(41,2%)	(67%)	(33%)
teacher's				
words				
Check the	<mark>178</mark>	<mark>60</mark>	<mark>67</mark>	<mark>36</mark>
prediction	<mark>(74,8%)</mark>	<mark>(25,2%)</mark>	<mark>(65%)</mark>	<mark>(35%)</mark>
Improve	103	135	31	72
hands-on	(43,3%)	(56,7%)	(30,1%)	(69,9%)
skills				
Develope	123	115	62	41
mathemati-	(51,7%)	(48,3%)	(60,2%)	(39,8%)
cal thinking				
Develope	238	20	76	27
physics	(91,6%)	(8,4%)	(73,8%)	(26,2%)
thinking				
Consolidate	182	56	72	31
the stu-	(76,5%)	(23,4%)	(69,9%)	(30,1%)
dents' be-				
lief on				
knowledge				

Tab. 4: Frequency of attribution about students choice

The result is significantly different between the conception of math and physics students. Physics teacher students believe more in the role "engage"; "check the prediction" and "improve hand on skills" more than math teacher students do (tab. 4). Regarding the conception of experiments function in teaching, there is a statistically significant difference in all factors apart from the conception of illustrating teacher's words. The most significant difference between math and physics students is the conception of the function for the development of thinking and hands-on skills. While math students believe in functions of illustrating knowledge and develop math thinking, physics students have more faith in the functions of enhancing the interest, checking the prediction, improving hands-on skills, developing physics thinking and consolidating belief on knowledge.

4.2. Compare the conceptions on the experiment of pedagogical students before and after learning fundamental of teaching

4.2.1. The conception of the experiment

Given in answer number 3, we see a clear difference between the 1st year and 3rd year students in items 1, 2, 3, 5, 6, 10. The remaining items are not quite different. The difference between the conception of an experiment of students in year 1 and year 3 is not enough (p = 0.068) in question number 4. The analysis showed that learning the subjects didactics just helps students realize the diversity of experiment in teaching but not change the students' understanding of the experiment.

Regarding the level of necessity which is got from the answer number 5, we realize that more and more pedagogical students comprehend the importance of physics after studying subjects' didactics. In contrast with what happened with physics, fewer students think that experiment is necessary for math after learning the subject. This proves that the subject fundamental theory of teaching in both mathematics and physics has not clearly clarified the important role of experiment in teaching math (tab. 5).

Group Statistics						
	year	N	Mean	Std. Devi-	Std. Er-	
				ation	ror	
					Mean	
5 nos moth	1	134	3.99	.649	.056	
5_nes_math	3	107	3.43	.741	.072	
5 nes phys	1	134	4.73	.462	.040	
5_nes_phys	3	107	4.81	.415	.040	

Tab. 5: Comparison mean between 1st year and 3th year students

Regarding the function of the experiment, students before and after learning the fundamental theory of teaching almost have no differences of perspective in functions of helping students understand the lesson, illustration the knowledge, improving hands-on skills, developing physics thinking. Meanwhile, the belief in the function of engaging students, checking the hypothesis and consolidating the belief of the experiment in teaching are significant different between two groups (p < 0.01). Just only the belief

of students in the function of mathematics thinking is decreased (p < 0.05).



Fig. 2: Comparison of frequency distribution of selection the type of experiment of 1st year and 3th year students

4.2.2. The Role of experiments in teaching

With the selection of students about the role of an experiment in teaching, we can find that students from 1st year and 3rd year are not a statistically significant difference. Their perception about the function of the experiment doesn't change in positive trend after studying subjects dicatics (tab. 6.). After studying subjects didactics, 3rd-year students just believe more in the rolls "engage" and "check the prediction" but less in the roll "develop mathematical thinking" (tab. 6).

Roll of experiment	1st year students		3rd year students	No No 10 (9,3 %) 55) 4 (3,7 %) 56) 34 (43,1 %) 50) 18 (16,8 %) 6) 66(61,7 %) 50) 50(46,7 %) 6) 14(13,1 %)
	Yes	No	Yes	No
Help students to understand the lesson	123(91,8%)	11 (8,2%)	97 (90,7 %)	10 (9,3 %)
Engage the students more	<mark>113 (84,3 %)</mark>	<mark>21 (15,7 %)</mark>	<mark>103(96,3%)</mark>	<mark>4 (3,7 %)</mark>
Illustrate the teacher's words	82 (61,2%)	52 (38,8%)	62 (57,9%)	34 (43,1 %)
Check the prediction	75(56%)	<mark>59 (44%)</mark>	<mark>89 (83,2%)</mark>	<mark>18 (16,8 %)</mark>
Improve hands-on skills	54 (40,3%)	80 (59,7%)	41(38,3%)	66(61,7%)
Develope mathematical thinking	87 (64,9 %)	47(35,1 %)	<mark>57 (53,3%)</mark>	<mark>50(46,7 %)</mark>
Develope physics thinking	111 (82,8%)	23 (17,2%)	93(86,9%)	14(13,1 %)
Consolidate the students' belief on knowledge	<mark>96 (71,6 %)</mark>	38 (28,4%)	83(77,6%)	<mark>24 (22,4 %)</mark>

Tab. 6: Selection of students about roll of experiment before and after studying subjects didactics

5. Conclusion

There is a difference in the perception of math and physics pedagogical students on an experiment. Physics students have the notion of considering an experiment almost as a part of the scientific method, and a way to check the hypothesis, while math students often emphasize the functions of illustration, experience and simulation.. There is no difference in the awareness of year 1 and year 3 students about an experiment, the fundamental theory of teaching subjects just help students understand the diversity of the experiment but not have the impact in changing students' conceptions on experiments. STEM integration requires physics and math pedagogical students to have the same conceptions about using experiments. A clear idea of experiments is an indispensable part of the scientific foundation for teacher students in mathematics and physics.

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Appendix: Perspectives of Experiments in Teaching

Below are some questions to gather demographic information. This data will not be used for any other purpose than for the research.

Personal Inf	formation				
Name (optional): Gender					
Age:					
Department: Year:					
University I	Entrance Exa	am Score			
Math	Physics	Chemistry	Biology		

1. Sketch an	2. Is there any Math
experiment which is	content considered as
the most typical one in	an experiment? If yes,
physics (explain if	please give an example
needed).	(explain if needed).

3. Which of the following actions is an experiment?

Action description	Yes	No	Other ideas
1. Lan supposes that drink-			
ing water frequently would			
benefit the skin. She tests			
this theory as following. In			
the first week, she drinks			
water as prescription and			
observes the skin, following			
by a week in which she			
drinks less water and ob-			
serves the skin again. Then			
she compares the results be-			
tween the two weeks.			
2. Lan goes to Hoa's house			
to take something for Hoa.			
So Hoa gives Lan her set of			
keys to open the door. Lan			

get in the house.Image: Content of the second
3. Lan tries to solve a problem calculating the velocity of a jumping parachutist. Lan applies different ways using different knowledge and manners such as computer modeling and formula transformation. 4. Lan rolls the dice to find out the rule that if we roll the dice so many times, it will be approximately the same probability for each face to perform. 5. From the Pythagoras' the- orem $a^2 + b^2 = c^2$, Lan de- duces that if we cut 3 squares having the sizes a, b, and c, respectively, from the same sheet and then weigh all of them, the mass of the c-size square is equal to the sum of the other two masses. Lan conducts the experiment to test and draws this conclusion. 6. Lan solves the quadratic equation $x^2 - 3x + 2 = 0$ and obtains roots as $x_1 = 1$ and $x_2 = 2$. After solving, she puts the two roots x_1, x_2 back into the equation to see if they match or not. 7. From the formula of the circumference of the circle, Lan measures by the following way. She draws a lot of circles of different diameter, uses a tape to measure the diameter and circumference of the circles and then calculates the ratio between the circumference and diameter. The average value of these ratios is π . 8. To find what the graph of equation $y = m / x^2 looks$ like, Lan in turn inputs dif- to the solution.
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transformation.Image: constraint of the second straint of the
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terent values of $m = 1, 2, 3,$
then draws the respective

graphs. From the graphs,		
she draws a preliminary		
conclusion about the behav-		
ior of the graph.		
9. Lan predicts that the		
larger the mass is, the more		
likely it would be sinked in		
water. Lan checks her		
prediction by putting objects		
of different masses in the		
water to see, if that is true or		
not.		
10. In the "Math is fun"		
book there is a guide on		
how to guess the age of a		
friend: First, multiply your		
own age by 2, add the prod-		
uct with 5, then multiply the		
sum with 5. Give me your		
result, then I will say your		
age. After several tries, Lan		
finds out the rule is that by		
subtracting the first 2 digits		
of the result by 2, we can		
have correct age.		

4. According to you, what is an experiment?

.....

5. The necessity of using experiments in teaching.

	Abso- lutely neces- sary	Very nec- es- sary	Of aver- age	Of little ne- ces- sity	Un- nec- es- sary
In					
Math					
In Sci-					
ence					

6. The role of experiment in teaching

- A. Help students to understand the lesson
- B. Engage the students more
- C. Ilustrate the teacher's words
- D. Check a prediction
- E. Improve hands-on skills
- F. Develope mathematical thinking
- G. Develope physics thinking
- H. Consolidate the students' belief on knowledge