U.S. Performance on the 2015 TIMSS Advanced Mathematics and Physics Assessments

A Closer Look

NCES 2020-051

U.S. DEPARTMENT OF EDUCATION

ies NATIONAL CENTER FOR EDUCATION STATISTICS
Institute of Education Sciences

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Executive Summary

With the increasing emphasis on science, technology, engineering, and mathematics—STEM—education and careers, it is important to understand how U.S. students are performing at the end of high school in the core subjects that are needed to prepare them to undertake more specialized STEM study in college and beyond. To this end, the United States participates in the Trends in International Mathematics and Science Study (TIMSS) Advanced, which measures the achievement of students in their final year of high school in advanced mathematics and physics in the United States and other countries. TIMSS Advanced was first administered in 1995 and most recently in 2015.

NCES first reported on TIMSS Advanced 2015 in Highlights from TIMSS and TIMSS Advanced 2015, which focused on the key findings of how U.S. students compare to students in the other participating countries (see textbox) in average scores and in the percentages reaching the TIMSS Advanced international benchmarks (*Advanced, High,* and *Intermediate*). As the *Highlights* showed, in terms of average scores, the United States performed relatively well in advanced mathematics (higher than five countries and lower than two) and less well in physics (higher than three countries and lower than four). To better understand these results, this "closer look" report further mines the TIMSS Advanced data to examine the performance of U.S. students in greater depth and explore how performance relates to students' opportunity to learn the content covered in the TIMSS Advanced assessment.

The report first examines the demographic and school characteristics of the U.S. TIMSS Advanced 2015 population, which in both subjects is the small subset of twelfth-graders who took eligible advanced mathematics and physics coursework (see textbox). It then describes the extent to which the topics assessed in TIMSS Advanced were covered in the curricula of the eligible U.S. courses and the percentage of U.S. TIMSS Advanced students overall and from different subgroups who were taking these courses.

Student performance is then described in terms of

- average scores in advanced mathematics and physics overall and by content domain (see textbox), and the percentage of students reaching international benchmarks—with new subgroup analyses that supplement the findings from the *Highlights;* and
- percent correct on the individual items (questions) across the content domains, topic areas, and topics that make up the advanced mathematics and physics assessments.

TIMSS Advanced 2015	
Participating countries	
France, Italy, Lebanon, Norway, Portugal, Slovenia, Sweden, Russian Federation, United States	
Student populations	
Nationally representative samples of students in their final year of secondary school who had taken or were currently taking eligible courses	
Eligible courses in the United States	
(AP stands for Advanced Placement. IB stands for International Baccalaureate.)	
In advanced mathematics:	**In physics:**
AP Calculus BC	AP Physics C – Electricity & Magnetism
AP Calculus AB	AP Physics C – Mechanics
IB Mathematics (higher level)	AP Physics B
IB Mathematics (standard level)	AP Physics 2
Other calculus courses	AP Physics 1
	IB Physics (higher level)
	IB Physics (standard level)
	Other physics courses
Content domains	
In advanced mathematics:	**In physics:**
Algebra	Mechanics and thermodynamics
Calculus	Electricity and magnetism
Geometry	Wave phenomena and atomic/nuclear physics

Item-level analyses are further used to explore U.S. performance in light of students' exposure to the TIMSS Advanced topics in their advanced mathematics and physics courses. Finally, 12 example items are included to illustrate some common approaches, misconceptions, and errors demonstrated by U.S. students in advanced mathematics and physics.

Who are TIMSS Advanced students in the United States and what are they taught?

- The U.S. TIMSS Advanced 2015 population is a select group of students. The students taking the advanced mathematics assessment represented 12.5 percent of U.S. twelfth-graders overall, and the students taking the physics assessment represented 5.3 percent. When viewed as a percentage of 18-year-old students, referred to as the "coverage index" for internationally comparative reporting in the *Highlights,* U.S. TIMSS Advanced students represented similarly small percentages: 11.4 percent for advanced mathematics and 4.8 percent for physics. The U.S. coverage index was in the middle of participating countries for advanced mathematics and among the countries with the lowest coverage indices for physics.
- Most U.S. TIMSS Advanced students took an AP course: 76 percent of students in the advanced mathematics assessment took AP calculus and 83 percent of students in the physics assessment took AP physics. Of those who had taken an AP course, the majority took the lowest level AP course (AP Calculus AB or the first-year AP Physics 1).
- U.S. students' opportunity to learn the advanced mathematics and physics content assessed in TIMSS Advanced varied by subject and the highest level course taken. Generally, coverage of advanced mathematics topics was more comprehensive than the coverage of physics topics.[1]
  - Across eligible U.S. advanced mathematics courses, all topics were covered in the AP and IB course curricula (or in a prerequisite course), except two topics that were not covered in the standard-level IB mathematics course. On average, across topics in all content domains, advanced mathematics teachers reported that TIMSS Advanced mathematics topics were taught to 98 percent of all U.S. TIMSS Advanced students by the time of the assessment (either in the current or a prior year).
  - In physics, there was considerable variation in the coverage of the TIMSS Advanced topics across the eligible AP and IB physics courses. In particular, the first-year AP Physics 1 course curriculum covered less than half of the TIMSS Advanced physics topics (mostly those related to *mechanics*), which means that U.S. students whose first high school physics course was AP Physics 1 would likely not have covered the majority of the TIMSS Advanced topics. The other AP and IB physics courses' curricula covered at least two-thirds of the topics in *mechanics and thermodynamics* and in *electricity and magnetism.* The topics in *wave phenomena and atomic/nuclear physics* were not included in the AP Physics C curriculum but may have been covered in prior courses. Physics teachers reported that TIMSS Advanced physics topics were taught to 73 percent of all U.S. TIMSS Advanced students on average, which reflects an average of 87 percent for *mechanics and thermodynamics,* 66 percent for *electricity and magnetism* and 62 percent for *wave phenomena and atomic/nuclear physics.*

[1] Although the TIMSS Advanced assessment was intended to include content covered across countries, some topics (particularly in physics) were not covered to the same extent in all countries. Higher coverage of advanced mathematics topics than physics topics was common across the countries participating in TIMSS Advanced 2015. See *TIMSS Advanced 2015 International Results for Advanced Mathematics and Physics* (exhibits M9.7 and P9.7).

How did U.S. TIMSS Advanced students perform in advanced mathematics and physics?

- Overall, the average scores for U.S. students were below the TIMSS Advanced scale centerpoints in both advanced mathematics and physics (by 15 and 63 score points, respectively).[2] U.S. students also performed, on average, below the centerpoints on the content domain subscales, except for the *calculus* subscale in advanced mathematics. In *calculus,* there was no measurable difference between the U.S. average score and the subscale centerpoint. In physics, average U.S. performance was especially low on the *electricity and magnetism* subscale (120 score points below the centerpoint).
- In both subjects, however, U.S. performance varied considerably depending on the specific courses taken, with AP students generally outperforming non-AP students. In particular, the average scores of students taking the highest level AP Calculus BC and AP Physics C-Electricity and Magnetism courses were higher than the overall U.S. averages in advanced mathematics and physics (by 71 and 100 score points, respectively) and on all content domain subscales (most notably on the *electricity and magnetism* subscale in physics). Additionally, students in these highest level courses were the only course subgroups to score higher than the international centerpoints in advanced mathematics and physics overall (by 56 and 37 points, respectively) and on one or more content subscale.
- In general, across subjects, U.S. males outperformed females and White students outperformed Black and Hispanic students (but not Asian students or students of Two or more races). The differences in average performance may be related to coursetaking patterns, as higher percentages of males and White students took the highest level AP courses—Calculus BC and Physics C—than their female and Black counterparts. Also, a higher percentage of Hispanic students took the lowest level AP Physics 1 course than White students.
- Additionally, U.S. students in suburban schools outperformed students in rural schools in advanced mathematics (though not in physics). This again may be related to coursetaking patterns, as a higher percentage of suburban students took AP Calculus BC than their rural (and town) counterparts. Average performance did not differ between students in public schools and those in private schools for either subject.
- The percentages of U.S. students reaching each of the three TIMSS Advanced 2015 international benchmarks—*Advanced, High,* and *Intermediate*—were higher than the respective international medians in advanced mathematics, but in physics the percentages were lower or not measurably different than the respective international medians.[3] In both subjects, the percentages of U.S. students overall reaching the *Advanced* benchmark reflected less than one-tenth of TIMSS Advanced students (7 percent for advanced mathematics and 5 percent for physics).
- Within the United States, however, some groups of students reached the *Advanced* level in greater proportions than U.S. students overall. Most notable were students in the highest level AP courses: 20 percent of AP Calculus BC students and 18 percent of AP Physics C-Electricity and Magnetism students reached this level. These groups of U.S. students also exceeded the international medians (2 percent for advanced mathematics and 5 percent for physics).

[2] The scale centerpoints represent the international means of the overall achievement distributions in the first TIMSS Advanced assessment year (1995). The score differences cited are based on scales from 0 to 1,000 with a fixed scale centerpoint of 500 and a standard deviation of 100.

[3] The international median is the middle percentage reaching each benchmark among the nine countries participating in TIMSS Advanced 2015.

How did U.S. students perform on TIMSS Advanced items across the advanced mathematics and physics content domains?

- In advanced mathematics, the U.S. average percent correct was higher on the items in *algebra* and *calculus* (46 and 47 percent, respectively) and lower on the items in *geometry* (38 percent) compared to advanced mathematics overall (44 percent). Performance on items in the *algebra* topic area of *functions* (53 percent correct) was notably higher than for advanced mathematics items overall.
- In physics, the U.S. average percent correct was higher on the items in *mechanics and thermodynamics* and in *wave phenomena and atomic/nuclear physics* (44 and 43 percent, respectively) and lower on the items in *electricity and magnetism* (36 percent) compared to physics items overall (42 percent). Performance on items in the *mechanics* topic areas of *forces and motion* and *laws of conservation* (48 and 49 percent correct, respectively) was notably higher than for physics items overall.

How did U.S. performance on TIMSS Advanced items relate to the level of topic coverage?

- U.S. performance on TIMSS Advanced mathematics and physics items ranged widely and was not strictly related to the level of topic coverage. Level of topic coverage (high, moderate, or low) was based on coverage in the curricula of eligible courses and whether the topic was reported as taught by the time of the assessment (exhibit A). Topic coverage was markedly greater for advanced mathematics than for physics.
    - Nearly three-quarters of the advanced mathematics topics (17 of 23) were in the high coverage category. Of the other six advanced mathematics topics, three were in the moderate coverage category and three were in the low coverage category.
    - In contrast, the majority of the 23 physics topics were in the low-coverage category (16), compared to 4 in the high and 3 in the moderate coverage categories.
    - Item performance in the United States ranged from 1 to 76 percent correct in advanced mathematics and from 5 to 85 percent correct in physics. For both subjects, the topics at each coverage level had examples of relatively higher item performance and relatively lower item performance. While there was wide-ranging item performance in many topics, in other topics item performance was more tightly clustered in the mid-performance range. Thus, strong patterns were generally not observed between topic coverage levels and item performance.
    - One apparent exception to the general lack of topic coverage-item performance patterns was for low-coverage physics topics not covered in AP Physics I (i.e., those related to *electricity and magnetism, wave phenomena and atomic/nuclear physics,* and *thermodynamics*). For these topics, the range of item performance tended to be lower than for topics at moderate- or high-coverage levels. Additionally, the low-coverage topics included most of the lowest-performing physics items. Because AP Physics I was the highest course taken by 42 percent of U.S. TIMSS Advanced students, this contributed to lower U.S. performance overall on items measuring these topics.

Exhibit A. Curriculum coverage of TIMSS Advanced mathematics and physics topics in the United States

Advanced Mathematics Topics		Physics Topics	
High Level of Topic Coverage *Covered in the intended curriculum for all AP and IB courses (or in a prior course), and taught to all or nearly all students (at least 99 percent).*			
Operations with exponential, logarithmic, polynomial, rational, and radical expressions Evaluating algebraic expressions Linear and quadratic equations and inequalities as well as systems of linear equations and inequalities Exponential, logarithmic, polynomial, rational, and radical equations Using equations and inequalities to solve contextual problems Equivalent representations of functions, including composite functions, as ordered pairs, tables, graphs, formulas, or words Properties of functions, including domain and range Limits of functions, including rational functions Differentiation of functions, products, quotients, and composite functions	Using derivatives to solve problems (optimization and rates of change) Using first and second derivatives to determine slope and local extrema, and points of inflection Using first and second derivatives to sketch and interpret graphs of functions Properties of geometric figures in two and three dimensions Using coordinate geometry to solve problems in two dimensions Trigonometric properties of triangles (sine, cosine, and tangent) Trigonometric functions and their graphs Solving problems involving trigonometric functions	Applying Newton's laws and laws of motion Forces, including frictional force, acting on a body	Kinetic and potential energy; conservation of mechanical energy Law of conservation of momentum; elastic and inelastic collisions
Moderate Level of Topic Coverage *At least partially covered in the intended curriculum for all AP and IB courses (or in a prior course), and taught to at least 85 percent of students.*			
The nth term of arithmetic and geometric sequences and the sums of finite and infinite series Integrating functions (polynomial, exponential, trigonometric, and rational)	Evaluating definite integrals, and applying integration to compute areas and volumes	Forces acting on a body moving in a circular path; the body's centripetal acceleration, speed, and circling time Law of gravitation in relation to movement of celestial objects	Mechanical waves; the relationship between speed, frequency, and wavelength
Low Level of Topic Coverage *Not covered in the intended curriculum for at least one AP or IB course, or taught to less than 85 percent of students.*			
Operations with complex numbers Conditions for continuity and differentiability of functions	Properties of vectors and their sums and differences	First law of thermodynamics Heat transfer and specific heat capacities Law of ideal gases; expansion of solids and liquids in relation to temperature change Electrostatic attraction or repulsion between isolated charged particles—Coulomb's law Charged particles in an electric field Electrical circuits; using Ohm's law and Joule's law Charged particles in a magnetic field Relationship between magnetism and electricity; magnetic fields around electric conductors; electromagnetic induction Faraday's and Lenz's laws of induction	Electromagnetic radiation; wavelength and frequency of various types of waves (radio, infrared, visible light, x-rays, gamma rays) Thermal radiation, temperature, and wavelength Reflection, refraction, interference, and diffraction Structure of the atom and its nucleus; atomic number and atomic mass; electromagnetic emission/absorption and the behavior of electrons Wave-particle duality and the photoelectric effect Nuclear reactions and their role in nature (stars) and society; radioactive isotopes Mass-energy equivalence in nuclear reactions and particle transformations

NOTES: All topics from the TIMSS Advanced 2015 Assessment Framework are included, but some have been abbreviated for this exhibit. "Intended curriculum" is based only on the AP and IB course guidelines, since other eligible courses differ across states and districts. "Covered in a prior course" reflects content expected to have been covered previously based on the AP and IB course guidelines and prerequisites. "Percent of students" is based on TIMSS Advanced students in all courses whose advanced mathematics or physics teachers reported that the topic had been taught by the time of the assessment (in the current year or a prior year).
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

What are some common approaches, misconceptions, and errors in advanced mathematics and physics demonstrated by U.S. TIMSS Advanced students?

- U.S. students demonstrated some common approaches, misconceptions, and errors on the TIMSS Advanced mathematics and physics items, including those assessing topics that had a high level of coverage across the U.S. TIMSS Advanced-eligible courses and where U.S. students performed relatively well on average.
    - In advanced mathematics, many U.S. students had difficulty solving problems in real-life contexts, demonstrating a deep understanding of some concepts and procedures needed to solve problems (e.g., derivatives, trigonometric functions, and simultaneous equations), and applying their knowledge of the properties of vectors.
    - In physics, many U.S. students had difficulty applying Newton's laws of motion in problem-solving situations, demonstrating an understanding of electric and magnetic fields, and correctly solving and showing their work on quantitative problems.
    - The prevalence of specific types of misconceptions and errors often varied based on the highest level advanced mathematics or physics course taken. For example, errors related to misunderstanding of concepts or problem-solving situations were less frequent among students taking the highest level AP calculus and physics courses than among U.S. students overall—especially in physics. In contrast, there were no differences in the frequency of some other types of errors among students taking the different courses. These included errors on computational items where students provided a correct answer but showed incomplete work (in physics) and on items where students used a correct method but made a computational error (in advanced mathematics).

The findings in this report relating student performance to curriculum coverage are not intended to be exhaustive or comprehensive of what can be learned from TIMSS Advanced. The diagnostic information provided by the example items is based on just a few of the hundreds of TIMSS Advanced items. Those analyzed in this report were selected to illustrate the sorts of findings from TIMSS Advanced item data that can help classroom teachers, researchers, and policymakers better understand the performance of U.S. students across advanced mathematics and physics topics. It is hoped that this report will spur additional research with TIMSS Advanced data to improve U.S. high school students' educational opportunities and college or career readiness in advanced mathematics and physics.

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Section 1: Introduction

With the increasing emphasis on science, technology, engineering, and mathematics—STEM—education and careers, it is important to understand how U.S. students are performing at the end of high school in the core subjects that are needed to prepare them to undertake more specialized STEM study in college and beyond. To this end, the United States participates in the Trends in International Mathematics and Science Study (TIMSS) Advanced, a comparative study that measures the advanced mathematics and physics achievement of students in their final year of high school who are taking or have taken advanced coursework in mathematics and physics. TIMSS Advanced is sponsored by the International Association for the Evaluation of Educational Achievement (IEA) and conducted, in the United States, by the National Center for Education Statistics (NCES) within the U.S. Department of Education's Institute of Education Sciences.

Like the other TIMSS assessments, which are given at the fourth and eighth grades, TIMSS Advanced is designed to align broadly with curricula in the participating education systems and, therefore, reflect students' school-based learning of advanced mathematics and physics. As such, TIMSS Advanced can inform policymakers, researchers, educators, and the public about the degree to which high school seniors in the United States excel in advanced mathematics and physics compared to their international peers.

Section 1 provides background on the focus and purpose of this report; an overview of the content and skills measured by TIMSS Advanced 2015 and how the student populations who took the TIMSS advanced assessments were defined; and briefly describes how the results are reported.

1.1 Background

TIMSS Advanced was administered most recently in 2015, in the United States and eight other education systems—France, Italy, Lebanon, Norway, Portugal, Slovenia, Sweden, and the Russian Federation. It was previously administered in 1995 and 2008, although the United States did not participate in 2008. Results from TIMSS Advanced 2015 in the United States were first reported in *Highlights from TIMSS and TIMSS Advanced 2015* (Provasnik et al. 2016). The *Highlights* report focused on how the performance of U.S. students compared to that of their counterparts in the other participating countries in terms of average scores and the percentages reaching the TIMSS Advanced international benchmarks (*Advanced, High,* and *Intermediate*) in advanced mathematics and physics. The *Highlights* showed that the United States overall performed relatively well in advanced mathematics and less well in physics (see sidebar on next page). To better understand these results, this "closer look" report further mines the TIMSS Advanced data to expand on the initial cross-national results presented in the *Highlights.*

The current report examines U.S. performance in more depth and analyzes students' opportunity to learn the TIMSS Advanced topics based on the curricula for the specific advanced mathematics and physics courses taken by students in the United States. It compares the performance and coursetaking patterns for key demographic subgroups (by gender, race/ethnicity, and school locale) and explores performance on individual mathematics and physics items (questions from the assessment) in light of students' exposure to

U.S. Standing in TIMSS Advanced 2015 Assessments

In 2015, the U.S. average score in advanced mathematics was higher than the average scores of students in five education systems and lower than the average scores of students in two education systems. The U.S. average score in physics was higher than the average scores of students in three education systems and lower than the average scores of students in four education systems. In neither subject were there measurable differences in the U.S. average scores from 1995 to 2015.

U.S. average score	Advanced mathematics	Physics
Higher than	France Italy Norway Sweden Slovenia	France Italy Lebanon
Not measurably different from	Portugal Russian Federation	Sweden
Lower than	Lebanon Russian Federation (intensive courses)*	Norway Portugal Russian Federation Slovenia

* Intensive courses are advanced mathematics courses that involve 6 or more hours per week. Results for the subset of Russian students in these courses are reported separately from the results for the Russian Federation overall, which also includes students taking courses that involve 4.5 hours per week.

These and other results from TIMSS Advanced are available on the NCES website at http://nces.ed.gov/timss/timss2015/). Users can also create customized tables and charts from TIMSS Advanced using the International Data Explorer at http://nces.ed.gov/surveys/international/ide/.

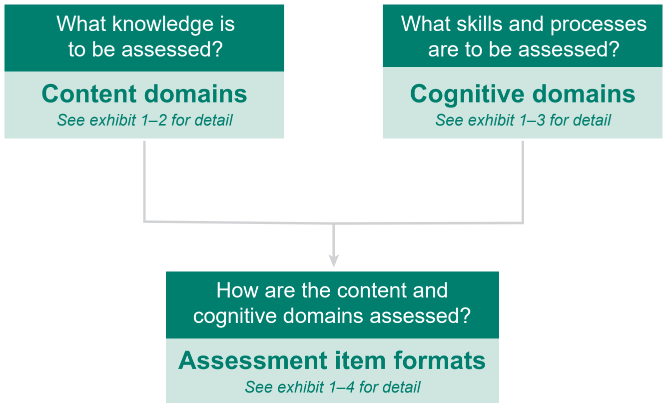
the relevant TIMSS Advanced topics in their advanced mathematics and physics courses. In addition, example items are used to illustrate some common approaches, misconceptions, and errors demonstrated by U.S. students in advanced mathematics and physics. These in-depth results are intended to expand the findings from TIMSS Advanced that can help classroom teachers, researchers, and policymakers better understand the performance of U.S. students.

1.2 Content and skills measured in TIMSS Advanced

The TIMSS Advanced assessments are developed through an international collaborative process involving input from U.S. and international experts in mathematics, science, and educational measurement. These experts develop assessment frameworks (Mullis and Martin 2015) that define the knowledge and skills to be assessed.

The TIMSS Advanced assessments measure students' knowledge and skills in advanced mathematics and physics and their ability to apply their knowledge in problem-solving situations. For each subject, the assessment framework describes the major content and cognitive domains to be covered in the assessment and how these will be assessed (exhibit 1-1). The content domains are further broken down into topic areas (exhibit 1-2) and, within topic areas, specific topics. Each assessment item (question) developed for TIMSS Advanced mathematics and physics measures knowledge of a specific topic from one of the content domains and requires students to demonstrate abilities, skills, and thinking processes from at least one of the cognitive domains. The framework also describes the item formats to be included in the assessments (i.e., multiple-choice and constructed-response items).

Exhibit 1-1. Overview of TIMSS Advanced 2015 assessment frameworks



SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

The frameworks specify targets for the percentage of total score points in the assessment to be devoted to each content domain, cognitive domain, and item format (exhibits 1-2, 1-3, and 1-4).[1] The assessments measure the full range of content and cognitive domains in the respective frameworks and reflect the different item formats. Participating students took TIMSS Advanced in only one subject—either advanced mathematics or physics.

Exhibit 1-2. TIMSS Advanced 2015 content domains and their target percentages for advanced mathematics and physics

Content domains	Topic areas	Target percentages[1]
Advanced mathematics		
Algebra	Expressions and operations Equations and inequalities Functions	35
Calculus	Limits Derivatives Integrals	35
Geometry	Noncoordinate and coordinate geometry Trigonometry	30
Physics		
Mechanics and thermodynamics	Forces and motion Laws of conservation Heat and temperature	40
Electricity and magnetism	Electricity and electrical circuits Magnetism and electromagnetic induction	25
Wave phenomena and atomic/nuclear physics	Wave phenomena Atomic and nuclear physics	35

[1] Target percentages reflect the intended percentage of total score points across all items in the assessment.
NOTE: The TIMSS Advanced frameworks also describe specific topics within each topic area. These are not shown in this exhibit; see section 3 tables.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

[1] See tables A-1 and A-2 in appendix A for detail about the distribution of items and score points by content domain, topic area, and topic. See the TIMSS Advanced 2015 international report for additional detail, including the distribution of item formats across the content and cognitive domains. See appendix C for details on how items are developed.

Exhibit 1-3. TIMSS Advanced 2015 cognitive domains and their target percentages for advanced mathematics and physics

Cognitive domains	General descriptions[1]	Target percentages[2]
Advanced mathematics		
Knowing	Knowledge of mathematics facts, concepts, and procedures	35
Applying	Ability to apply mathematical knowledge, or understanding of mathematical concepts, to create representations and solve problems in familiar situations (whether purely mathematical or real-life)	35
Reasoning	Ability to draw on knowledge and understanding from different areas of mathematics to formulate conjectures, make logical deductions based on specific assumptions and rules, and justify results in novel or complex problem-solving situations	30
Physics[3]		
Knowing	Knowledge of physics facts, relationships, processes, concepts, and equipment	30
Applying	Ability to use physics knowledge and methods to generate explanations and solve problems in contexts likely to be familiar in the teaching and learning of physics (including both quantitative and qualitative problems)	40
Reasoning	Ability to engage in scientific reasoning to develop hypotheses, design investigations, analyze data, draw conclusions, solve problems (in unfamiliar or more complex contexts), and extend understandings to new situations	30

[1] These general descriptions summarize the skills and processes to be assessed in advanced mathematics and physics. More detail is included in the TIMSS Advanced framework, which provides lists of specific behaviors to be elicited by items that are aligned with each cognitive domain.
[2] Target percentages reflect the intended percentage of total score points across all items in the advanced mathematics and physics assessments.
[3] The TIMSS Advanced physics framework also describes key science inquiry practices to be assessed that draw upon the range of processes specified in the cognitive domains and include skills from across mathematics and science coursework that students use in a systematic way to conduct scientific inquiry of physical phenomena. These practices include asking questions based on evidence, generating evidence, working with data, answering the research question, and making an argument from evidence. Targets are not provided for the coverage of the science inquiry practices and results are not reported separately by science practice.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Exhibit 1-4. TIMSS Advanced 2015 item formats and their target percentages for advanced mathematics and physics

Item formats	Descriptions	Target percentages[1]
Multiple choice	Provide students with a question and four response options (one correct and three incorrect), and students select the correct response. All multiple-choice items are worth one score point. At least 50 percent of the total assessment score points are devoted to multiple-choice items.	≥ 50
Constructed response	Require students to generate a written response and include both short-answer items (worth one score point) and more extended items (worth two score points). A scoring guide developed for each item specifies the maximum number of score points and the criteria for a fully or partially correct response.	≤ 50

[1] Target percentages reflect the intended percentage of total score points across all items in the assessment.
NOTE: Examples of both item formats can be found in sections 5.3 and 6.3.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

1.3 Defining the student populations who take TIMSS Advanced

The TIMSS Advanced assessments in advanced mathematics and physics are taken by nationally representative samples of students in their final year of secondary school who meet the study's eligibility criteria. To be eligible, students need to be taking or have previously taken either advanced mathematics or physics courses at the time of the study. Internationally, the TIMSS Advanced-eligible courses are defined as those that cover most of the topics outlined in the respective TIMSS Advanced assessment framework. In all participating countries, the TIMSS Advanced populations reflect a relatively small subset of students. In 2015, the coverage indices (or, percentages of the corresponding age cohort) ranged from 1.9 to 34.4 in advanced mathematics and from 3.9 to 21.5 in physics (Provasnik et al. 2016).[2] With coverage indices of 11.4 and 4.8 percent, respectively, the United States was in the middle of participating countries for advanced mathematics and among the countries with the lowest coverage indices for physics.

In the United States, the TIMSS Advanced samples consisted of twelfth-grade students who had taken eligible advanced mathematics and physics courses in the current or a prior year (described in appendix C). This advanced subset of U.S. students in 2015 is the focus of this report (described in more detail in section 2).

1.4 How results are reported

The results presented in this report come from the TIMSS Advanced 2015 student assessments and from the various context questionnaires that are also administered as part of TIMSS Advanced to collect information that may be related to students' achievement. The context questionnaires include a student questionnaire, a school questionnaire, a teacher questionnaire, and a country-level curriculum questionnaire.

The student assessments provide several measures to examine U.S. TIMSS Advanced students' performance, including scale scores, international benchmarks of achievement, and item-level statistics. The scale scores and international benchmarks are used to provide a general description of U.S. TIMSS Advanced students' performance in advanced mathematics (in section 5.1) and physics (in section 6.1). The item-level statistics provide student performance data on individual assessment items from across the topics assessed in TIMSS Advanced to explore U.S. performance in more detail (in sections 5.2 and 6.2). In addition, these item-level data are used to illustrate common approaches, misconceptions, and errors demonstrated by U.S. students on specific example items (in sections 5.3 and 6.3).

Data from the student and school questionnaires are used primarily to conduct subgroup analyses, allowing student performance to be examined not just overall but also by course type (the highest level advanced mathematics or physics course taken), sex, race/ethnicity, and school locale.[3]

Data collected for the U.S. national curriculum questionnaire are used to determine if the TIMSS Advanced topics are covered in the curriculum guides of the eligible U.S. courses. The curriculum questionnaire data are presented alongside data from the teacher questionnaire on the extent to which the TIMSS Advanced

[2] International reporting describes the TIMSS Advanced populations as a percentage of the corresponding age cohort or "coverage index." Reporting the coverage index allows for fair international comparisons because it accounts for the fact that education systems vary in both structure and the percentage of students who take advanced courses. Coverage indices in the United States are based on the percentage of all 18-year-olds.

[3] Results are not shown by school control, which is one of the variables used in section 2 to describe U.S. TIMSS Advanced students, because there were no measurable differences in scores between students from public schools and those from private schools for either advanced mathematics or physics. Any observed differences were not statistically significant due to large standard errors.

topics are taught to the students taking the assessments. Together, the data from the curriculum and teacher questionnaires provide a picture of TIMSS Advanced topic coverage in both the intended and implemented curricula as experienced by the students who took the TIMSS Advanced assessments (see section 3). These topic coverage data are later combined with the item performance data to examine U.S. performance in light of students' opportunity to learn the advanced mathematics and physics topics assessed in TIMSS Advanced (in sections 5.2 and 6.2).

Where to Find More Information

In addition to the information and data presented in the exhibits, figures, and tables in the main body of the report, supplemental tables in appendix A provide disaggregated data for select report tables, and appendix B provides the data used in figures in table format. Tables providing standard errors for the data cited in the report can be found online at the national website for TIMSS Advanced 2015 (https://nces.ed.gov/timss/timss2015/).

Additional technical notes and details about the data sources and methodologies used in TIMSS Advanced 2015 and this report can be found in appendix C, as well as online at the national (https://nces.ed.gov/timss/timss15technotes.asp) and international (https://timss.bc.edu/publications/timss/2015-a-methods.html) TIMSS Advanced websites.

While this report focuses primarily on the performance of U.S. TIMSS Advanced students, there are some sections that compare U.S. performance to international performance. Specifically, it examines how the average scores of U.S. students overall and in different subgroups compare to the international scale centerpoints, as well as how the percentages of U.S. students reaching each international benchmark, overall and in different subgroups, compare to the international median percentages (in sections 5.1 and 6.1). These analyses demonstrate if and how subgroup performance varied from overall U.S. performance in the international context. In addition, the international average percent correct is shown on the example items (in sections 5.3 and 6.3).

A detailed description of the methods used to analyze and report the student performance data is provided in section 4.

Section 2: TIMSS Advanced Students in the United States

This first part of this section describes the U.S. TIMSS Advanced population by comparing the percentages of students in various demographic and school categories in this population with these percentages for the grade 12 population overall.[4] The second part of this section overviews the different advanced mathematics and physics courses taken by the U.S. TIMSS Advanced population and describes the percentages of advanced students who took these courses.

2.1 Characteristics of students and their schools

The U.S. TIMSS Advanced 2015 population was a select group of students. Of the total grade 12 population in the United States, 12.5 percent of students were eligible for the advanced mathematics assessment and 5.3 percent were eligible for the physics assessment (table 2-1).

Sex

The U.S. TIMSS Advanced 2015 population for advanced mathematics was about evenly split between males and females (51 vs. 49 percent), but for physics it was disproportionately male (61 percent vs. 39 percent female).[5] In the grade 12 population overall, males represented 51 percent and females represented 49 percent of all students.

[4] All characteristics of the U.S. students reported here are estimates based on a nationally representative sample as described in section 1 which are used to make generalized statements about all U.S. twelfth-graders who had taken advanced mathematics and physics courses. This report focuses on the sample in relation to the grade 12 population overall, as it is more conventional and helpful in understanding the national context than the coverage indices described earlier, which are used for international comparisons.

[5] The apparent difference between males and females in advanced mathematics was not statistically significant.

Table 2-1. Number and percentage distribution of U.S. students in the total grade 12 population and in TIMSS Advanced mathematics and physics, by selected characteristics: 2015

Selected characteristics	Total U.S. grade 12 population[1]	U.S. TIMSS Advanced populations[2] Advanced mathematics	Physics
Total number of students	**3,798,601**	**473,405**	**199,944**
Percentage of total grade 12 population	100	12.5	5.3
Percentage of students by characteristics			
Sex			
Male	51	51	61 ▲
Female	49	49	39 ▼
Race/ethnicity[3]			
White	55	61 ▲	54
Black	14	4 ▼	8 ▼
Hispanic	22	15 ▼	15 ▼
Asian	5	13 ▲	18 ▲
Native Hawaiian/Pacific Islander	#	#	#
American Indian/Alaska Native	1	# ▼	# ▼
Two or more races	2	5 ▲	6 ▲
School control			
Public	92	83 ▼	84 ▼
Private	8	17 ▲	16 ▲
School locale[4]			
Urban	31	19 ▼	21 ▼
Suburban	41	38	47
Town	11	35 ▲	26 ▲
Rural	18	8 ▼	6 ▼

▲ Percentage is higher than the percentage of the total grade 12 population.
▼ Percentage is lower than the percentage of the total grade 12 population.
Rounds to zero.
[1] Data are based on the total grade 12 population, as reported by the 2014–15 Common Core of Data and the 2013–14 Private School Survey. In contrast, the coverage index in the TIMSS international report is calculated based on the population of 18-year-olds in 2015. In the United States, the TIMSS coverage index was 11.4 percent for advanced mathematics and 4.8 percent for physics.
[2] The numbers and percentages of U.S. students in the TIMSS Advanced mathematics and physics populations are based on the weighted counts of students in the samples. The unweighted counts are 2,954 students who took the advanced mathematics assessment and 2,932 students who took the physics assessment.
[3] Black includes African American and Hispanic includes Latino. Racial categories exclude Hispanic origin.
[4] Urban is defined as territories inside an urbanized area and inside a principal city. Suburban is defined as territories inside an urbanized area but outside a principal city. Town is defined as territories inside an urban cluster but outside an urbanized area. Rural is defined as territories that are not in an urbanized area or urban cluster. For additional information, see https://nces.ed.gov/programs/edge/docs/NCES_LOCALE_USERSMANUAL_2016012.pdf.
NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Common Core of Data (CCD) 2014–15, Private School Survey 2013–14; International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Race/ethnicity

The U.S. TIMSS Advanced 2015 population for advanced mathematics was 61 percent White, 15 percent Hispanic, 13 percent Asian, 5 percent Two or more races, 4 percent Black, and less than 1 percent Native Hawaiian/Pacific Islander or American Indian/Alaska Native. In comparison to the grade 12 population overall, the percentages of TIMSS Advanced students who are White, Asian, or Two or more races were disproportionately higher, while the percentages of Black, Hispanic, and American Indian/Alaska Native students were disproportionately lower.

The U.S. TIMSS Advanced 2015 population for physics was 54 percent White, 18 percent Asian, 15 percent Hispanic, 8 percent Black, 6 percent Two or more races, and less than 1 percent Native Hawaiian/Pacific Islander or American Indian/Alaska Native. In comparison to the grade 12 population overall, the percentages of TIMSS Advanced students who are Asian or of Two or more races were disproportionately higher, while the percentages of for Black, Hispanic, and American Indian/Alaska Native students were disproportionately lower.

White students were represented at higher rates in the advanced mathematics assessment (61 percent) than in the physics assessment (54 percent), whereas Black students were represented at higher rates in the physics assessment (8 percent) than in the advanced mathematics assessment (4 percent).

School control

A greater proportion of the U.S. TIMSS Advanced 2015 population was enrolled in public schools than in private schools (83 vs. 17 percent for advanced mathematics and 84 vs. 16 percent for physics). However, in the grade 12 population overall, 92 percent of students were enrolled in public schools and 8 percent in private schools. Thus, students in both subjects disproportionately came from private schools compared to the grade 12 population overall.

School locale

The U.S. TIMSS Advanced 2015 population came from schools in all types of locales.[6] In both subjects, the largest percentages of students were from suburban schools (38 percent in advanced mathematics and 47 percent in physics), and the smallest percentages were from rural schools (8 percent in advanced mathematics and 6 percent in physics).

The prevalence of suburban students in the U.S. TIMSS Advanced population reflects the predominance of these students in the grade 12 population overall (41 percent). There were differences, however, in the percentages of students from the other school locales, with disproportionately lower percentages of rural and urban students and disproportionately higher percentages of town students than in the grade 12 population overall. Specifically, 8 percent of U.S. students in advanced mathematics and 6 percent in physics were from rural schools compared to 18 percent in the grade 12 population overall. Likewise, 19 percent of students in advanced mathematics and 21 percent in physics were from urban schools compared to 31 percent in the grade 12 population overall. Conversely, 35 percent of students in advanced mathematics and 26 percent in physics were from town schools compared to 11 percent in the grade 12 population overall.

2.2 Advanced mathematics and physics courses taken by U.S. students

In the United States, TIMSS Advanced-eligible courses included Advanced Placement (AP) courses, International Baccalaureate (IB) courses, and other advanced courses with curricula expected to cover most of the topics outlined in the TIMSS Advanced assessment frameworks (exhibit 2-1). Tables 2-2 and 2-3 show the percentages of U.S. students in TIMSS Advanced 2015 who took each of the eligible advanced mathematics and physics courses, indicating the highest level course taken.[7] Of course, it is important to keep in mind that coursetaking patterns in the U.S. reflect both the specific advanced mathematics and physics courses that are

[6] See the notes in table 2-1 for definitions of locales.

[7] The order of AP courses in these tables and the other data tables and figures throughout the report reflect the hierarchy in the level of advanced mathematics courses (i.e., AP Calculus BC is a higher level course than AB) and physics courses (i.e., AP Physics C is the highest level, B and 2 are medium level, and 1 is the lowest level). Students' highest level course was determined based on transcript data obtained from sampled schools.

offered by schools (student access) across the states and districts in the TIMSS Advanced sample,[8] as well as the level of student enrollment in these different courses, both of which may differ for different groups of students (as described in section 3). Considering student access, the number of high schools offering AP courses has increased over the past decade, but recent data show that of those offering AP courses in 2015, less than half offered exams in the higher level AP Calculus BC and AP Physics C courses (College Board 2015a). Also, the IB program in the United States is relatively small compared with the AP program, and many students may not have access to IB mathematics and physics courses in their schools (College Board 2015a, IB Program 2019).

Advanced mathematics

For advanced mathematics, the U.S. TIMSS Advanced 2015 population included all students who had taken or were taking an AP calculus course (including both the AB and BC levels); an IB mathematics course (including both standard- and higher-level courses); or some other state-, district-, or school-specific calculus course (exhibit 2-1).

The majority of the advanced mathematics students had taken an AP course in calculus as their highest mathematics course—76 percent of students had taken an AP calculus course compared to 24 percent who had not (table 2-2). Of those who had taken an AP calculus course, more than twice as many had taken the lower level AP Calculus AB course as had taken the higher level AP Calculus BC course. Looking at the non-AP calculus coursetaking patterns, overall 1 percent of U.S. TIMSS Advanced students had taken IB Mathematics—nearly all of whom had taken the standard-level IB course—and 23 percent had taken other, non-AP, non-IB calculus courses as their highest mathematics course.

Physics

For physics, the TIMSS Advanced population included all students who had taken or were taking an AP physics course (including Physics B, 1, 2, and C); an IB physics course (including both standard- and higher-level courses); or some other state-, district- or school-specific second-year physics course (exhibit 2-1). The AP physics program was revised starting in the 2014-15 school year, with AP Physics B (a 1-year algebra-based physics course offered prior to 2014–15) being replaced with a 2-year algebra-based physics course sequence (AP Physics 1 and 2, starting in 2014–15).

Similar to advanced mathematics, the majority of the physics students had taken an AP course as their highest physics course—83 percent of students had taken an AP physics course compared to 17 percent who had not (table 2-3). Of those who had taken an AP physics course, the largest percentage had taken the lowest level AP Physics 1 course, which was new in 2015 (42 percent of U.S. TIMSS Advanced students overall).[9] This is more than 1.5 times the percentage of students who had taken one or both of the highest level AP Physics C courses (25 percent), the majority of whom had taken only AP Physics C-Mechanics.[10] A small percentage of students (4 percent) had taken AP Physics 2, the second course in the new two-course sequence that started in 2014–15; and 12 percent of students had taken AP Physics B, which was discontinued in 2014–15. Thus, U.S. seniors taking the TIMSS Advanced physics assessment in spring 2015 included those who had taken AP Physics B in their junior year or prior, those taking AP Physics 1 and/or 2 in their senior year, and those taking one or both of the AP Physics C courses in their senior year or prior. Looking at the non-AP coursetaking patterns, 6 percent of U.S. TIMSS Advanced students overall had taken IB Physics—most of whom had taken the standard-level IB course—and 12 percent had taken other non-AP, non-IB second-year physics courses as their highest physics course.

[8] Schools were eligible for inclusion in the 2015 TIMSS Advanced sample if they offered at least one eligible advanced mathematics or physics course. See appendix C for more information about the TIMSS Advanced samples.

[9] For students whose highest level course was AP Physics 1, this may be their first high school physics course, since 2014–15 was the first time that AP offered a first-year course.

[10] The AP Program Summary Report (College Board 2015a) shows that of the schools offering AP Physics C in 2015, less than one-quarter offered exams in AP Physics C-Electricity and Magnetism.

Exhibit 2-1. Overview of courses taken by U.S. TIMSS Advanced students: 2014–15

Advanced mathematics courses	Descriptions
AP Calculus AB	AP Calculus AB is a 1-year course designed to correspond to a first-semester college calculus course. It focuses on differential and integral calculus.
AP Calculus BC	AP Calculus BC is a 1-year course designed to correspond to a first- and second-semester college calculus course. It is an accelerated version of AP Calculus AB (above), covering additional topics.
IB Mathematics	IB Mathematics is a 2-year comprehensive mathematics course, offered at either a *standard level* (requiring 150 hours of instruction) or a *higher level* (requiring 240 hours of instruction).
Other calculus courses[1]	Other calculus courses include state-, district-, and school-specific calculus courses, including those identified as "honors" or "regents" courses.
Physics courses	**Descriptions**
AP Physics B *(Prior to 2014–15)*	AP Physics B was a 1-year physics course (offered prior to the 2014–15 school year) designed to correspond to a first- and second-semester algebra-based, introductory college physics course. It was intended to follow an introductory high school physics course, and was considered a second-year course. AP Physics B was replaced with the 2-year (and consequently more in-depth) sequential course, AP Physics 1 and 2 starting in the 2014–15 school year (see below).
AP Physics 1 *(2014–15)*	AP Physics 1 is the first of the two courses that were designed to replace Physics B beginning in the 2014–15 school year. It is a 1-year course designed to correspond to the first semester of an algebra-based, introductory college physics course focused primarily on mechanics. AP Physics 1 is considered a first-year physics course and no prior high school physics is required.
AP Physics 2 *(2014–15)*	AP Physics 2 is the second of the two courses that were designed to replace Physics B beginning in the 2014–15 school year. It is a 1-year course designed to correspond to the second semester of an algebra-based, introductory college physics course that covers more advanced topics than AP Physics 1. AP Physics 2 is considered a second-year physics course intended to follow an introductory physics course (such as AP Physics 1 or a comparable course).
AP Physics C	AP Physics C courses include two, typically sequential courses, each designed to correspond to a semester of a first-year calculus-based college physics course: • *AP Physics C-Mechanics (AP Physics C-M);* and • *AP Physics C-Electricity and Magnetism (AP Physics C-E/M).* Both courses are considered second-year physics courses. Some schools may offer combined courses, and, in some cases, schools may offer AP Physics C as a first-year course for eligible students (typically, a full-year mechanics course).
IB Physics	IB Physics is a 2-year algebra-based physics course, offered at either a *standard level* (requiring 150 hours of instruction) or a *higher level* (requiring 240 hours of instruction).
Other physics courses[1]	Other physics courses include state-, district- and school-specific second-year physics courses, including those identified as "honors" or "regents" courses.

[1] Other calculus and physics courses were identified using the definitions from the School Codes for the Exchange of Data (SCED) course classification system. Descriptions of courses and their content in school catalogues were reviewed to determine course eligibility based on the courses covering most of the TIMSS Advanced topics.

NOTE: AP stands for Advanced Placement. IB stands for International Baccalaureate. AP and IB courses have specific curricula that are intended to be taught to all students regardless of the state, district, or school in which they take them. Additionally, AP and higher level IB courses enable students passing the associated exam to earn college credit and/or qualify for more advanced college courses. The TIMSS Advanced assessments were administered in the spring of the 2014–15 school year; thus, the courses taken by students represent what was offered in schools in the 2014–15 school year or the prior year.

SOURCE: International Association for the Evaluation of Educational Achievement (IEA)'s Trends in International Mathematics and Science Study (TIMSS) Advanced 2015; The College Board, http://apcentral.collegeboard.com/apc/public/courses/descriptions/index.html; International Baccalaureate, http://www.ibo.org/programmes/diploma-programme/curriculum/.

Table 2-2. Number and percentage distribution of U.S. TIMSS Advanced mathematics students, by course type: 2015

Course type[1]	U.S. TIMSS Advanced mathematics students[2]	
	Number	Percent
All courses	**473,405**	**100**
Total AP calculus courses	**357,446**	**76**
AP Calculus BC	89,977	19
AP Calculus AB	267,470	56
Total non-AP mathematics courses	**115,959**	**24**
IB Mathematics	6,198	1
Higher level	951	#
Standard level	5,247	1
Other calculus courses[3]	109,761	23

Rounds to zero.
[1] Course type reflects the highest level mathematics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other calculus courses" category only if they have not taken an AP or IB course.
[2] The numbers and percentages of U.S. TIMSS Advanced mathematics students are based on the weighted counts of students in the sample. The unweighted count is 2,954 students who took the TIMSS Advanced mathematics assessment.
[3] Includes other calculus courses (including "honors" or "regents" courses).
NOTE: Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table 2-3. Number and percentage distribution of U.S. TIMSS Advanced physics students, by course type: 2015

Course type[1]	U.S. TIMSS Advanced physics students[2]	
	Number	Percent
All courses	**199,944**	**100**
Total AP physics courses[3]	**165,243**	**83**
AP Physics C	49,045	25
AP Physics C-E/M[4]	19,091	10
AP Physics C-M	29,954	15
AP Physics B	23,636	12
AP Physics 2	8,802	4
AP Physics 1	83,760	42
Total non-AP physics courses	**34,701**	**17**
IB Physics	11,486	6
Higher level	744	#
Standard level	10,742	5
Other physics courses[5]	23,215	12

Rounds to zero.
[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other physics courses" category only if they have not taken an AP or IB course.
[2] The numbers and percentages of U.S. TIMSS Advanced physics students are based on the weighted counts of students in the sample. The unweighted count is 2,932 students who took the TIMSS Advanced physics assessment.
[3] AP Physics B was a 1-year physics course that was discontinued after the 2013–-14 school year and was replaced with the 2-year course sequence AP Physics 1 and 2 beginning in 2014–15. Data in the AP Physics B category are generally for students who took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering a 1-year course identified as AP Physics B in 2014–15. AP Physics C includes courses in mechanics (C-M) and electricity and magnetism (C-E/M).
[4] A large majority of students whose highest physics course was AP Physics C-E/M had also taken AP Physics C-M (89 percent), either sequentially or in a combined course.
[5] Includes other types of second-year physics courses (including "honors" and "regents" courses).
NOTE: Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Section 3:
What U.S. Students Are Taught in Advanced Mathematics and Physics

The TIMSS Advanced 2015 Assessment Framework specifies the advanced mathematics and physics content that participating countries agreed should be the focus of an international assessment. Although the TIMSS Advanced assessment was intended to include content covered across countries, some topics (particularly in physics) were not covered to the same extent in all countries.[11]

To provide context for examining U.S. student performance in depth, this section describes (1) the *content included* in the TIMSS Advanced-eligible advanced mathematics and physics courses in the United States, (2) the extent of coverage of the TIMSS Advanced topics in the *intended* and *implemented* curricula of these courses, and (3) the *coursetaking patterns* of U.S. TIMSS Advanced students overall and by sex, race/ethnicity, and school locale. There are separate subsections for advanced mathematics and physics.

Terminology and Data Sources for Section 3

The descriptions of the *content included* in the TIMSS Advanced-eligible advanced mathematics and physics courses in the United States were generated from (a) a review of school course catalogs, (b) definitions in the School Codes for the Exchange of Data system, and (c) the AP and IB course descriptions available from the College Board and IB Program, respectively. The TIMSS Advanced 2015 assessments were administered in the spring of the 2014–15 school year; thus, the courses represent what was offered in schools in the 2014–15 school year or the prior year.

The *intended curriculum* is based on (a) the curriculum frameworks for AP calculus and AP physics courses available from the College Board[12] and (b) the core curricula specified for the IB mathematics and IB physics standard-level and higher-level courses available from the IB Program. The intended curriculum is not indicated for the non-AP, non-IB courses eligible for TIMSS Advanced, because the curricula for these courses vary across states, districts, and schools. These results use research conducted during the completion of the U.S. national curriculum questionnaire, which asked countries to indicate whether, according to their curricula, students in TIMSS Advanced-eligible advanced mathematics or physics courses have been taught the topic by the end of the 2014–15 school year (in the current course or before). (See the technical notes in appendix C for more detail about how the United States completed the topic coverage questions in the curriculum questionnaire.)

The *implemented curriculum* is indicated by the percentage of U.S. TIMSS Advanced students whose advanced mathematics or physics teachers reported that the TIMSS Advanced topics were “mostly taught this year” or “mostly taught before this year” (i.e., by the time of the assessment). This section uses the national results from the teacher questionnaire. Percentages reflect all students in the TIMSS Advanced population, including those taking AP, IB, and other non-AP, non-IB advanced mathematics or physics courses. (See the technical notes in appendix C for more detail about the teacher questionnaire data.)

U.S. TIMSS Advanced students’ *coursetaking patterns* are based on transcript data obtained from schools. Students were assigned to the highest level advanced mathematics or physics course they had taken by the time of the assessment.[13] Results are reported as percentages of U.S. TIMSS Advanced students who had taken the various TIMSS Advanced-eligible courses (overall and by subgroups).

[11] See *TIMSS Advanced 2015 International Results for Advanced Mathematics and Physics* (exhibits M9.7 and P9.7).

[12] AP course guides describe the topics that must be covered for students to perform well on the corresponding AP exams. However, additional topics may be added to meet individual state curriculum standards for their high school students.

[13] Students were also asked to identify the high school mathematics and physics courses they had taken in the student questionnaire, but these data were not used for the analyses in this report.

3.1 Advanced mathematics

Content included in the intended curricula of the TIMSS Advanced-eligible advanced mathematics courses in the United States

The TIMSS Advanced-eligible courses in advanced mathematics are shown in exhibit 2-1. The differences in their focus and specific content covered are described below. (See exhibit A-1 in appendix A for more detail about the content covered in each course.)

AP Calculus AB

AP Calculus AB is a 1-year course designed to correspond to a first-semester college calculus course. AP Calculus AB covers a range of mathematics topics typically covered in first-year, introductory calculus courses, including limits, conditions of continuity and differentiability, and differential and integral calculus. Students are expected to have taken or demonstrated mastery of the equivalent of 4 years of high school mathematics designed for college-bound students, including prior courses covering algebra, geometry, trigonometry, analytic geometry, and elementary functions.[14] In particular, students must be familiar with the properties, algebra, and graphs of functions. Students must also understand the language of functions and know the values of the trigonometric functions at common angles and their multiples.

AP Calculus BC

AP Calculus BC is a 1-year course designed to correspond to a first- and second-semester college calculus course. It covers all the topics of AP Calculus AB (see above), but at a faster pace and including some additional topics: the analysis of parametric, polar, and vector functions; and polynomial approximations and series, including the use of the Taylor and Maclaurin series to approximate other functions; and the use of techniques to determine convergence or divergence. AP Calculus BC also covers the evaluation and application of integrals in more depth than AP Calculus AB. The requirement for enrollment in AP Calculus BC is the same as for AP Calculus AB: the equivalent of 4 years of high school mathematics designed for college-bound students.

IB Mathematics

IB Mathematics is a 2-year comprehensive mathematics course, offered at either a standard level or a higher level. The core curriculum for both levels covers topics in algebra, functions and equations, circular functions and trigonometry, vectors, statistics and probability, and calculus. The higher level IB mathematics course curriculum includes additional topics in four areas from which students can choose one to study. Both IB mathematics courses aim to introduce mathematical concepts in a comprehensible and coherent way, with the higher level course requiring more mathematical rigor. Both IB mathematics courses require prior learning in the areas of numbers, sets and numbers, algebra, geometry, coordinate geometry, trigonometry, and statistics and probability.

Other calculus courses

Other calculus courses (non-AP, non-IB) include state-, district-, and school-specific calculus courses, including those identified as “honors” or “regents” courses. The specific content varies across states, districts, and schools but generally covers topics in differential and integral calculus and analytic geometry.

[14] Elementary functions include linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise-defined functions.

Extent of coverage of TIMSS Advanced mathematics topics in U.S. advanced mathematics courses

Table 3-1 summarizes the extent of coverage of the TIMSS Advanced topics across the U.S. TIMSS Advanced-eligible advanced mathematics courses. This table shows the

1. number of specific mathematics topics within each topic area and content domain that are assessed in TIMSS Advanced (as described in the framework);
2. extent to which these TIMSS Advanced topics are covered in the *intended* curricula of the eligible AP calculus and IB mathematics courses, indicating both the extent of coverage (aggregated at the topic area level) and the proportion of topics within the content domain and overall that are at least partially covered in the intended curriculum for each course (based on topic-level data provided in tables A-3a, A-3b, and A-3c); and
3. overall percentage of U.S. TIMSS Advanced students who had been taught the TIMSS Advanced topics by the time of the assessment (either in the current or a prior year) based on their teachers' reports (i.e., *implemented* curriculum). These data are presented as the average and range across topics in the topic area and content domain.

It should be noted that the data on the *intended* curricula (described in 2 above) are not available for other (non-AP, non-IB) advanced mathematics courses because these vary across states, districts, and schools. In contrast, the teacher questionnaire data on the *implemented* curriculum (described in 3 above) reflect teachers of all TIMSS Advanced-eligible advanced mathematics courses. For this and other reasons discussed below, patterns reflected in the two sources of data may differ slightly.[15]

In general, the TIMSS Advanced mathematics topics were well covered in the intended curricula of the eligible courses, with some variations by course type and by content domain. AP Calculus AB, AP Calculus BC, and the higher level IB mathematics course generally covered all the TIMSS Advanced mathematics topics (either in the current course or in a prerequisite course). In contrast, the standard-level IB mathematics course covered about 91 percent (21 of 23) of TIMSS Advanced mathematics topics. On average, across the topics in all content domains, 98 percent of U.S. TIMSS Advanced students were reported by their advanced mathematics teachers to have been taught the TIMSS Advanced mathematics topics by the time of the assessment (either in the current or a prior year).[16]

Findings about the coverage of TIMSS Advanced mathematics topics in each content domain are described below.

Algebra

- All the topics in algebra—with one exception—were covered in the intended curricula (or were a prerequisite) for all of the eligible AP and IB mathematics courses (tables 3-1 and A-3a). In the AP courses, these topics were considered foundational knowledge. In the IB mathematics courses, these topics were an explicit part of the curriculum. The exception was the *expressions and operations* topic of *operations with complex numbers,* which was not explicitly covered in the standard-level IB Mathematics curriculum.

[15] Data cited in this section for specific advanced mathematics topics are not shown in table 3-1, which summarizes the data at the topic area level. Coverage of the specific TIMSS Advanced mathematics topics in each topic area and the percentages of U.S. TIMSS Advanced students taught these topics in the different TIMSS Advanced-eligible courses are provided in the supplemental tables in appendix A (A-3a, A-3b, and A-3c).

[16] The average percentage of students taught the TIMSS Advanced mathematics topics was at least 90 percent in all countries. See TIMSS Advanced 2015 International Results for Advanced Mathematics and Physics (exhibit M9.8).

Table 3-1. Extent of TIMSS Advanced mathematics topic coverage in the intended curriculum by course type and the overall percentage of U.S. TIMSS Advanced students who were taught these topics, by content domain and topic area: 2015

Content domains and topic areas	Number of mathematics topics assessed in TIMSS Advanced[1]	Extent of TIMSS Advanced topic coverage in the *intended* curriculum by course type[2]				Overall percentage of U.S. students taught TIMSS Advanced mathematics topics[3]	
		AP Calculus		IB Mathematics			
		BC	AB	Higher level	Standard level	Average across topics	Range across topics
Total advanced mathematics (all content domains)	**23**	**23/23**	**23/23**	**23/23**	**21/23**	**98**	**81–100**
Algebra	**9**	**9/9**	**9/9**	**9/9**	**8/9**	**99**	**93–100**
Expressions and operations	4	⊙	⊙	●	◖	98	93–100
Equations and inequalities	3	⊙	⊙	●	●	100	100–100
Functions	2	⊙	⊙	●	●	100	100–100
Calculus	**8**	**8/8**	**8/8**	**8/8**	**7/8**	**98**	**95–100**
Limits	2	●	●	●	◖	100	100–100
Derivatives	4	●	●	●	●	100	99–100
Integrals	2	●	●	●	●	95	95–95
Geometry	**6**	**6/6**	**6/6**	**6/6**	**6/6**	**96**	**81–100**
Noncoordinate and coordinate geometry	3	⊙	⊙	●	●	93	81–99
Trigonometry	3	⊙	⊙	●	●	100	99–100

● Topic area is covered in the course curriculum (all topics are fully or partially covered in the TIMSS Advanced-eligible mathematics course or are expected to have been covered in a prerequisite mathematics course).

◖ Topic area is partially covered (at least one topic is not covered in the course curriculum nor specified as prerequisite knowledge).

⊙ Topic area reflects foundational knowledge expected to have been covered in a prerequisite mathematics course.

[1] The number of topics assessed in TIMSS Advanced reflects the specific advanced mathematics topics included in each topic area and content domain as shown in table A-1.

[2] “Extent of TIMSS Advanced topic coverage in the intended curriculum” is based on the overlap of TIMSS Advanced mathematics topics with Advanced Placement (AP) Calculus course descriptions available from the College Board and the core curriculum specified for the International Baccalaureate (IB) Mathematics standard-level and higher-level courses. The intended curriculum is not indicated for other TIMSS Advanced-eligible non-AP, non-IB advanced mathematics courses, since the curricula for these courses vary across states, districts, and schools. The ratios shown under each course type reflect the proportion of TIMSS Advanced topics in a given content domain (and overall) that are at least partially covered in the intended curriculum for that course (or are considered foundational knowledge expected to have been covered in a prerequisite mathematics course). The Common Core State Standards in Mathematics were used to determine foundational knowledge covered in prerequisite mathematics courses.

[3] “Overall percentage of U.S. students taught TIMSS Advanced mathematics topics” reflects the implemented curriculum. It is based, for each topic, on the number of students whose teachers reported that the students in their advanced mathematics class were “mostly taught this year” or “mostly taught before this year” that respective topic. These data reflect all U.S. TIMSS Advanced students, including those taking AP, IB and other non-AP, non-IB advanced mathematics courses. For students with more than one advanced mathematics teacher, students are counted as “taught” if any of their teachers indicated that the topic was taught in the current or a prior year. Percentages shown reflect the average and the range across the topics in each topic area, content domain, and overall (all content domains).

NOTE: Coverage of the specific TIMSS Advanced mathematics topics in each topic area and the percentage of U.S. students taught these topics by course type is provided in the supplemental tables A-3a, 3b, and 3c. Detail may not sum to totals because of rounding.

SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015; The College Board, http://apcentral.collegeboard.com/apc/public/courses/descriptions/index.html; International Baccalaureate, http://www.ibo.org/programmes/diploma-programme/curriculum/; The Common Core State Standards Initiative, http://www.corestandards.org/wp-content/uploads/Math_Standards1.pdf.

- Teacher data generally confirm the coverage patterns indicated by the review of course curricula, showing that nearly all U.S. TIMSS Advanced students were reported to have been taught the algebra topics by the time of the assessment. Two areas of divergence were in the *expressions and operations* topic area. First, the topic on *operations with complex numbers* (not covered in the standard-level IB mathematics course curriculum) was reported by teachers to have been taught to essentially all students;[17] and, second, the topic relating to *finite and infinite series* (indicated as prerequisite knowledge for AP calculus and explicitly covered in both IB Mathematics courses) was reported to have been taught to less than 100 percent of students taking AP Calculus AB, AP Calculus BC, or other non-AP, non-IB calculus courses. Although these students would likely have covered *finite and infinite series* in a prior course, some teachers may not consider this topic to be prerequisite knowledge for their calculus courses or do not know whether students had covered this in previous courses.[18]

Calculus

- All the topics in calculus—with one exception—were covered in the intended curricula of the eligible AP and IB mathematics courses (tables 3-1 and A-3b). In all AP and IB courses, these topics were explicit parts of the curricula. The exception was the *limits* topic on *conditions for continuity* and *differentiability of functions,* which was not covered by the standard-level IB mathematics course.
- Teacher data are generally consistent with the intended curricula, with nearly all U.S. TIMSS Advanced students reported to have been taught the calculus topics. One exception was the *limits* topic on *conditions for continuity and differentiability of functions,* which was not covered in the standard-level IB course curriculum but was reported by teachers as taught to all students. Two other exceptions to the observed coverage patterns were the topics in *integrals,* which are included in the curricula of all eligible AP and IB courses but reported as taught to less than 100 percent of students taking AP Calculus AB or other, non-AP or non-IB calculus courses (98 and 83 percent, respectively). Because integrals are the last topic covered in AP Calculus AB, some teachers may not have covered the topic at the time of the TIMSS Advanced assessment. This may also be a topic not yet covered for teachers of students taking non-AP, non-IB courses.

[17] IB mathematics teachers often teach both the standard-level and higher-level courses in their schools, and their responses to the questionnaire are likely based on the full set of topics covered in the higher-level course. Therefore, some topics indicated as not included in the standard-level curriculum may be identified as having been taught to all IB mathematics students.

[18] AP Calculus BC includes more advanced topics related to *finite and infinite series* (exhibit A-1) that go beyond what is assessed in TIMSS Advanced. Topics taught toward the end of the year may have been reported by teachers as "not yet taught or just introduced."

Geometry

- All the topics in *geometry* were covered in the intended curricula (or were prerequisites) for all the eligible AP and IB courses (tables 3-1 and A-3c). Except for three, these geometry topics are considered foundational knowledge for both the AP calculus and IB mathematics courses. The exceptions were the topic related to *vectors* in the *noncoordinate and coordinate geometry* topic area and the two topics related to *trigonometric functions* in the *trigonometry* topic area, all of which are covered explicitly in the both the standard- and higher-level IB mathematics course curricula.
- The teacher data confirm the coverage of all topics in *trigonometry* and two of the three topics in *noncoordinate and coordinate geometry,* with nearly all U.S. TIMSS Advanced students reported to have been taught these topics. The one exception was the topic on *properties of vectors and their sums and differences* (which is foundational to both AP courses and covered in both IB course curricula) was reported by teachers as taught to 83 percent of students taking AP Calculus AB, 87 percent taking AP Calculus BC, and 66 percent of those taking other calculus courses. Some calculus teachers may not consider the geometry topic related to vectors to be prerequisite knowledge for their calculus courses or they may not know what students had been taught about this topic in previous courses. Teachers may also have interpreted the *vectors* topic to include more than what is assessed in TIMSS Advanced. This also may be related to variation across states in the requirements in prior mathematics courses.

Coursetaking patterns of U.S. TIMSS Advanced mathematics students

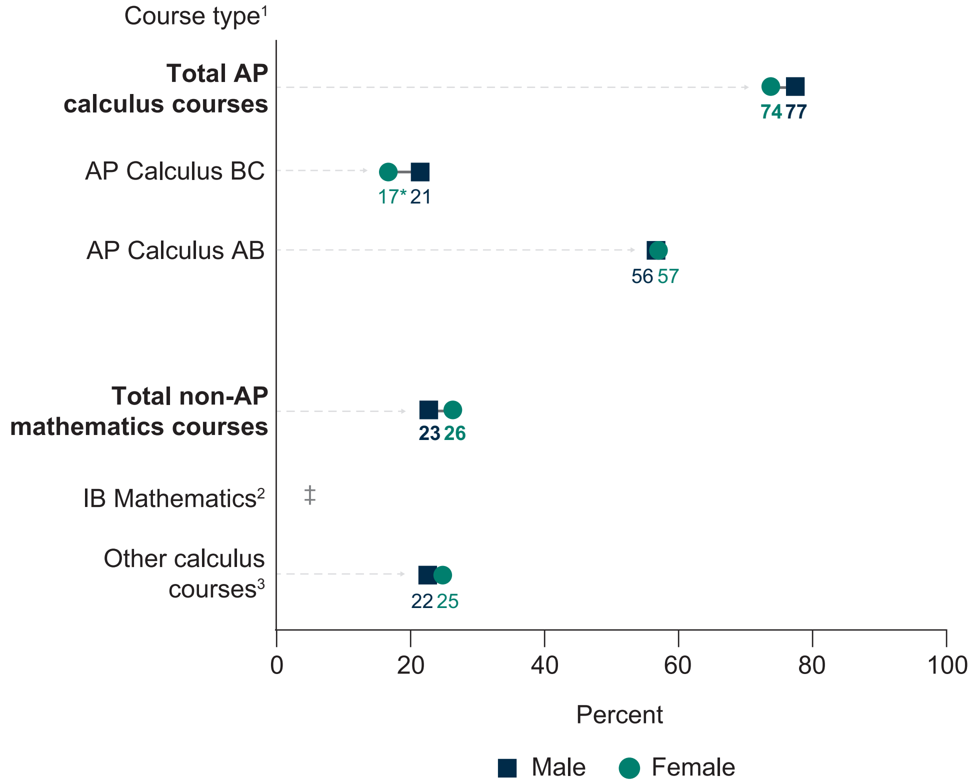
About three-quarters of U.S. TIMSS Advanced mathematics students (76 percent) had taken an AP mathematics course as their highest mathematics course, consisting of 56 percent who had taken AP Calculus AB and 19 percent who had taken AP Calculus BC (table 2-2). About one-quarter of U.S. students had taken only non-AP mathematics courses, with 1 percent having taken IB Mathematics and 23 percent having taken other calculus courses.

Given the aforementioned differences in TIMSS Advanced content coverage among the eligible U.S. advanced mathematics courses, it is also important to examine the coursetaking patterns for U.S. TIMSS Advanced students in different subgroups to determine if some students were more or less likely than others to have had access to the assessed content.

Differences by sex

Overall, among U.S. TIMSS Advanced students, there were generally no measurable differences between the percentages of male and female students who had taken AP calculus as their highest mathematics course (figure 3-1 and table B-1). However, within the AP courses a higher percentage of male students (21 percent) had taken the higher level AP course, Calculus BC, than female students (17 percent).

Figure 3-1. Percentage of U.S. TIMSS Advanced mathematics students, by course type and sex: 2015

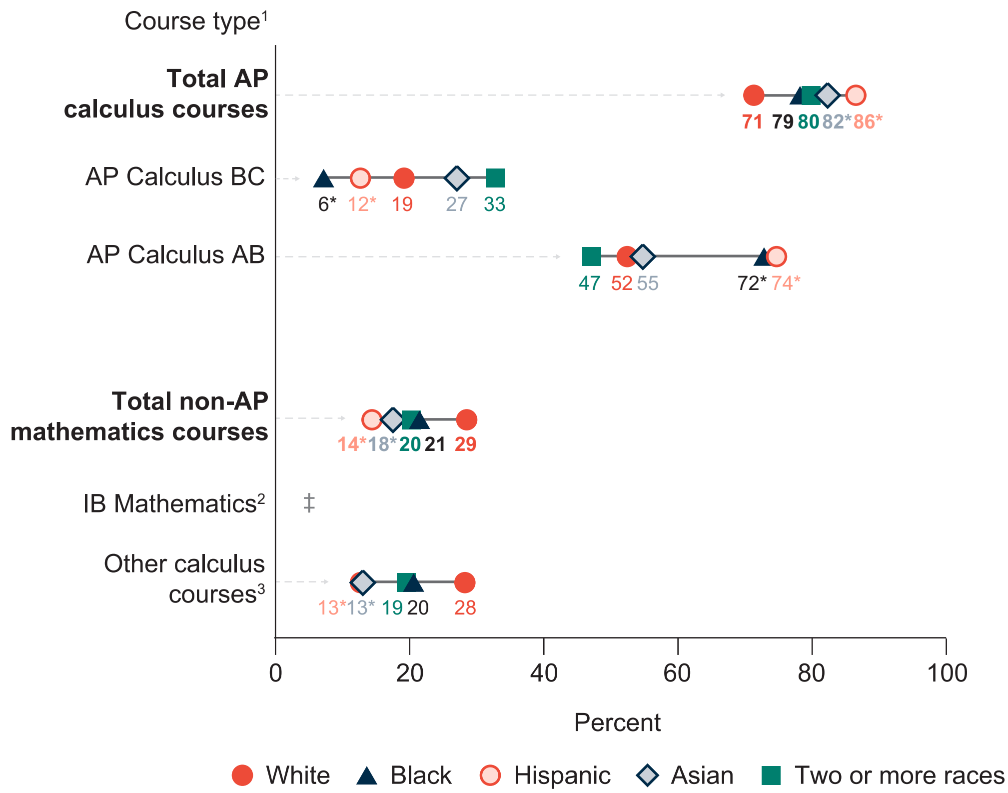


‡ Reporting standards not met (sample size < 62).
* $p < .05$. Female percentage is significantly different from male percentage.
[1] Course type reflects the highest level mathematics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other calculus courses" category only if they have not taken an AP or IB course.
[2] Includes higher-level and standard-level IB mathematics courses.
[3] Includes other calculus courses (including "honors" and "regents" courses).
NOTE: Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Differences by race/ethnicity

Overall, higher percentages of Hispanic and Asian students than White students had taken AP calculus courses (86 and 82 percent compared to 71 percent, respectively), and lower percentages of Hispanic and Asian students than White students had taken non-AP courses (14 and 18 percent compared to 29 percent, respectively) (figure 3-2 and table B-2). However, there were differences by AP course type. Higher percentages of Black (72 percent) and Hispanic (74 percent) students than White students (52 percent) had taken AP Calculus AB. Conversely, lower percentages of Black (6 percent) and Hispanic (12 percent) students than White students (19 percent) had taken AP Calculus BC.

Figure 3-2. Percentage of U.S. TIMSS Advanced mathematics students, by course type and race/ethnicity: 2015

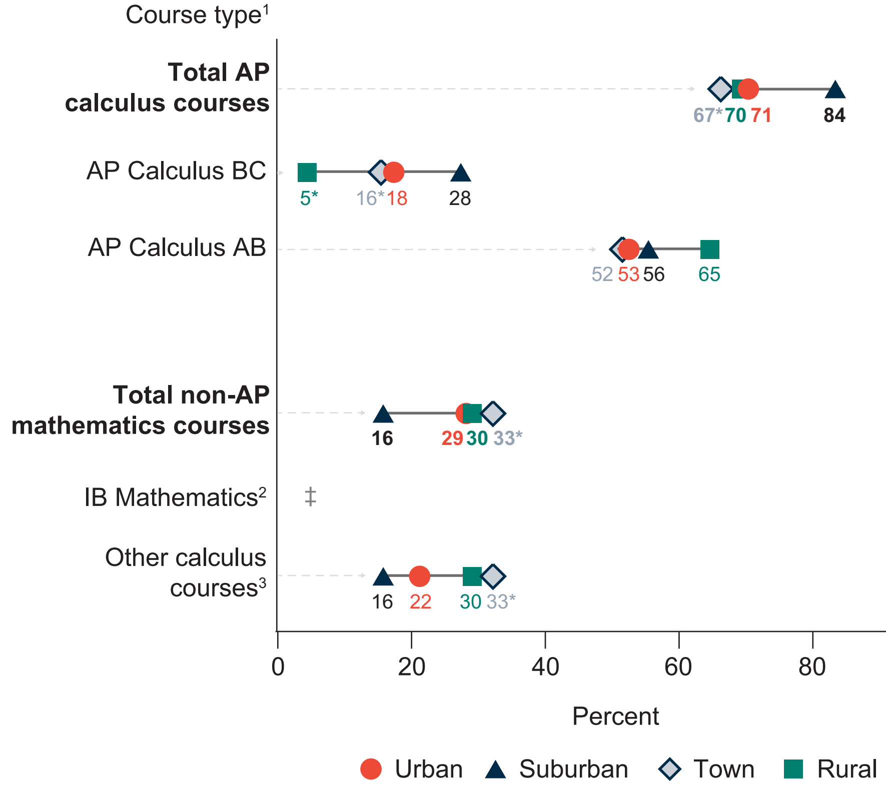


‡ Reporting standards not met (sample size < 62).
* *p* < .05. The subgroup percentage is significantly different from the percentage of White students.
[1] Course type reflects the highest level mathematics course taken. If students took another course in addition to an Advanced Placement (AP) or IB course, they are included in the AP or IB category. Students are included in the “other calculus courses” category only if they have not taken an AP or IB course.
[2] Includes higher-level and standard-level IB mathematics courses.
[3] Includes other calculus courses (including “honors” and “regents” courses).
NOTE: Reporting standards were not met for Native Hawaiian/Pacific Islander and Native American/Alaska Native students (sample size < 62). Data for these students are not shown separately in the figure. Black includes African American and Hispanic includes Latino. Racial categories exclude Hispanic origin. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Differences by school locale

Lower percentages of rural and town students than suburban students had taken AP Calculus BC (5 and 16 percent compared to 28 percent, respectively) (figure 3-3 and table B-3). Additionally, a higher percentage of town students than suburban students had taken other (non-AP, non-IB) calculus courses (33 percent vs. 16 percent, respectively).

Figure 3-3. Percentage of U.S. TIMSS Advanced mathematics students, by course type and school locale: 2015



‡ Reporting standards not met (sample size < 62).
* *p* < .05. The subgroup percentage is significantly different from the percentage of suburban students.
[1] Course type reflects the highest level mathematics course taken. If students took another course in addition to an Advanced Placement (AP) or IB course, they are included in the AP or IB category. Students are included in the "other calculus courses" category only if they have not taken an AP or IB course.
[2] Includes higher-level and standard-level IB mathematics courses.
[3] Includes other calculus courses (including "honors" and "regents" courses).
NOTE: Urban is defined as territories inside an urbanized area and inside a principal city. Suburban is defined as territories inside an urbanized area but outside a principal city. Town is defined as territories inside an urban cluster but outside an urbanized area. Rural is defined as territories that are not in an urbanized area or urban cluster. For additional information, see https://nces.ed.gov/programs/edge/docs/NCES_LOCALE_USERSMANUAL_2016012.pdf. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

In sum, these differences in coursetaking patterns mean that not all U.S. students had the same exposure to the advanced mathematics topics assessed in TIMSS Advanced. In particular, those students whose highest course was AP Calculus AB (56 percent of the U.S. TIMSS Advanced mathematics population, table 2-2)—and the relatively large proportions of Black (72 percent) and Hispanic (74 percent) students in this course (figure 3-2)—are not likely to have had the same extent of coverage of the topics related to *finite and infinite series* (table A-3a) and *vectors* (table A-3c) as those whose highest course was AP Calculus BC.[19]

[19] The *intended* curricula for both AP Calculus AB and BC include these topics as foundational knowledge expected to have been covered in a prerequisite mathematics course. However, AP Calculus BC covers additional and more advanced topics related to *finite and infinite series* and to *vectors* not included in AP Calculus AB.

3.2 Physics

Content included in the intended curricula of the TIMSS Advanced-eligible physics courses in the United States

The TIMSS Advanced-eligible courses in physics are shown in exhibit 2-1. The differences in their focus and specific content covered are described below. (See exhibit A-2 in appendix A for more detail about the content covered in each course.)

AP Physics B

AP Physics B was a 1-year algebra-based physics course offered prior to the 2014–15 school year that often provided a foundation in physics for students majoring in the life sciences, pre-medicine, or other applied sciences in college. It was designed as a second-year high school course covering a range of physics topics that are typically covered in a first-year introductory, algebra-based college physics course, including Newtonian mechanics, fluid and thermal physics, electricity and magnetism, waves and optics, and atomic and nuclear physics. As such, AP Physics B students were generally expected to have taken a prior introductory high school physics course and to have knowledge of algebra and basic geometry and trigonometry. Because TIMSS Advanced was administered in the spring of 2015 (when this course had been largely discontinued), most U.S. TIMSS Advanced students whose highest course was AP Physics B completed this course in their junior year.[20]

AP Physics 1 and 2

AP Physics 1 and 2 is a 2-year sequence that replaced AP Physics B in the 2014–15 school year. These two courses are the equivalent of the first and second semesters of an introductory, algebra-based college physics course that together cover the same topics as AP Physics B, but in more depth. The AP Physics 1 and 2 sequence provides a foundation in physics to support future advanced coursework in the sciences. Since 2014–15 was the first year that AP Physics 1 and 2 were offered, schools were in transition and the specific courses that students took varied. The majority of students took AP Physics 1 in their senior year. Others took both AP Physics 1 and 2, which some schools offered as one-semester courses to provide equivalent topic coverage as AP Physics B in the same time span, while other students took only AP Physics 2 in their senior year.

AP Physics 1 does not require any prior high school physics and may be a first-year physics course for many students. It focuses on mechanics, with some coverage of introductory topics related to wave properties, electrostatics, and electric circuits. AP Physics 2 is intended as a second-year physics course to follow AP Physics 1 or an equivalent first-year physics course. Since the 2014–15 school year was the first year that AP Physics 2 was offered, students who took only Physics 2 in their senior year had likely taken a previous physics course. The mechanics topics covered in AP Physics 1 are considered foundational knowledge for AP Physics 2, which focuses on more advanced physics topics covering thermodynamics, electricity and magnetism, wave phenomena, and atomic and nuclear physics. For AP Physics 1, students are expected to have completed geometry and be concurrently enrolled in algebra II or its equivalent. For AP Physics 2, students are expected to have taken or be concurrently enrolled in pre-calculus or its equivalent.

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[20] Based on information received from schools participating in TIMSS Advanced, some schools continued to offer a 1-year algebra-based course identified as AP Physics B in 2014–15, and some U.S. TIMSS Advanced students took this in their senior year.

Enrollment in AP physics courses increased in the 2014–15 school year, due in large part to higher enrollment in AP Physics 1 compared to AP Physics B in the previous year (Heitin 2015).[21] With the new AP Physics 1 course, students could get AP credit for a first-year physics course in their senior year, even those who had not taken physics previously.

AP Physics C

There are two AP Physics C courses: Physics C-Mechanics and Physics C-Electricity and Magnetism. Each corresponds to one semester of an introductory, calculus-based college physics course that is focused on a specific area of physics (mechanics or electricity and magnetism). AP Physics C is typically taken as a two-course sequence, with Physics C-Mechanics taken prior to Physics C-Electricity and Magnetism. The AP Physics C sequence provides a physics foundation for physical science and engineering majors in college and was designed as a second-year high school physics course. Although there are no prerequisite physics courses specified, completion of a prior introductory high school physics course is recommended in the AP course guide. Some schools offer AP Physics C as two separate one-semester courses, while others offer a single, combined 1-year course. Some students may take a single AP Physics C-Mechanics course, which may be offered as a full-year course. AP Physics C courses cover the topics in mechanics and in electricity and magnetism that are included in the AP Physics 1 and 2 sequence, but in more depth and with calculus applications. Students are expected to have completed or be concurrently enrolled in calculus.

IB Physics

IB Physics is a 2-year algebra-based physics course, offered at either a standard level or a higher level. The IB physics courses provide students with a broad, general understanding of the physics principles covered in an introductory college course. The core curriculum for both courses covers eight topic areas: physics and physical measurement; mechanics; thermal physics; oscillations and waves; electric currents; fields and forces; atomic and nuclear physics; and energy, power, and climate change. The higher level IB physics course includes additional topics in six areas. Both courses also include instruction in two other topic areas chosen by the student from a set of options that varies between the standard-level and higher-level courses. The standard-level IB course does not require any prior high school physics, but the higher-level course would typically follow a prior introductory physics course. Knowledge of algebra as well as some geometry and trigonometry topics is required.

Other physics courses

Other physics courses (non-AP, non-IB) include state-, district-, and school-specific second-year physics courses, including those identified as “honors” or “regents” courses. The specific content varies across states, districts, and schools but generally covers Newtonian mechanics; heat, temperature, and thermodynamics; electricity and magnetism; wave phenomena; and atomic and nuclear physics.

[21] The AP report on exam volume changes (2005–2015) (College Board 2015b) shows that the number of AP physics exams given in 2015 increased by 66 percent compared to 2014, and that the majority of these exams were in AP Physics 1.

Extent of coverage of TIMSS Advanced physics topics in U.S. physics courses

Table 3-2 summarizes the extent of coverage of the TIMSS Advanced topics across the U.S. TIMSS Advanced-eligible physics courses. This table shows

1. the number of specific physics topics within each topic area and content domain that are assessed in TIMSS Advanced (as described in the framework);
2. the extent to which these TIMSS Advanced topics are covered in the *intended* curricula of the eligible AP and IB physics courses, indicating both the extent of coverage (aggregated at the topic area level) and the proportion of topics within the content domain and overall that are at least partially covered in the intended curriculum for each course (based on topic-level data provided in tables A-4a, A-4b, and A-4c); and
3. the overall percentage of U.S. TIMSS Advanced students who had been taught the TIMSS Advanced physics topics by the time of the assessment (either in the current or a prior year) based on their teachers' reports (i.e., *implemented* curriculum). These data are presented as the average and range across topics in the topic area and content domain.

It should be noted that the data on the *intended* curricula (described in 2 above) are not available for other (non-AP, non-IB) physics courses because these vary across states, districts, and schools. In contrast, the teacher questionnaire data on the *implemented* curriculum (described in 3 above) reflect teachers of all TIMSS Advanced-eligible physics courses. For this and other reasons discussed below, patterns reflected in the two sources of data may differ slightly.[22]

Overall, the extent of coverage of TIMSS Advanced topics in the eligible physics courses varied by course type and content domain. AP Physics B and the higher level IB physics course generally covered all or nearly all of the TIMSS Advanced topics. AP Physics C courses (mechanics, and electricity and magnetism) covered the TIMSS Advanced topics in their corresponding content areas, with the topics covered in AP Physics C-Mechanics considered foundational knowledge for AP Physics C-Electricity & Magnetism. However, coverage of other topics depended on which prior physics course(s) students had taken previously.[23]

Like its predecessor AP Physics B, the curriculum for the new AP Physics 1 and 2 course sequence covers nearly all of the TIMSS Advanced topics as previously reported (Lazzaro et al. 2016). Although AP Physics 2 would not alone cover all the TIMSS Advanced topics, accompanied by AP Physics 1 or an equivalent course in a prior year (which would have been typical), the AP Physics 2 students taking the 2015 TIMSS Advanced assessment would likely have covered nearly all the topics. In contrast, the standard-level IB physics course covered about four-fifths of TIMSS Advanced physics topics (19 of 23), and AP Physics 1 covered less than half of the topics (10 of 23). This means that students whose first high school physics course was AP Physics 1 would not likely have covered the majority of the TIMSS Advanced topics. On average across the TIMSS Advanced topics in all content domains, 73 percent of U.S. students overall were reported by their physics teachers to have been taught the topics by the time of the assessment (either in the current or a prior year).[24]

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[22] Data cited in this section for specific physics topics in each content domain are not shown in table 3-2, which summarizes the data at the topic area level. Coverage of the specific TIMSS Advanced physics topics in each topic area and the percentage of U.S. TIMSS Advanced students taught these topics in the different TIMSS Advanced-eligible courses is provided in the supplementary tables in appendix A (A-4a, A-4b, and A-4c).

[23] The AP Physics C course guides do not specify particular prerequisites, so table 3-2 only indicates if the TIMSS Advanced topics are included in the associated AP course guidelines. However, because both AP Physics C courses are designed as second-year, specialist physics courses, topics included in a first-year physics course would likely have been covered prior to Physics C.

[24] Average topic coverage in the U.S. was generally low compared to other countries (most at or above 80 percent); however, there were other countries with similarly low coverage to the U.S. for topics in *electricity and magnetism* and *wave phenomena and atomic/nuclear physics* (between 50 and 60 percent). See *TIMSS Advanced 2015 International Results for Advanced Mathematics and Physics* (exhibit P9.8).

Table 3-2. Extent of TIMSS Advanced physics topic coverage in the intended curriculum by course type and the overall percentage of U.S. TIMSS Advanced students who were taught these topics, by content domain and topic area: 2015

Content domains and topic areas	Number of physics topics assessed in TIMSS Advanced[1]	Extent of TIMSS Advanced topic coverage in the *intended* curriculum by course type[2]							Overall percentage of U.S. students taught TIMSS Advanced physics topics[4]	
		AP Physics C[3]		AP Physics 1 & 2			IB Physics			
		Electricity & magnetism	Mechanics	Physics 2	Physics 1	AP Physics B	Higher level	Standard level	Average across topics	Range across topics
Total physics(all content domains)	**23**	**12/23**	**6/23**	**22/23**	**10/23**	**22/23**	**23/23**	**19/23**	**73**	**45–100**
Mechanics and thermodynamics	**9**	**6/9**	**6/9**	**9/9**	**6/9**	**9/9**	**9/9**	**8/9**	**87**	**62–100**
Forces and motion	4	⊙	●	⊙	●	●	●	●	98	95–100
Laws of conservation	3	⊙	◖	●	◖	●	●	◖	87	63–100
Heat and temperature	2	○	○	●	○	●	●	●	63	62–64
Electricity and magnetism	**6**	**6/6**	**0/6**	**6/6**	**2/6**	**6/6**	**6/6**	**4/6**	**66**	**49–82**
Electricity and electric circuits	3	●	○	●	◖	●	●	●	79	74–82
Magnetism and electromagnetic induction	3	●	○	●	○	●	●	◖	54	49–58
Waves phenomena and atomic/nuclear physics	**8**	**0/8**	**0/8**	**7/8**	**2/8**	**7/8**	**8/8**	**7/8**	**62**	**45–87**
Wave phenomena	4	○	○	◖	◖	◖	●	●	69	53–87
Atomic and nuclear physics	4	○	○	●	○	●	●	◖	54	45–73

● Topic area is covered in the course curriculum (all topics are fully or partially covered in the TIMSS Advanced-eligible physics course or are expected to have been covered in a prior physics course).
◖ Topic area is partially covered (at least one topic is not covered in the course curriculum nor specified as prerequisite knowledge).
⊙ Topic area reflects foundational concepts expected for AP Physics 2 that are covered in AP Physics 1 and those expected for AP Physics C-Electricity & Magnetism that are covered in C-Mechanics.
○ Topic area is not included in the course curriculum.

[1] The number of topics assessed in TIMSS Advanced reflects the specific physics topics included in each topic area and content domain as shown in table A-2.

[2] "Extent of TIMSS Advanced topic coverage in the intended curriculum" is based on the overlap of TIMSS Advanced physics topics with Advanced Placement (AP) Physics course descriptions available from the College Board and the core curriculum specified for the International Baccalaureate (IB) Physics standard-level and higher-level courses. The intended curriculum is not indicated for other TIMSS Advanced-eligible non-AP, non-IB physics courses, since the curricula for these courses vary across states, districts, and schools. The ratios shown under each course type reflect the proportion of TIMSS Advanced topics in a given content domain (and overall) that are at least partially covered in the intended curriculum for that course (or are considered foundational knowledge expected to have been covered in a prior physics course).

[3] The AP Physics C course curriculum covers a specific set of topics in mechanics and in electricity and magnetism. The extent to which other TIMSS Advanced topics have been covered in AP Physics C or prior physics courses varies across states, districts, and schools.

[4] "Overall percentage of U.S. students taught TIMSS Advanced physics topics" reflects the implemented curriculum. It is based, for each topic, on the number of students whose teachers reported that the students in their physics class were "mostly taught this year" or "mostly taught before this year" that respective topic. These data reflect all U.S. TIMSS Advanced students, including those taking AP, IB and other non-AP, non-IB physics courses. For students with more than one physics teacher, students are counted as "taught" if any of their teachers indicated that the topic was taught in the current or a prior year. Percentages shown reflect the average and the range across the topics in each topic area, content domain, and overall (all content domains).

NOTE: Coverage of the specific TIMSS Advanced physics topics in each topic area and the percentage of U.S. students taught these topics by course type is provided in the supplemental tables A-4a, 4b, and 4c. Detail may not sum to totals because of rounding.

SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015; The College Board, http://apcentral.collegeboard.com/apc/public/courses/descriptions/index.html; International Baccalaureate, http://www.ibo.org/programmes/diploma-programme/curriculum/.

Findings about the coverage of TIMSS Advanced physics topics in each content domain are described below.

Mechanics and thermodynamics

- Most of the topics in *mechanics and thermodynamics*—including all four of the topics related to *forces and motion* (or, Newtonian mechanics) and two related to the *laws of conservation* (i.e., those on mechanical energy and on momentum)—are covered in all the TIMSS Advanced-eligible AP and IB physics courses (tables 3-2 and A-4a). These topics are included in the curricula for AP Physics C-Mechanics, AP Physics B, and AP Physics 1, and are considered foundational knowledge for the second-year AP Physics 2 course. Students whose highest course was AP Physics C-Electricity and Magnetism would likely have covered these mechanics topics in AP Physics C-Mechanics or another prior physics course.
- There were three topics in *mechanics and thermodynamics,* however, with lower coverage in the AP and IB physics course curricula: both topics in the *heat and temperature* topic area and the topic on the *first law of thermodynamics* in the *laws of conservation* topic area. These topics are only fully covered in AP Physics B, AP Physics 2, and higher level IB Physics.
- The teacher data generally confirmed the coverage patterns indicated by the review of curricula, showing that nearly all U.S. TIMSS Advanced students were reported to have been taught the topics in *forces and motion* and the two topics in *laws of conservation* related to mechanical energy and momentum by the time of the assessment (97 to 100 percent). In comparison, from 62 to 64 percent of students had been taught the two topics in *heat and temperature* and the topic on the *first law of thermodynamics* in *laws of conservation* by the time of the assessment. The coverage across these latter three topics ranged from about half of students who had taken AP Physics 1, to at least three-quarters of those who had taken AP Physics B or AP Physics 2, to all students who had taken either of the IB physics courses.[25] Coverage of these three topics for students taking other non-AP, non-IB courses ranged from 66 percent having been taught the topic on *law of ideal gases* to 86 percent having been taught the *first law of thermodynamics.* Teachers also reported that a relatively low percentage of AP Physics C students had been taught these topics (from 48 to 58 percent).[26] For the AP Physics 1 students, the teacher data indicating that about half of these students have been taught these topics (from 48 to 52 percent) appears to be in contrast with the reports on the intended curriculum, which show that the topics are covered in AP Physics 2 but not AP Physics 1. However, as mentioned in the technical notes for this section, these additional topics may also have been taught to many AP Physics 1 students in order to meet state curriculum standards for grade 12. Also, since the 2014–15 school year was the first year that the AP Physics 1 and 2 course sequence was offered, some schools may have included some of the AP Physics 2 topics as part of what was identified as an AP Physics 1 course in the transcript data in order to provide better topic coverage for their seniors who previously would have taken AP Physics B.[27]

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[25] As noted previously for IB mathematics, IB physics teachers often teach both the standard-level and higher-level courses in their schools, and their responses to the questionnaire are likely based on the topics covered in the higher-level course. Therefore, some topics indicated as not included in the standard-level curriculum may be identified as having been taught to all IB physics students.

[26] As noted previously, AP Physics C courses are specialist courses that cover specific topics in mechanics and in electricity and magnetism. Teachers of AP Physics C may not know what topics students were taught in their prior physics courses, although students may have covered these topics previously.

[27] This applies to multiple topics across the physics content domains.

Electricity and magnetism

- All topics in *electricity and magnetism* were covered in the intended curricula for most of the higher-level AP courses—AP Physics B, Physics 2 and Physics C-Electricity and Magnetism—as well as the higher level IB physics course (tables 3-2 and A-4b). In contrast, in the *electricity and electric circuits* topic area, AP Physics 1 only covers the topics related to *electrostatics and electrical circuits* at an introductory level and does not cover the topic of *charged particles in an electric field* at all. In the *magnetism and electromagnetic induction* topic area, AP Physics 1 covers none of the three topics, and the standard-level IB physics course covers only one (*charged particles in a magnetic field*).
- The relatively lower curriculum coverage of *electricity and magnetism* topics and the variation in coverage across courses was generally reflected in the teacher data as well, with correspondingly low percentages of U.S. TIMSS Advanced students reported to have been taught these topic areas by the time of the assessment (66 percent overall and driven by lower percentages for students taking AP Physics 1 and other non-AP, non-IB physics courses). For example, the lowest coverage was of the topic on *Faraday's and Lenz's laws of induction* in the *magnetism and electromagnetic induction* topic area, which was reported as taught to 49 percent of students overall and just 25 percent of AP Physics I students. Notably, even in the most advanced AP Physics C-electricity and magnetism course, about one-quarter of students were not reported to have been taught this topic by the time of the assessment. (This more advanced topic may be covered toward the end of the AP Physics C course and, thus, reported by some teachers as "not yet taught or just introduced.") Of the other topics not covered in the AP Physics 1 curriculum, the two topics in the *magnetism and induction* topic area were reported as taught to 31 percent of AP Physics 1 students, but the other topic (*charged particles in an electric field*) was taught to over half the students (59 percent). Again, these may be topics required to meet state curriculum standards or included to provide increased topic coverage for seniors taking the new AP Physics 1 course.

Wave phenomena and atomic/nuclear physics

- All the topics in *wave phenomena and atomic/nuclear physics* are covered in the intended curriculum for the higher-level IB physics course, and most (7 of 8) are covered in the curricula for the standard-level IB physics course, AP Physics B, and AP Physics 2 (or across the AP Physics 1 and 2 sequence) (tables 3-2 and A-4c). Few topics (2 of 8) are covered by AP Physics 1 alone, and none are covered by AP Physics C courses. Students in AP Physics C courses in their senior year would only have had these topics in a prior physics course.
  - In *wave phenomena,* the topic on *mechanical waves* is covered in AP Physics 1, and the topic on *reflection, refraction, interference, and diffraction* of waves is partially covered (and continued in AP Physics 2), but the other two topics (on *electromagnetic radiation* and on *thermal radiation, temperature, and wavelength*) are not covered in AP Physics 1. The topic on *thermal radiation, temperature, and wavelength* is explicitly covered only in the intended curricula for IB physics courses. Although the AP course curricula do not explicitly cover this topic, learning objectives in AP Physics 2 related to energy transfer and emission spectra build on these concepts.
  - In *atomic and nuclear physics,* all four topics are included in AP Physics B, AP Physics 2, and higher-level IB Physics; and all but one (*wave-particle duality*) are covered in standard-level IB Physics. AP Physics C and AP Physics 1 course curricula do not cover any of the topics in this topic area.

- The teacher data were more consistent with the curriculum coverage patterns for the topic area of *wave phenomena* than for the topic area of *atomic and nuclear physics.* The average percentage of U.S. TIMSS Advanced students reported to have been taught the topics in *wave phenomena* was 69 percent overall (ranging from 56 percent of AP Physics 1 students to all or nearly all IB physics students), which appears to be consistent with the coverage of topics in the course curricula. In contrast, the average percentage of students reported to have been taught the topics in *atomic and nuclear physics* was 54 percent overall, including 36 percent of students in the AP Physics 1, which does not cover any of these topics. Clearer examples of differences between the intended and implemented curriculum data are found at the individual topic level. The topic related to *mechanical waves* was reported as taught to the highest percentage of students (87 percent overall), reflecting at least 80 percent of students in all eligible courses, including the AP Physics C courses in which the topic is not covered (although it is likely to have been taught in a prior course). In another example, teachers reported that about half of U.S students (53 percent) had been taught the topic on *thermal radiation, temperature, and wavelength,* which contrasts with the curriculum coverage results indicating that this topic is covered only in IB physics courses (representing 6 percent of the U.S. TIMSS Advanced population, table 2-3). It is possible that the description of this topic was interpreted by the U.S. physics teachers to include content beyond what is assessed in TIMSS Advanced. In the *atomic and nuclear physics* topic area, 73 percent of students overall were reported to have been taught the topic on *structure of the atom and its nucleus* by the time of the assessment compared to no more than half who were reported to have been taught the more advanced topics in modern physics related to *wave-particle duality, types of nuclear reactions,* and *mass-energy equivalence.* The 73 percent of students overall reported by teachers to have been taught the topics on *electromagnetic radiation* and *structure of the atom and its nucleus* includes more than half of AP Physics 1 students. As noted previously, these topics (covered in the intended curriculum for AP Physics 2 but not AP Physics 1) may have been added for the AP Physics 1 course first offered to seniors in the 2014–15 school year.

Coursetaking patterns of U.S. TIMSS Advanced physics students

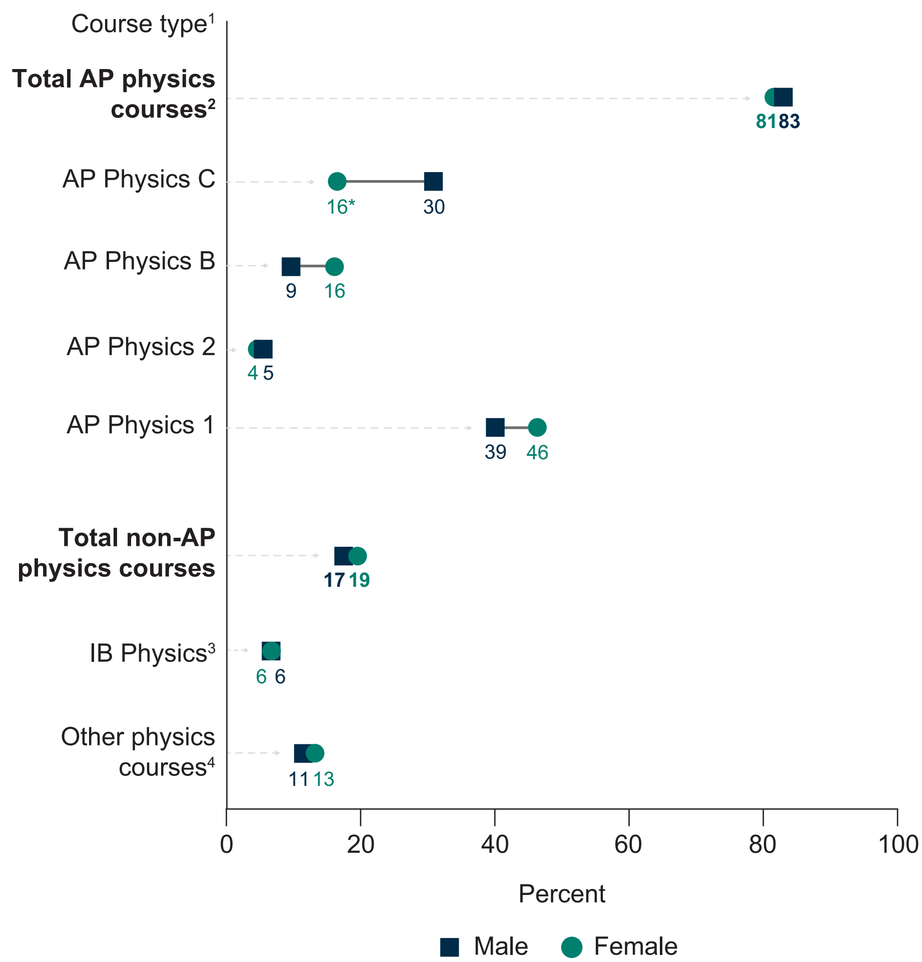
Most U.S. TIMSS Advanced physics students (83 percent) had taken an AP physics course as their highest physics course, with a greater percentage (42 percent) having taken the lowest level AP Physics I compared to 25 percent having taken one or both of the highest level AP Physics C courses (table 2-3). Another 16 percent had taken AP Physics B or AP Physics 2 as their highest physics course. About one-fifth of U.S. students had taken only non-AP physics courses, with 6 percent having taken IB Physics and 12 percent having taken other (non-AP, non-IB) second-year physics courses.

Given the aforementioned differences in TIMSS Advanced content coverage among the eligible U.S. physics courses, it is also important to examine the coursetaking patterns for U.S. TIMSS Advanced students in different subgroups to determine if some students are more or less likely than others to have had access to the assessed content.

Differences by sex

Overall, among U.S. TIMSS Advanced students, there were generally no measurable differences between the percentages of male and female students who had taken the different AP or non-AP physics courses (figure 3-4 and table B-4). There was one notable exception, however, with nearly twice the percentage of males as females (30 vs. 16 percent) having taken AP Physics C as their highest physics course. Enrollment of male students in the other AP physics courses was 39 percent in AP Physics 1 and 14 percent across AP Physics B and 2 combined. Enrollment of female students in the other AP physics courses was 46 percent in AP Physics 1 and 20 percent across AP Physics B and 2.

Figure 3-4. Percentage of U.S. TIMSS Advanced physics students, by course type and sex: 2015

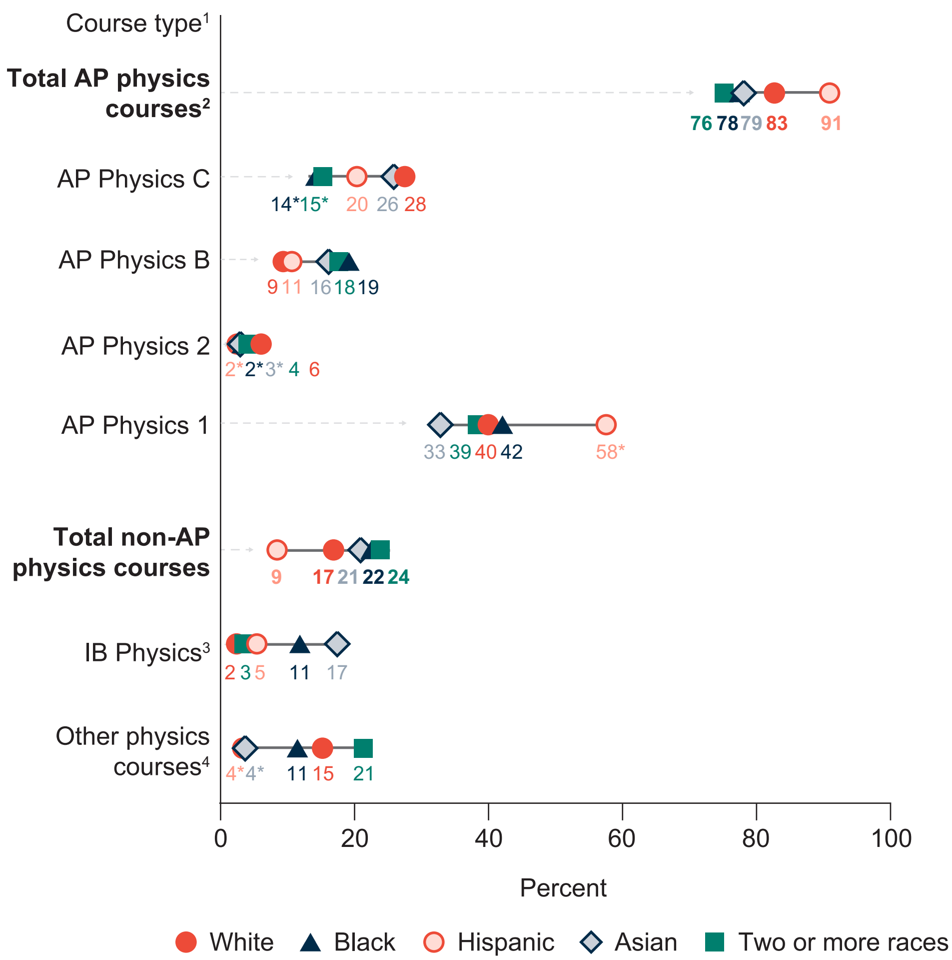


* $p < .05$. Female percentage is significantly different from male percentage.
[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the “other physics courses” category only if they have not taken an AP or IB course.
[2] AP Physics C includes courses in mechanics (C-M) and electricity and magnetism (C-E/M). AP Physics B was a 1-year physics course, which was discontinued after the 2013–14 school year and was replaced with the 2-year course sequence AP Physics 1 and 2 beginning in 2014–15. Data in the AP Physics B category are generally for students who took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering a 1-year course identified as AP Physics B in 2014–15.
[3] Includes higher-level and standard-level IB physics courses.
[4] Includes other types of second-year physics courses (including “honors” and “regents” courses).
NOTE: Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Differences by race/ethnicity

Overall, there were no measurable differences between the percentages of students in different racial/ethnic groups who had taken AP courses versus non-AP courses (figure 3-5 and table B-5). Within the AP courses, however, there were some differences. A higher percentage of White students than of Black students or students of Two or more races had taken an AP Physics C course (28 percent compared to 14 and 15 percent, respectively). There was also a higher percentage of White students than Black, Hispanic, and Asian students who had taken AP Physics 2 (6 percent compared to 2, 2, and 3 percent, respectively). Hispanic students were more likely than White students to have taken the lowest level AP course, Physics 1 (58 percent compared to 40 percent).

Figure 3-5. Percentage of U.S. TIMSS Advanced physics students, by course type and race/ethnicity: 2015



* $p < .05$. The subgroup percentage is significantly different from the percentage of White students.

[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other physics courses" category only if they have not taken an AP or IB course.

[2] AP Physics C includes courses in mechanics (C-M) and electricity and magnetism (C-E/M). AP Physics B was a 1-year physics course, which was discontinued after the 2013–14 school year and was replaced with the 2-year course sequence AP Physics 1 and 2 beginning in 2014–15. Data in the AP Physics B category are generally for students who took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering a 1-year course identified as AP Physics B in 2014–15.

[3] Includes higher-level and standard-level IB physics courses.

[4] Includes other types of second-year physics courses (including "honors" and "regents" courses).

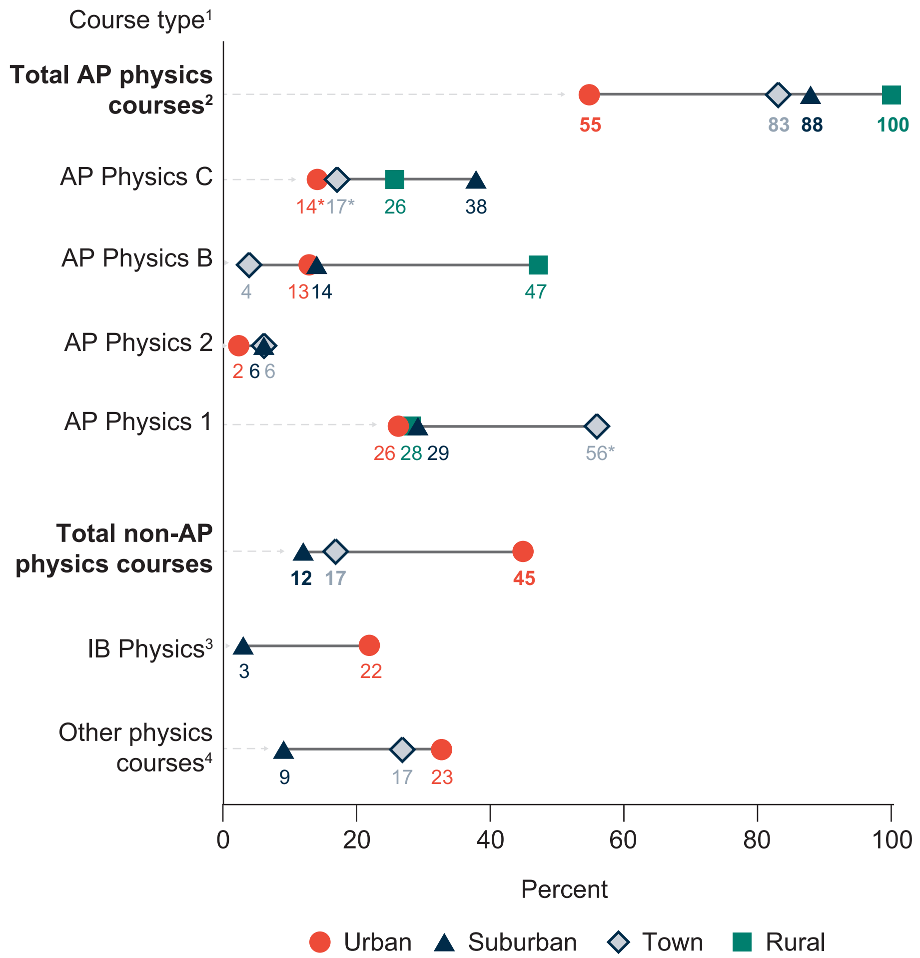
NOTE: Reporting standards were not met for Native Hawaiian/Pacific Islander and Native American/Alaska Native students (sample size < 62). Data for these students are not shown separately in the figure. Black includes African American and Hispanic includes Latino. Racial categories exclude Hispanic origin. Detail may not sum to totals because of rounding.

SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Differences by school locale

Lower percentages of urban and town students than suburban students had taken one or both of the highest-level AP Physics C courses (14 and 17 percent vs. 38 percent, respectively) (figure 3-6 and table B-6). In addition, a higher percentage of town students than suburban students had taken the first-year AP Physics 1 course (56 percent vs. 29 percent, respectively).

Figure 3-6. Percentage of U.S. TIMSS Advanced physics students, by course type and school locale: 2015



* *p* < .05. The subgroup percentage is significantly different from the percentage of suburban students.
[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other physics courses" category only if they have not taken an AP or IB course.
[2] AP Physics C includes courses in mechanics (C-M) and electricity and magnetism (C-E/M). AP Physics B was a 1-year physics course that was discontinued after the 2013–14 school year and was replaced with the 2-year course sequence AP Physics 1 and 2 beginning in 2014–15. Data in the AP Physics B category are generally for students who took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering a 1-year course identified as AP Physics B in 2014–15.
[3] Includes higher-level and standard-level IB physics courses.
[4] Includes other types of second-year physics courses (including "honors" and "regents" courses).
NOTE: Urban is defined as territories inside an urbanized area and inside a principal city. Suburban is defined as territories inside an urbanized area but outside a principal city. Town is defined as territories inside an urban cluster but outside an urbanized area. Rural is defined as territories that are not in an urbanized area or urban cluster. For additional information, see https://nces.ed.gov/programs/edge/docs/NCES_LOCALE_USERSMANUAL_2016012.pdf. There were no rural students who had taken AP Physics 2 or non-AP physics courses (including IB Physics and other physics courses) as their highest course. Nor were there town students who had taken IB Physics. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

In sum, these differences in coursetaking patterns mean that not all U.S. students have had the same exposure to the physics topics assessed in TIMSS Advanced. In particular, those students whose highest physics course was AP Physics 1 (42 percent of TIMSS Advanced students overall, table 2-3)—and the relatively large proportion of Hispanic students (58 percent) and town students (56 percent) in this course (figures 3-5 and 3-6)—are less likely to have covered the more advanced topics in *electricity and magnetism* (table A-4b) and in *wave phenomena and atomic/nuclear physics* (table A-4c), as well as the topics related to *thermodynamics* in the *mechanics* and *thermodynamics* content domain (table A-4a) than students in higher level AP physics courses.

Section 4: Methods Used to Analyze and Report Results From Student Performance Data

This section describes how the TIMSS Advanced student performance data were analyzed and how they are reported in the results sections for advanced mathematics and physics (sections 5 and 6, respectively). All calculations are based on unrounded data and thus, in some cases, differences cited in text may differ slightly from calculations based on the rounded data that are presented in tables and figures. All differences cited in the text were tested using *t* tests and are significant at the 0.05 level. No adjustments were made for multiple comparisons. All of the figures presented in sections 5 and 6 are accompanied by supplemental tables in appendix B that show the corresponding data in table format. (See Technical Notes in appendix C for more information about statistical comparisons.)

4.1 Analysis and reporting for subsections 5.1 and 6.1: student performance results based on scale scores and international benchmarks

Subsections 5.1 and 6.1 describe U.S. TIMSS Advanced students' performance in advanced mathematics and physics, respectively, using scale scores and international benchmarks.

Scale scores

TIMSS Advanced achievement results are reported on a scale from 0 to 1,000, with a fixed scale centerpoint of 500 and a standard deviation of 100. The **scale centerpoints** represent the international means of the overall achievement distributions for advanced mathematics and physics in the first TIMSS Advanced assessment year (1995). TIMSS Advanced provides average overall scale scores for advanced mathematics and physics, as well as subscale scores for each content and cognitive domain.

This report describes these average overall scale scores and subscale scores for advanced mathematics and physics—both for U.S. TIMSS Advanced students overall and by course type, sex, race/ethnicity, and school locale.[28] It also compares the U.S. averages (overall and by course type) to the TIMSS Advanced scale centerpoints for advanced mathematics and physics.

[28] Course type analyses are based on the highest course taken in advanced mathematics or physics (see section 2). Results are not shown by school control because there were no measurable differences in scores between students from public schools and those from private schools for either advanced mathematics or physics. Any observed differences were not statistically significant due to large standard errors.

International benchmarks

TIMSS Advanced international benchmarks provide a way to interpret the scale scores and to understand how students' proficiency varies at different points on the scales. Each successive point, or benchmark, is associated with the knowledge and skills that students taking the TIMSS Advanced assessments successfully demonstrate at each level. TIMSS Advanced describes three benchmarks of achievement: *Advanced, High,* and *Intermediate.* (See exhibits 5-1 and 6-1 for detailed descriptions of the skills and knowledge demonstrated by students reaching each benchmark for advanced mathematics and physics, respectively.)

This report presents the percentages of U.S. TIMSS Advanced students reaching each benchmark (i.e., students who scored at or above the benchmark) by course type, sex, race/ethnicity, and school locale and compares these percentages to (a) the percentages of U.S. students who reach them overall and (b) the international median percentages reaching each benchmark among the countries participating in TIMSS Advanced 2015.

In addition, the report examines the coursetaking patterns of U.S. students at different achievement levels. To do this, the percentage distributions by course type among the subset of students reaching each international benchmark are compared with the distributions for U.S. students overall.

4.2 Analysis and reporting for subsections 5.2 and 6.2: U.S. item-level performance across TIMSS Advanced content domains

Subsections 5.2 and 6.2 explore the performance of U.S. students on the TIMSS Advanced assessment items in advanced mathematics and physics, respectively, and relate item performance to the level of topic coverage. All item-level statistics, category cutpoints, and comparisons in these subsections are U.S.-focused and are based only on the 2015 item pool. Therefore, conclusions about relative U.S. students' performance across items at the content domain, topic area, and topic levels may not be generalizable beyond the 2015 assessment.

Item-level statistics

Percent correct

The starting point, or building block, for the analyses in sections 5.2 and 6.2 is the "percent correct," which is the percentage of students receiving credit on each item. For multiple-choice and short constructed-response items (each worth one score point), this reflects the percentage of students who provided a correct answer. For extended constructed-response items, this reflects the weighted percentage of students receiving full credit (2 points) or partial credit (1 point). Thus, the higher the percent correct is for any given item, the greater the proportion of students who received credit on that item.[29]

Average percent correct

"Average percent correct" is the "percent correct" averaged across items in a given set. The higher the "average percent correct" is for the set, the greater the proportion of students who received credit on the items in that set. In this report, item "percent correct" is averaged across (a) all advanced mathematics and all physics items, (b) all items in each content domain, and (c) all items in each topic area within the content domains. Comparisons are then made between the averages for each content domain and topic area and the average across all advanced mathematics or physics items.[30]

[29] Thus, an item with a high percent correct is a relatively easier item and an item with a low percent correct is a relatively more difficult item.

[30] Averages are not computed at the topic level due to the small number of items in each topic.

Relating item performance to the level of topic coverage

The percent correct data on the individual items in each advanced mathematics and physics topic are used in conjunction with the topic coverage data from section 3 to examine U.S. performance in light of students' opportunity to learn advanced mathematics and physics content. In this analysis, each of the TIMSS Advanced topics is categorized into three levels of topic coverage: high, moderate, and low. The categorizations rely on the curriculum and teacher questionnaire data that reflect the intended and implemented curriculum, respectively. The cutpoints for high, moderate, and low coverage were set at the same level for advanced mathematics and physics to best reflect the range of U.S. topic coverage across both subjects, with the "high" category reflecting topics that had been taught to all or nearly all students and the "low" category reflecting topics taught to less than 85 percent of students overall by the time of the assessment.[31]

The cutpoints for the level of topic coverage are defined as follows:

High coverage indicates that the topic

a. is covered in the intended curriculum for all AP and IB courses, or reflects foundational knowledge covered in a prior course,[32] *and*
b. was taught to all or nearly all students (99.0–100 percent of students overall) by the time of the assessment.[33]

Moderate coverage indicates that the topic

a. is at least partially covered in the intended curriculum for all AP and IB courses, or reflects foundational knowledge covered in a prior course, *and*
b. was taught to at least 85 percent of students overall (85.0–98.9 percent).

Low coverage indicates that the topic

a. is *not* covered in the intended curriculum for at least one of the AP or IB courses *or*
b. was taught to less than 85 percent of students overall.

4.3 Analysis and reporting for subsections 5.3 and 6.3: example item performance demonstrating common approaches, misconceptions, and errors

Subsections 5.3 and 6.3 explore U.S. TIMSS Advanced students' common approaches, misconceptions, and errors on a set of 12 example items—6 items each for advanced mathematics and physics.[34]

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[31] Assignment to the level of topic coverage categories were based on unrounded figures.

[32] As described in section 3, topic coverage in the intended curricula is indicated for AP and IB courses but not for other TIMSS Advanced-eligible non-AP, non-IB calculus and physics courses, because the curricula for these latter courses vary across states, districts, and schools. "Covered in a prior course" reflects content expected to have been covered previously based on the AP and IB course guidelines and prerequisites. The Common Core State Standards in Mathematics were used to determine foundational knowledge for AP Calculus courses covered in prerequisite mathematics courses covering algebra and geometry topics.

[33] The percentage of students taught the topics reflects all U.S. TIMSS Advanced students, including those who had taken AP, IB, or other non-AP, non-IB calculus or physics courses. When teacher data were not available for a specific topic, the categorization relied on the curricular data for that topic and the teacher data for closely-related topics (see tables A-3a, 3b, 3c and A-4a, 4b, 4c).

[34] These items are designated by the TIMSS & PIRLS International Study Center for use as examples in the international report as well as by participating countries in their national reports. Example items from TIMSS Advanced 2015 are used in this report with prior permission from the International Association for the Evaluation of Educational Achievement (IEA).

These results again draw upon item-level statistics, including the percent correct for multiple-choice and short constructed-response items worth one score point, the percent full credit and percent partial credit for extended constructed-response items worth two score points, and the percentage of incorrect responses and blank responses. To further differentiate within partially correct or incorrect responses, this report also draws upon diagnostic scoring data obtained using special codes that were included in TIMSS Advanced 2015 constructed-response item scoring guides to track particular misconceptions or errors.[35] The percentage distributions of U.S. students providing specific response types (i.e., those selecting the different incorrect response options for multiple-choice items and the different types of partial and incorrect response types for constructed-response items) are used to report on the prevalence of certain types of errors and misconceptions in the United States. For each example item, the percent correct (for multiple-choice and short constructed-response items) or percent full credit (for extended constructed-response items) are reported for U.S. TIMSS Advanced students overall and by course type. These are compared to the international average item performance (percent correct and percent partial averaged across countries participating in TIMSS Advanced 2015). Finally, patterns in U.S. student performance are described based on the percentage distributions across response types for U.S. TIMSS Advanced students overall and by course type.

[35] The use of diagnostic codes in TIMSS Advanced scoring guides is illustrated for the constructed-response example items presented in sections 5.3 and 6.3. Further description of diagnostic scoring guides and the two-digit scoring system used in TIMSS Advanced is found in appendix C and the TIMSS technical documentation (http://timss.bc.edu/publications/timss/2015-a-methods.html).

Section 5: Advanced Mathematics Results

This section answers the following questions: (1) how did U.S. TIMSS Advanced students, who were described in section 2, perform in advanced mathematics; (2) how did their performance on TIMSS Advanced mathematics items vary across content domains and relate to the level of topic coverage that was described in section 3; and (3) what are their common approaches, misconceptions, and errors in advanced mathematics?

5.1 How did U.S. TIMSS Advanced students perform in advanced mathematics?

U.S. performance in advanced mathematics is described in terms of the average scores and percentages of students reaching the international benchmarks overall and by course type, as well as by student and school characteristics.

Average advanced mathematics performance overall

U.S. students taking the TIMSS Advanced mathematics assessment, who represent 12.5 percent of twelfth-graders overall, had an average score of 485, which was 15 points below the TIMSS Advanced scale centerpoint of 500 (table 5-1). In *calculus,* U.S. students' average score was 504, which was not measurably different from the scale centerpoint, whereas it was 478 in *algebra* and 455 in *geometry* (22 and 45 points below the scale centerpoint, respectively) (table 5-2).

U.S. TIMSS Advanced students' average scores on the cognitive subscales ranged from 480 in *applying* to 488 in *knowing.* On all three subscales (including *reasoning*), average scores were below the scale centerpoint (table 5-3). The remainder of the advanced mathematics results explore the performance of U.S. students in more detail.

Average advanced mathematics performance by course type

In the United States, average advanced mathematics performance varied by course type. Students who had taken one of the AP calculus courses as their highest mathematics course performed, on average, above the U.S. average (497 for AP total vs. 485), whereas those who had not taken an AP calculus course performed, on average, below the U.S. average (448 vs. 485) (table 5-1).[36] The same generalization holds for all three content subscales (table 5-2). However, there were differences based on the level of AP course taken. (See table 5.1 for the percentage of TIMSS Advanced students in each course type described in section 2.)

[36] In tables 5-1, 5-2 and 5-3, performance of students who had taken an AP calculus course (AB or BC) is indicated as "Total AP calculus courses."

Table 5-1. Percentage distribution and average advanced mathematics scores of U.S. TIMSS Advanced students, by course type: 2015

Course type[1]	Percentage of students in the U.S. TIMSS Advanced mathematics population[2]	Average score	Score difference from U.S. average[3]	Score difference from TIMSS Advanced scale centerpoint[4]
All course types (U.S. average)	**100**	**485**	**†**	**-15***
TIMSS Advanced scale centerpoint	**†**	**500**	**15***	**†**
Total AP calculus courses	**76**	**497**	**12***	**-3**
AP Calculus BC	19	556	71*	56*
AP Calculus AB	56	477	-8*	-23*
Total non-AP mathematics courses	**24**	**448**	**-37***	**-52***
IB Mathematics[5]	‡	‡	‡	‡
Other calculus courses[6]	23	447	-38*	-53*

† Not applicable.
‡ Reporting standards not met.
* $p < .05$. Subgroup average score is significantly different from U.S. average/TIMSS Advanced scale centerpoint.
[1] Course type reflects the highest level mathematics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other calculus courses" category only if they have not taken an AP or IB course.
[2] The percentage of students in the mathematics population is based on the weighted counts of students in the sample.
[3] The score difference is calculated by subtracting the subgroup average score from the U.S. average score.
[4] The score difference is calculated by subtracting the subgroup average score from the TIMSS Advanced scale centerpoint. The TIMSS Advanced scale centerpoint represents the international mean of the overall achievement distribution in the first assessment year (1995).
[5] Includes both higher-level and standard-level IB mathematics courses.
[6] Includes other calculus courses (including "honors" and "regents" courses).
NOTE: Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Students who had taken AP Calculus BC (the higher level AP course) tended to pull up the average score for AP calculus students overall (556 vs. 497) (table 5-1). TIMSS Advanced students who had taken AP Calculus BC (19 percent) scored, on average, above the U.S. average overall (556 vs. 485) and on all three content subscales (table 5-2). They also were the only specific course group to score on average above the scale centerpoint overall or on any of the subscales. The score differences between the U.S. students who had taken AP Calculus BC and U.S. students on average ranged from 69 points on the *algebra* subscale to 79 points on the *calculus* subscale.

In contrast, the 56 percent of TIMSS Advanced students who had taken the lower level AP Calculus AB as their highest mathematics course scored, on average, below the U.S. average overall (477 vs. 485) and on the *geometry* subscale (445 vs. 455) and not measurably different from the U.S. average on the other subscales (471 vs. 478 in *algebra* and 497 vs. 504 in *calculus*) (tables 5-1 and 5-2). Like U.S. TIMSS Advanced students overall, the students who had taken AP Calculus AB were below the scale centerpoint overall and on the *algebra* and *geometry* subscales. AP Calculus AB courses had a lower level of coverage of the topics related to *finite and infinite series* in *algebra* (table A-3a) and *vectors* in *geometry* (table A-3c) compared to AP Calculus BC.

Among students taking non-AP mathematics courses, those who had taken other calculus courses (i.e., non-AP, non-IB) scored, on average, below the U.S. average overall (447 vs. 485) and on all three content subscales, with differences ranging from 33 points below average on the *geometry* subscale to 46 points below average on the *calculus* subscale (tables 5-1 and 5-2).[37] Like AP Calculus AB courses, non-AP, non-IB calculus courses also had lower levels of coverage of the topics related to *finite and infinite series* and *vectors* and also had lower levels of coverage of the topics related to *integrals* (tables A-3a, A-3b, and A-3c).[38]

In terms of the cognitive domains (i.e., *knowing, applying,* and *reasoning*), the patterns in performance by course type were generally similar to those for the content domains. Students who had taken an AP calculus course as their highest course, on average, performed above the U.S. average on all three cognitive subscales, whereas those who had not taken an AP course performed, on average, below the U.S. average on all three cognitive subscales (table 5-3). Again, however, the results were driven by the strong performance of AP Calculus BC students—who scored from 71 points higher in *applying* and *reasoning* to 76 points higher in *knowing* than U.S. TIMSS Advanced students on average. These students also scored, on average, above the scale centerpoint on all three cognitive subscales.

In contrast, students who had taken AP Calculus AB were not measurably different from U.S. students, on average, on any of the cognitive subscales. They also scored below the scale centerpoints on all three. Students who had taken other calculus courses (i.e., non-AP, non-IB) as their highest mathematics course scored lower on average on all three cognitive subscales than the U.S. averages, ranging from 38 points (in *reasoning*) to 43 points (in *knowing*) lower, and also lower than the scale centerpoint.

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[37] Data on students in IB Mathematics are not reported separately because reporting standards were not met for this group (i.e., the sample size was less than 62 students).

[38] The teacher data show that lower percentages of U.S. TIMSS Advanced students were taught these topics (from 81 to 95 percent of students) than were taught any other advanced mathematics topics (at least 99 percent of students) (table A-3a, A-3b, and A-3c).

Table 5-2. Average advanced mathematics scores of U.S. TIMSS Advanced students, by content domain and course type: 2015

Course type[1]	Algebra			Calculus			Geometry		
	Average score	Score difference from U.S. average[2]	Score difference from TIMSS Advanced scale centerpoint[3]	Average score	Score difference from U.S. average[2]	Score difference from TIMSS Advanced scale centerpoint[3]	Average score	Score difference from U.S. average[2]	Score difference from TIMSS Advanced scale centerpoint[3]
All course types (U.S. average)	**478**	**†**	**-22***	**504**	**†**	**4**	**455**	**†**	**-45***
TIMSS Advanced scale centerpoint	**500**	**22***	**†**	**500**	**-4**	**†**	**500**	**45***	**†**
Total AP calculus courses	**490**	**12***	**-10**	**519**	**15***	**19***	**465**	**10***	**-35***
AP Calculus BC	547	69*	47*	583	79*	83*	526	71*	26*
AP Calculus AB	471	-7	-29*	497	-7	-3	445	-10*	-55*
Total non-AP mathematics courses	**440**	**-38***	**-60***	**459**	**-46***	**-41***	**423**	**-32***	**-77***
IB Mathematics[4]	‡	‡	‡	‡	‡	‡	‡	‡	‡
Other calculus courses[5]	439	-39*	-61*	458	-46*	-42*	422	-33*	-78*

† Not applicable.
‡ Reporting standards not met.
* $p < .05$. Subgroup average score is significantly different from U.S. average score/TIMSS Advanced scale centerpoint.
[1] Course type reflects the highest level mathematics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the “other calculus courses” category only if they have not taken an AP or IB course.
[2] The score difference is calculated by subtracting the subgroup average score from the U.S. average score.
[3] The score difference is calculated by subtracting the subgroup average score from the TIMSS Advanced scale centerpoint. The TIMSS Advanced scale centerpoint represents the international mean of the overall achievement distribution in the first assessment year (1995).
[4] Includes both higher-level and standard-level IB mathematics courses.
[5] Includes other calculus courses (including “honors” and “regents” courses).
NOTE: TIMSS Advanced produces separate subscales for each content domain. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table 5-3. Average advanced mathematics scores of U.S. TIMSS Advanced students, by cognitive domain and course type: 2015

Course type[1]	Knowing			Applying			Reasoning		
	Average score	Score difference from U.S. average[2]	Score difference from TIMSS Advanced scale centerpoint[3]	Average score	Score difference from U.S. average[2]	Score difference from TIMSS Advanced scale centerpoint[3]	Average score	Score difference from U.S. average[2]	Score difference from TIMSS Advanced scale centerpoint[3]
All course types (U.S. average)	**488**	**†**	**-12***	**480**	**†**	**-20***	**484**	**†**	**-16***
TIMSS Advanced scale centerpoint	**500**	**12***	**†**	**500**	**20***	**†**	**500**	**16***	**†**
Total AP calculus courses	**502**	**14***	**2**	**492**	**13***	**-8**	**497**	**12***	**-3**
AP Calculus BC	564	76*	64*	551	71*	51*	556	71*	56*
AP Calculus AB	481	-7	-19*	472	-7	-28*	477	-8	-23*
Total non-AP mathematics courses	**445**	**-42***	**-55***	**441**	**-39***	**-59***	**447**	**-37***	**-53***
IB Mathematics[4]	‡	‡	‡	‡	‡	‡	‡	‡	‡
Other calculus courses[5]	445	-43*	-55*	440	-39*	-60*	446	-38*	-54*

† Not applicable.
‡ Reporting standards not met.
* *p* < .05. Subgroup average score is significantly different from U.S. average score/TIMSS Advanced scale centerpoint.
[1] Course type reflects the highest level mathematics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the “other calculus courses” category only if they have not taken an AP or IB course.
[2] The score difference is calculated by subtracting the subgroup average score from the U.S. average score.
[3] The score difference is calculated by subtracting the subgroup average score from the TIMSS Advanced scale centerpoint. The TIMSS Advanced scale centerpoint represents the international mean of the overall achievement distribution in the first assessment year (1995).
[4] Includes both higher-level and standard-level IB mathematics courses.
[5] Includes other calculus courses (including “honors” and “regents” courses).
NOTE: TIMSS Advanced produces separate subscales for each cognitive domain. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

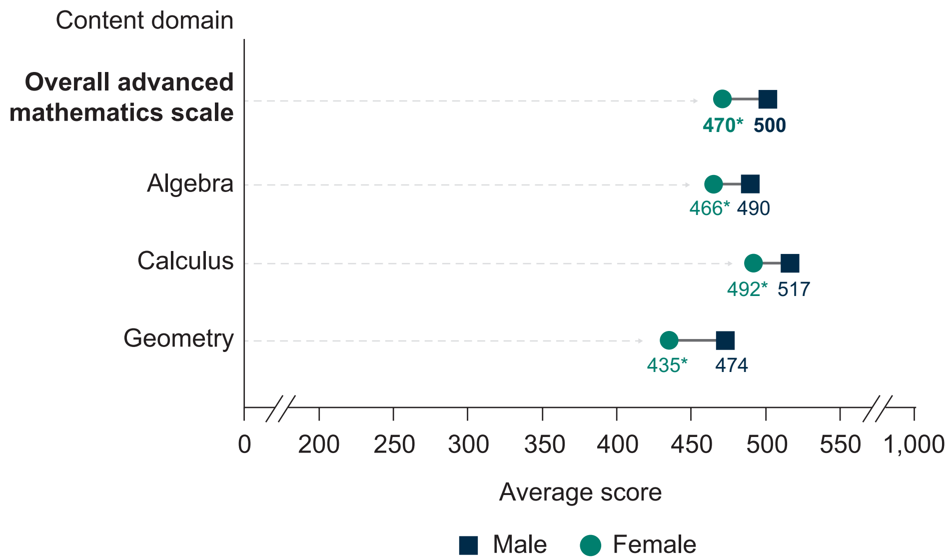
Average advanced mathematics performance by student and school characteristics

Sex

In the United States, males outperformed females in advanced mathematics overall and on all three content subscales (figure 5-1 and table B-7). U.S. males' average advanced mathematics score was 500, 30 score points higher than females' average score of 470. On the subscales, male-female average differences were 24 points in *algebra,* 25 points in *calculus,* and 39 points in *geometry.*

These differences may be related to the coursetaking patterns observed in section 3, which showed that a somewhat higher percentage of males than females (21 vs. 17 percent) had taken the more rigorous AP Calculus BC as their highest mathematics course (figure 3-1).

Figure 5-1. Average advanced mathematics scores of U.S. TIMSS Advanced students, by content domain and sex: 2015



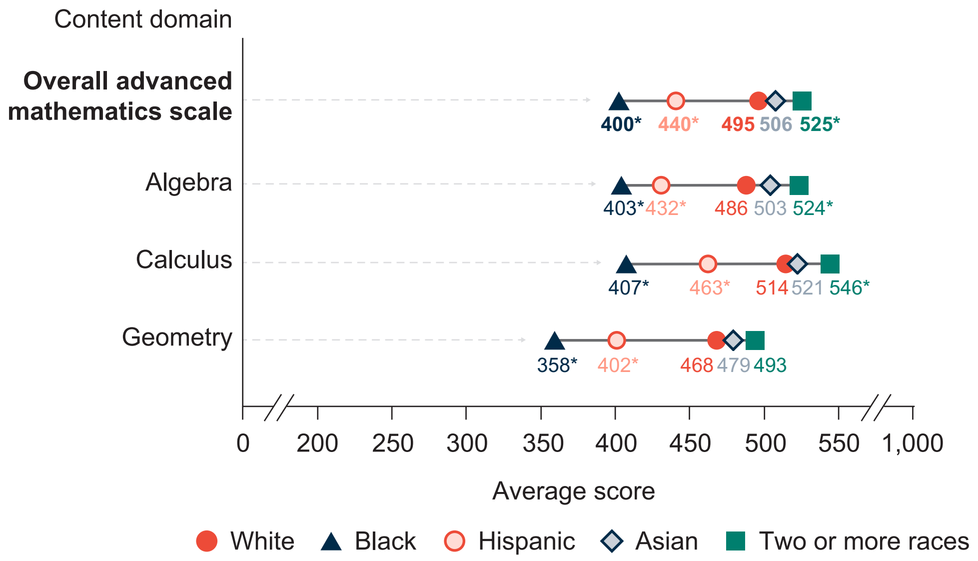
* $p < .05$. Female average score is significantly different from the male average score.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Race/ethnicity

In the United States, average performance in advanced mathematics differed across racial/ethnic groups overall and on all three content subscales (figure 5-2 and table B-8). The average advanced mathematics score for White students was 495, which was higher than the average scores for Black (400) and Hispanic (440) students, but lower than the average score for students of Two or more races (525) and not measurably different from the average score for Asian students (506).[39] White students also scored higher, on average, than Black and Hispanic students on the three content subscales, while White students scored lower, on average, than students of Two or more races in *algebra* and *calculus*. Across content domains, the differences in Hispanic students' average scores and the U.S. average ranged from 41 points (in *calculus*) to 52 points (in *geometry*), and for Black students these average differences ranged from 75 points (in *algebra*) to 97 points (in *calculus* and *geometry*), which is nearly one standard deviation.

As with performance differences by sex, differences by racial/ethnic groups may be related to the coursetaking patterns observed in section 3 (figure 3-2 and table B-2). These patterns indicated that lower percentages of Black and Hispanic students had taken AP Calculus BC than White students had (by 13 and 7 percentage points, respectively). Conversely, they indicate that there were higher percentages of Black and Hispanic students who had taken AP Calculus AB than White students (by 20 and 21 percentage points, respectively).

Figure 5-2. Average advanced mathematics scores of U.S. TIMSS Advanced students, by content domain and race/ethnicity: 2015



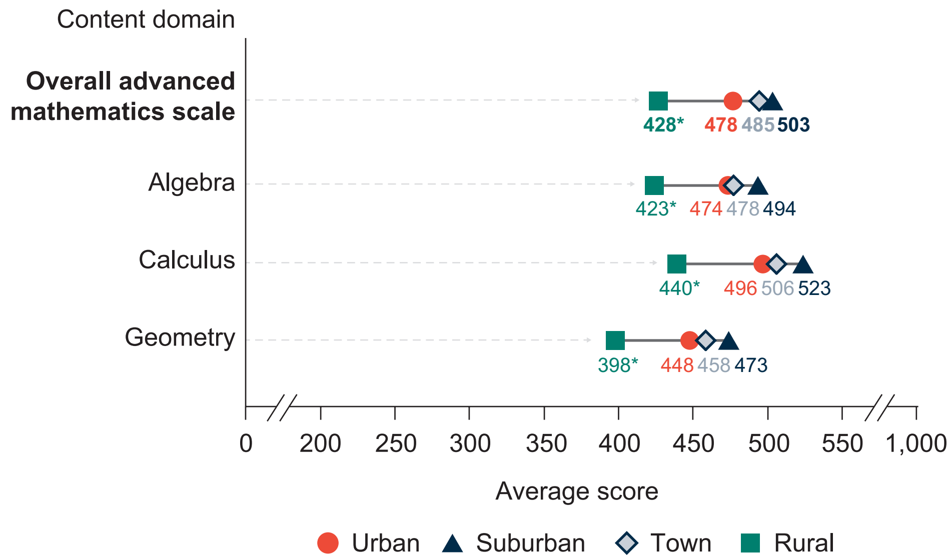
* *p* < .05. Subgroup average score is significantly different from the average score of White students.
NOTE: Reporting standards were not met for Native Hawaiian/Pacific Islander and Native American/Alaska Native students (sample size < 62). Data for these students are not shown separately in the figure, but are included in the U.S. average. Black includes African American and Hispanic includes Latino. Racial categories exclude Hispanic origin.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

[39] Data for Native Hawaiian/Pacific Islander and American Indian/Alaska Native are not reported separately because reporting standards were not met for these groups (i.e., the sample sizes were less than 62 students each).

School locale

In the United States, suburban students outperformed rural students in advanced mathematics overall and on all three content subscales (figure 5-3 and table B-9). Suburban students' average advanced mathematics score was 503 points, 75 score points higher than rural students' average score of 428 points. On the subscales, suburban-rural average differences were 71 points in *algebra,* 83 points in *calculus,* and 76 points in *geometry.* As described in section 3 (figure 3-3), a higher percentage of suburban students (28 percent) had taken AP Calculus BC than rural students (5 percent).

Figure 5-3. Average advanced mathematics scores of U.S. TIMSS Advanced students, by content domain and school locale: 2015



* *p* < .05. Subgroup average score is significantly different from the average score of suburban students.
NOTE: Urban is defined as territories inside an urbanized area and inside a principal city. Suburban is defined as territories inside an urbanized area but outside a principal city. Town is defined as territories inside an urban cluster but outside an urbanized area. Rural is defined as territories that are not in an urbanized area or urban cluster. For additional information, see https://nces.ed.gov/programs/edge/docs/NCES_LOCALE_USERSMANUAL_2016012.pdf. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Percentage of students reaching TIMSS Advanced international benchmarks in advanced mathematics

This section describes the percentage of students reaching each of the three TIMSS Advanced international benchmarks: *Advanced, High,* and *Low* (exhibit 5-1). The percentage reaching each benchmark includes the students who reached any higher benchmarks.

Exhibit 5-1. Description of TIMSS Advanced international benchmarks in advanced mathematics: 2015

Benchmarks	Descriptions
Advanced (625)	Students demonstrate a thorough understanding of concepts, mastery of procedures, and mathematical reasoning skills. They can solve problems in complex contexts in algebra, calculus, geometry, and trigonometry. • In *algebra,* students can reason with functions to solve pure mathematical problems; demonstrate a facility with complex numbers and permutations; and find sums of algebraic and infinite geometric series. • In *calculus,* students demonstrate a thorough understanding of continuity and differentiability; solve problems about optimization in different contexts and justify their solutions; and use definite integrals to calculate the area between the curves. • In *geometry,* students use geometric reasoning to solve complex problems; properties of vectors to express relationships among vectors; and trigonometric properties, including the sine and cosine rules, to solve nonroutine problems about geometric figures.
High (550)	Students can apply a broad range of mathematical concepts and procedures in algebra, calculus, geometry, and trigonometry to analyze and solve multistep problems set in routine and nonroutine contexts. • In *algebra,* students analyze and solve problems, including problems set in a practical context and those requiring interpretation of information related to functions and graphs of functions; determine a sum of an arithmetic sequence and solve quadratic and other inequalities; and simplify logarithmic expressions and multiply complex numbers. • In *calculus,* students have a basic understanding of continuity and differentiability. They can analyze equations of functions and graphs of functions and relate the graphs of functions to graphs and signs of their first and second derivatives. They also show some conceptual understanding of definite integrals. • In *geometry,* students can use trigonometric properties to solve a variety of problems involving trigonometric functions and geometric figures and use the Cartesian plane to solve problems, identify a vector perpendicular to a given vector, and prove that a quadrilateral given in the coordinate system is a parallelogram.
Intermediate (475)	Students demonstrate a basic knowledge of concepts and procedures in algebra, calculus, and geometry to solve routine problems. • In *algebra,* students apply and transform a formula to solve a word problem; determine a term in a geometric sequence and analyze a proposed solution of a simple logarithmic equation; and recognize a graph of the absolute value of a function and identify and evaluate composite functions. • In *calculus,* students find the derivative of exponential, trigonometric, and simple rational functions; find limits of rational and exponential functions; and make connections between the sign of the derivative and the graph of a function. • In *geometry,* students use their knowledge of basic properties of geometric figures and the Pythagorean theorem to solve problems; and add and subtract vectors in coordinate form.

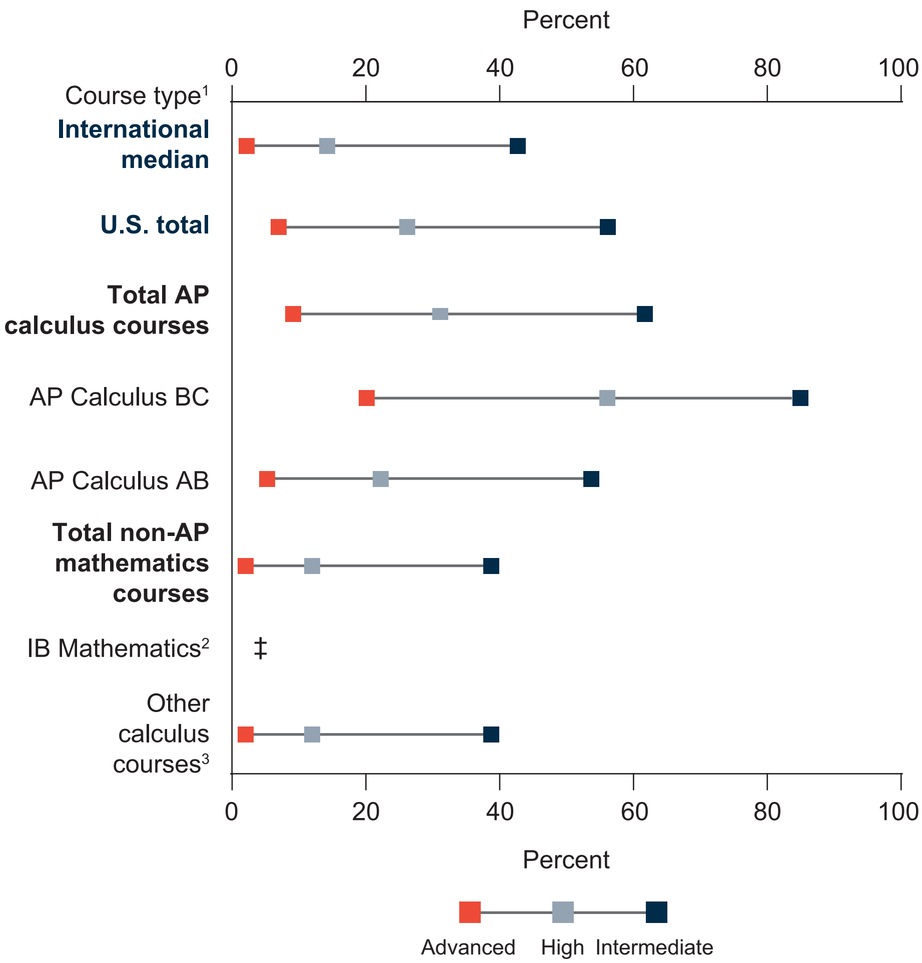
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science (TIMSS) Advanced, 2015.

Overall, 7 percent of U.S. students reached the *Advanced* international benchmark in advanced mathematics, 26 percent reached the *High* benchmark, and 56 percent reached the *Intermediate* benchmark (figure 5-4). The percentage of U.S. students reaching each of these benchmarks was higher than the international medians (2, 14, and 43 percent, respectively).[40]

Course type

As with average scores, the percentages of U.S. TIMSS Advanced students reaching the international benchmarks in advanced mathematics varied by course type (figure 5-4). Higher percentages of U.S. students who had taken an AP calculus course reached each of the three international benchmarks in advanced mathematics than did U.S. TIMSS Advanced students overall (i.e., U.S. total), although there were differences by AP course level.

Figure 5-4. Percentage of U.S. TIMSS Advanced students reaching TIMSS Advanced international benchmarks in advanced mathematics, by course type: 2015



Course type[1]	Advanced (625)	High (550)	Intermediate (475)
International median	**2***	**14***	**43***
U.S. total	**7**	**26**	**56**
Total AP calculus courses	**9***	**31***	**62***
AP Calculus BC	20*	56*	85*
AP Calculus AB	5*	22*	54
Total non-AP mathematics courses	**2***	**12***	**39***
IB Mathematics[2]	‡	‡	‡
Other calculus courses[3]	2*	12*	39*

‡ Reporting standards not met (sample size < 62).
* *p* < .05. Percentage is statistically different from the U.S. total percentage at the same benchmark.
[1] Course type reflects the highest level mathematics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other calculus courses" category only if they have not taken an AP or IB course.
[2] Includes higher-level and standard-level IB mathematics courses.
[3] Includes other calculus courses (including "honors" and "regents" courses).
NOTE: The percentages of U.S. TIMSS Advanced students whose highest course was an AP calculus course who reached each of the benchmarks were higher than the international medians; this was true for both AP Calculus BC and AP Calculus AB students.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

[40] The international medians represent the middle percentage reaching each benchmark among the countries participating in TIMSS Advanced 2015.

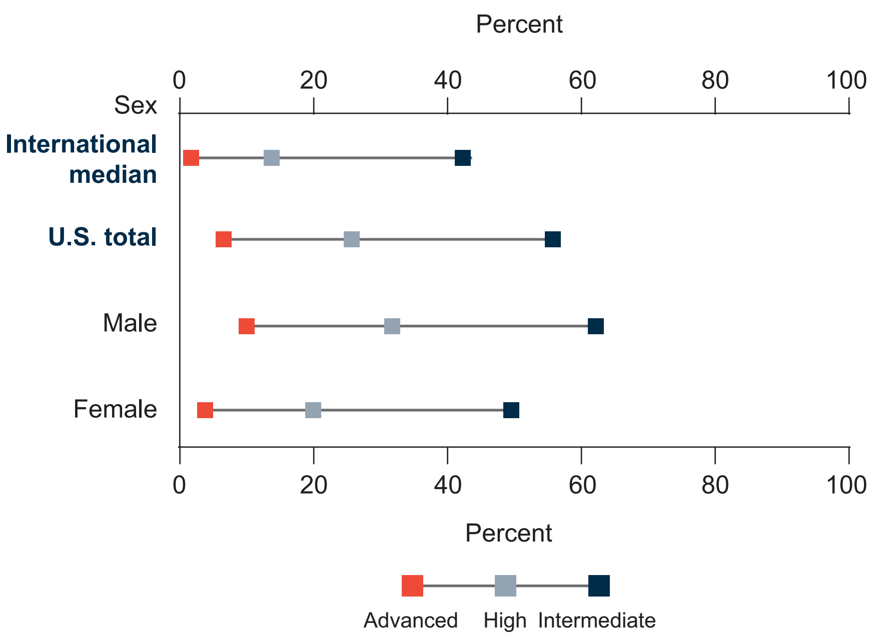
Among U.S. students who had taken AP Calculus BC as their highest mathematics course, 20 percent of students reached the *Advanced* benchmark, 56 percent reached the *High* benchmark, and 85 percent reached the *Intermediate* benchmark. These percentages were higher than the U.S. total (7, 26, and 56 percent, respectively) and the international medians (2, 14, and 43 percent, respectively). In contrast, among students who had taken AP Calculus AB, 5 percent of students reached the *Advanced* benchmark, 22 percent reached the *High* benchmark, and 54 percent reached the *Intermediate* benchmark. These percentages were lower than the U.S. total reaching the *Advanced* and *High* benchmarks and not measurably different from the U.S. total reaching the *Intermediate* benchmark, but higher than the international medians for each benchmark.

Among U.S. students who had taken a non-AP course as their highest mathematics course, lower percentages of students reached the *Advanced* (2 percent), *High* (12 percent) and *Intermediate* benchmarks (39 percent) than the U.S. total.[41]

Sex

The percentages of U.S. TIMSS Advanced students reaching the international benchmarks in advanced mathematics varied by sex (figure 5-5). Ten percent of male students reached the *Advanced* benchmark, 32 percent reached the *High* benchmark, and 63 percent reached the *Intermediate* benchmark; these percentages were higher than the U.S. total (7, 26, and 56 percent, respectively). In contrast, 4 percent of female students reached the *Advanced* benchmark, 20 percent reached the *High* benchmark, and 50 percent reached the *Intermediate* benchmark; these percentages were lower than the U.S. total. However, for females (as well as males), the percentages reaching each of the benchmarks were higher than the respective international medians.

Figure 5-5. Percentage of U.S. TIMSS Advanced students reaching TIMSS Advanced international benchmarks in advanced mathematics, by sex: 2015



Advanced (625)	High (550)	Intermediate (475)
2*	**14***	**43***
7	**26**	**56**
10*	32*	63*
4*	20*	50*

* $p < .05$. Percentage is statistically different from the U.S. total percentage at the same benchmark.
NOTE: The percentages of U.S. TIMSS Advanced students reaching each benchmark were higher than the international medians for both males and females.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

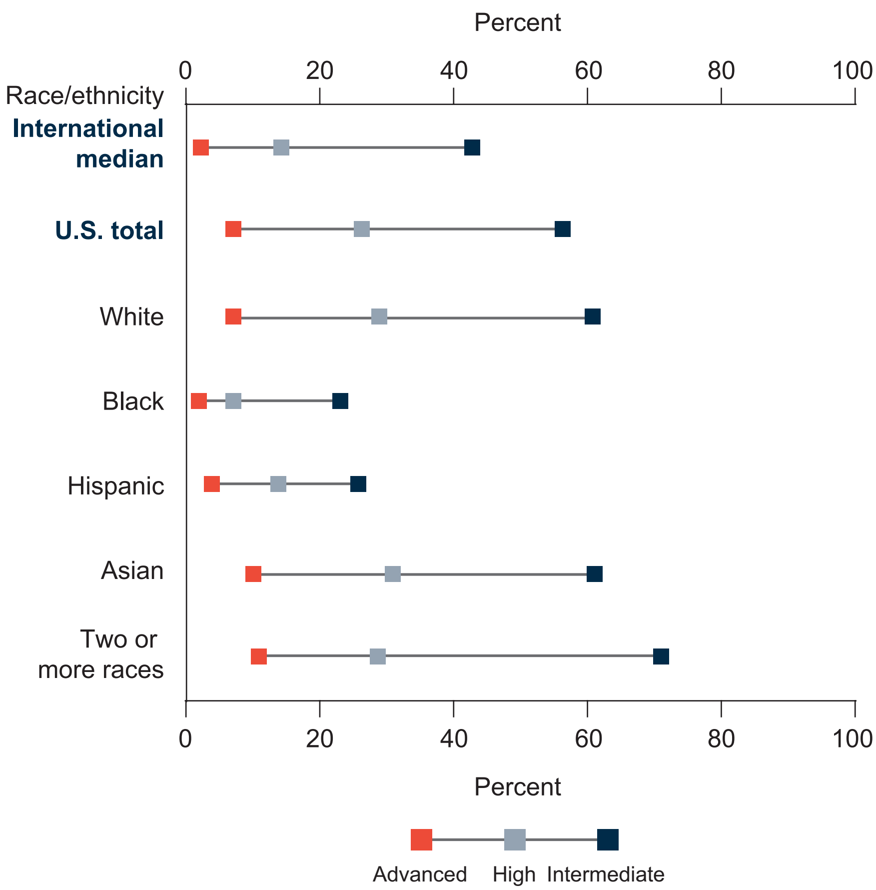
[41] This was also the case for the subgroup of students whose highest course was, specifically, an "other calculus course" (i.e., non-AP, non-IB). Data for non-AP students in IB Mathematics are not reported separately because reporting standards were not met for this group (i.e., the sample size was less than 62 students).

Race/ethnicity

Differences were also seen across racial/ethnic groups (figure 5-6). Higher percentages of White U.S. TIMSS Advanced students reached the *High* (29 percent) and *Intermediate* (61 percent) international benchmarks in advanced mathematics than the U.S. total (26 and 56 percent, respectively). The 77 percent of students of Two or more races who reached the *Intermediate* benchmark also was higher than the U.S. total. White and Asian students also exceeded the international medians for each benchmark, as did students of Two or more races for the *High* and *Intermediate* benchmarks.

In contrast, lower percentages of Black students reached each of the three benchmarks (1, 7, and 23 percent at *Advanced, High,* and *Intermediate,* respectively) than the U.S. total, and lower percentages of Hispanic students reached the *High* (14 percent) and *Intermediate* (36 percent) benchmarks than the U.S. total. The percentages of Black students reaching the *High* and *Intermediate* benchmarks were also lower than the international medians.

Figure 5-6. Percentage of U.S. TIMSS Advanced students reaching TIMSS Advanced international benchmarks in advanced mathematics, by race/ethnicity: 2015



Race/ethnicity	Advanced (625)	High (550)	Intermediate (475)
International median	**2***	**14***	**43***
U.S. total	**7**	**26**	**56**
White	7	29*	61*
Black	1*	7*	23*
Hispanic	4	14*	36*
Asian	10	31	61
Two or more races	11	39	77*

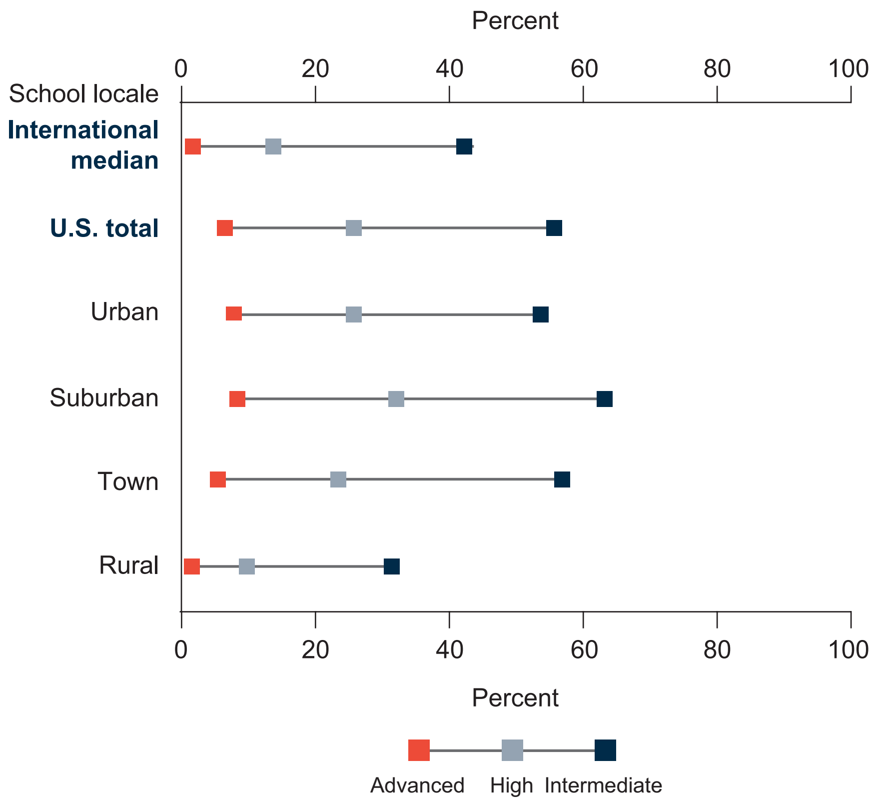
* *p* < .05. Percentage is statistically different from the U.S. total percentage at the same benchmark.
NOTE: Reporting standards were not met for Native Hawaiian/Pacific Islander and Native American/Alaska Native students (sample size < 62). Data for these students are not shown separately in the figure, but are included in the U.S. average. Black includes African American and Hispanic includes Latino. Racial categories exclude Hispanic origin. The percentages of White and Asian U.S. TIMSS Advanced students reaching each of the benchmarks were higher than the international medians. The percentages of students of Two or more races reaching the *High* and *Intermediate* benchmarks were also higher than the international medians. In contrast, the percentages of Black students reaching the *High* and *Intermediate* benchmarks were lower than the international medians.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

School locale

Differences were also seen by school locale (figure 5-7). Higher percentages of suburban students reached the *High* (33 percent) and *Intermediate* (64 percent) international benchmarks in advanced mathematics than the U.S. total (26 and 56 percent, respectively). The percentages of suburban students (along with town students) also exceeded the international medians for each benchmark, as did urban students for the *Intermediate* benchmark.

In contrast, lower percentages of rural students reached any of the benchmarks than the U.S. total, with 1, 10, and 32 percent of rural students reaching the *Advanced, High,* and *Intermediate* benchmarks, respectively.

Figure 5-7. Percentage of U.S. TIMSS Advanced students reaching TIMSS Advanced international benchmarks in advanced mathematics, by school locale: 2015



School locale	Advanced (625)	High (550)	Intermediate (475)
International median	**2***	**14***	**43***
U.S. total	**7**	**26**	**56**
Urban	8	26	54
Suburban	9	33*	64*
Town	6	24	57
Rural	1*	10*	32*

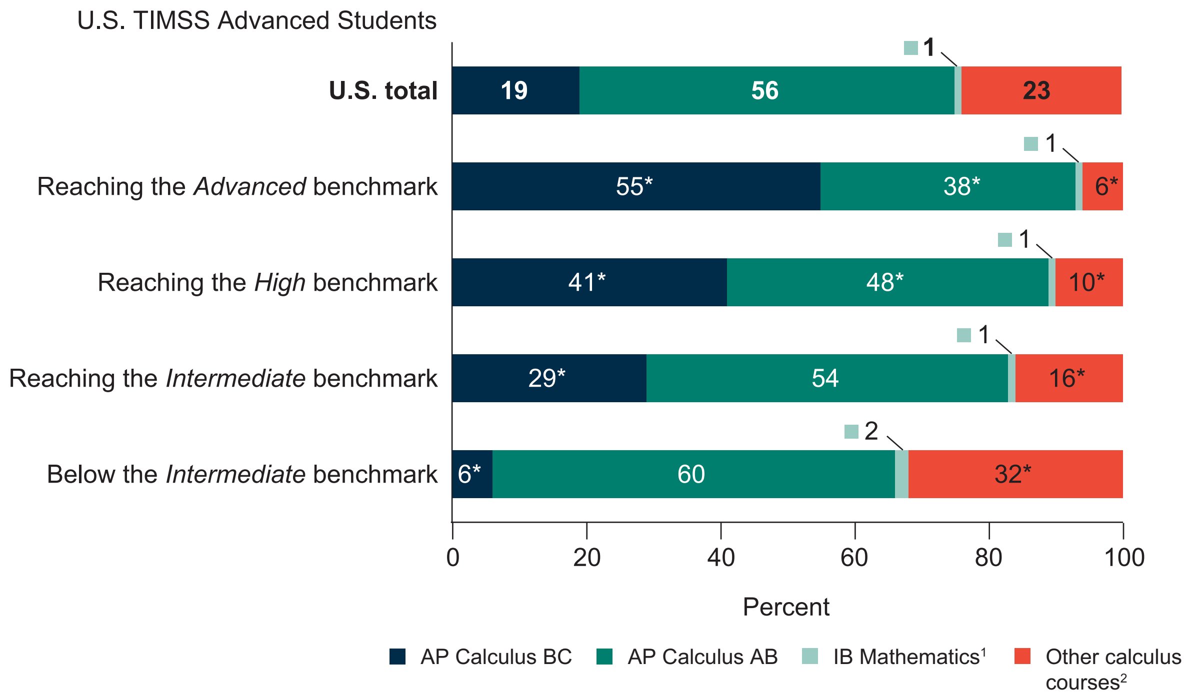
* $p < .05$. Percentage is statistically different from the U.S. total percentage at the same benchmark.
NOTE: Urban is defined as territories inside an urbanized area and inside a principal city. Suburban is defined as territories inside an urbanized area but outside a principal city. Town is defined as territories inside an urban cluster but outside an urbanized area. Rural is defined as territories that are not in an urbanized area or urban cluster. For additional information, see https://nces.ed.gov/programs/edge/docs/NCES_LOCALE_USERSMANUAL_2016012.pdf. The percentages of U.S. TIMSS Advanced suburban and town students were higher than the international medians at all three benchmarks. The percentage of urban students was higher than the international median at the *Intermediate* benchmark.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Coursetaking patterns of students reaching each international benchmark in advanced mathematics

To further understand U.S. performance in advanced mathematics, this subsection compares the coursetaking patterns of students at different achievement levels to the coursetaking patterns of the TIMSS Advanced population overall (i.e., U.S. total) (figure 5-8 and table B-10). In the overall population (table 2-2), the majority of students (56 percent) had taken the lower level AP Calculus AB course compared to just 19 percent who had taken the higher level AP Calculus BC, and another 24 percent who had taken non-AP courses (i.e., IB Mathematics or other calculus courses). This was not the coursetaking pattern observed for the subsets of students who reached each of the international benchmarks in advanced mathematics.

Among students who reached the *Advanced* international benchmark in advanced mathematics (7 percent of U.S. TIMSS Advanced students overall) (figure 5-4), 55 percent had taken the higher level AP Calculus BC, 38 percent who had taken AP Calculus AB, and very small percentages had taken non-AP courses (1 percent taking IB Mathematics, and 6 percent taking other calculus courses) (figure 5-8). This distribution across the different course types differed markedly from the average distribution for U.S. TIMSS Advanced students overall. Specifically, the 55 percent of students reaching the *Advanced* benchmark who had taken AP Calculus BC was higher than the U.S. total (19 percent), whereas the percentages of students reaching the *Advanced* benchmark who had taken AP Calculus AB or other non-AP, non-IB courses were lower than the U.S. totals (56 and 23 percent, respectively).

Figure 5-8. Percentage distribution of U.S. TIMSS Advanced students reaching each TIMSS Advanced international benchmark in advanced mathematics compared to the U.S. total, by course type: 2015



* *p* < .05. Percentage is statistically different from the U.S. total percentage.

¹ Includes higher-level and standard-level International Baccalaureate (IB) mathematics courses.

² Includes other calculus courses (including “honors” and “regents” courses).

NOTE: Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or IB course, they are included in the AP or IB category. Students are included in the “other calculus courses” category only if they have not taken an AP or IB course. This figure compares the coursetaking patterns of students reaching each benchmark with the coursetaking patterns of U.S. students overall. Detail may not sum to totals because of rounding.

SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

The patterns observed among students who reached the *High* and *Intermediate* benchmarks (26 and 56 percent of U.S. TIMSS Advanced students overall, respectively) were generally similar to those among the *Advanced* students. For example, the percentages of students reaching the *High* and *Intermediate* benchmarks who had taken AP Calculus BC were higher than the U.S. totals, and the percentages who had taken other non-AP, non-IB courses were lower.

In contrast, among students who did not reach the *Intermediate* benchmark (44 percent of U.S. TIMSS Advanced students overall), a higher percentage (32 percent) had taken other non-AP, non-IB calculus courses and a lower percentage (6 percent) had taken AP Calculus BC than the U.S. totals.

5.2 How did U.S. students' performance on TIMSS Advanced mathematics items vary across content domains and relate to the level of topic coverage?

This subsection examines U.S. TIMSS Advanced students' performance on individual advanced mathematics items. The first part describes the average U.S. performance across items by content domain and broad topic area within each content domain. The second part describes how item performance relates to the level of topic coverage in the intended and implemented curriculum. The following textbox summarizes the item statistics used to produce the results in this section and relates these to the specific tables and figures shown. (See section 4 for a more complete description of the analyses used to produce the results shown in this section.)

Item Statistics Used in Section 5.2

Percent correct is the weighted percentage of students receiving full or partial credit on each item. This report shows item percent correct for each item within the individual TIMSS Advanced mathematics topics.

Average percent correct is the percent correct averaged across items. In this report, averages are computed for different item sets to provide results for

a. advanced mathematics overall (across all advanced mathematics items),
b. each content domain, and
c. each broad topic area within a content domain.

Relative performance by content domain and topic area is determined by comparison to the average percent correct for mathematics overall.

Average U.S. performance on advanced mathematics items by content domain and broad topic area within each content domain

In the United States, the average percent correct overall (i.e., across all TIMSS Advanced mathematics items) was 44 percent (table 5-4). This percentage is used as the reference point for identifying content domains and topics areas in which item performance was relatively higher or lower than advanced mathematics overall (i.e., those with a higher or lower average percent correct than the overall 44 percent).

Looking across content domains, the U.S. average percent correct was relatively higher on the items in *algebra* (46 percent correct) and *calculus* (47 percent correct) and relatively lower on the items in *geometry* (38 percent correct), compared to advanced mathematics items overall (44 percent correct). Within content domains, there was also variation in the average item percent correct across topic areas:

- In *algebra,* the U.S. average percent correct for two topic areas—*functions* (53 percent) and *equations and inequalities* (45 percent)—was higher than for advanced mathematics overall (44 percent). In contrast, in the topic area *expressions and operations,* U.S. students had a lower average percent correct (42 percent) than in advanced mathematics overall.
- In *calculus,* the U.S. average percent correct for all three topic areas—*limits* (49 percent), *derivatives* (47 percent), and *integrals* (46 percent)—was higher than for advanced mathematics overall.
- In *geometry,* the U.S. average percent correct for both topic areas—*noncoordinate and coordinate geometry* (40 percent) and *trigonometry* (36 percent)—was lower than for advanced mathematics overall.

Table 5-4. Average U.S. performance on TIMSS Advanced mathematics items, by content domain and topic area: 2015

Content domain and topic area	Number of items	U.S. average percent correct[1]
Advanced mathematics (all content domains and topic areas)	**101**	**44**
Algebra	**37**	**46** ▲
Expressions and operations	13	42 ▼
Equations and inequalities	15	45 ▲
Functions	9	53 ▲
Calculus	**34**	**47** ▲
Limits	8	49 ▲
Derivatives	18	47 ▲
Integrals	8	46 ▲
Geometry	**30**	**38** ▼
Noncoordinate and coordinate geometry	16	40 ▼
Trigonometry	14	36 ▼

▲ Content domain/topic area average is higher than advanced mathematics (all content domains and topic areas) average ($p < .05$).

▼ Content domain/topic area average is lower than advanced mathematics (all content domains and topic areas) average ($p < .05$).

[1] Average percent correct is the weighted percentage of students receiving full credit (1 point on multiple-choice items, 1 point on short constructed-response items, and 2 points on extended constructed-response items) or partial credit (1 point on extended constructed-response items), averaged across items.

SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.(TIMSS) Advanced, 2015.

How U.S. performance on TIMSS Advanced mathematics items relates to the level of topic coverage

The prior results showed how U.S. student performance on advanced mathematics items varied across the content domains and broad topic areas within each content domain. The results presented here show how student performance on the set of items in each individual topic relates to the level of topic coverage in their advanced mathematics courses (from section 3).

To examine how U.S. student performance relates to students' opportunity to learn, TIMSS Advanced topics are organized into three levels of topic coverage in the United States—high, moderate, and low—based on both the intended and implemented curriculum data (tables A-3a, A-3b, and A-3c).[42] For each topic, a graph is included that shows the distribution of percent correct for the set of items in that topic and identifies the item type: multiple-choice, short constructed-response, or extended constructed-response (figure 5-9 and table B-11). These distributions show how tightly clustered or how widely spread items are in terms of percent correct and thus indicates the range of U.S. performance across the specific set of items developed to assess each topic. Even for topics reported as taught to most students, performance across the individual items within and across topics still varies for a variety of reasons, including how challenging the content covered by the topic is, the breadth of content covered by the topic, and the specific requirements of the items and their contexts.

Over half of the advanced mathematics topics (17 of 23) were in the high coverage category. These topics spanned all content domains, including seven of nine topics in *algebra,* five topics in *calculus* related to *limits* and *derivatives* and five of six topics in *geometry.* Three advanced mathematics topics were in the moderate coverage category—the two *calculus* topics from the topic area of *integrals* and one *algebra* topic involving arithmetic and geometric series. Three topics—one from each content domain—were in the low coverage category.

The sections below describe the range of U.S. item performance on TIMSS Advanced mathematics topics in the high, moderate, and low coverage categories.

High level of topic coverage

All seven of the high-coverage *algebra* topics were reported by teachers to be universally taught,[43] and all were covered in IB mathematics course curricula or considered foundational knowledge for AP calculus covered in prerequisite courses (table A-3a). Still, there was a range of performance across the items assessing these *algebra* topics. For example, item performance in the topic of *equivalent representations of functions* tended to be on the higher end of the performance range (54 to 76 percent correct) whereas item performance in the topics of *using equations and inequalities to solve contextual problems* (36 to 61 percent correct); *properties of functions, including domain and range* (26 to 52 percent correct); and *linear and quadratic equations and inequalities* (26 to 58 percent) tended to be in the mid-performance range of U.S. students on the TIMSS Advanced mathematics items (figure 5-9). With the exception of one item which asks students to find the value of a compound exponent expression (71 percent correct), item performance in the topic of *evaluating algebraic expressions* did not exceed 50 percent correct.

[42] See definitions of the high, moderate, and low categories for the level of topic coverage in section 4 and in the notes on figure 5-9. Percent correct reflects the weighted percentage of students receiving full or partial credit on each item.

[43] The one exception is *using equations and inequalities to solve contextual problems,* which was not asked on the teacher questionnaire and thus there are no data on the percentage of students taught the topic. This topic was classified at the same level of topic coverage as other topics on equations and inequalities.

All five of the high-coverage *calculus* topics were covered across the TIMSS Advanced-eligible courses, including both AP calculus and IB mathematics courses, with teachers reporting that all or nearly all students had been taught these topics (table A-3b).[44] Performance on items in *differentiation of functions* tended to reflect this high coverage level and be on the higher end of the performance range (48 to 71 percent correct), as did performance on three of the four items in *limits of functions* (56 to 59 percent correct) (figure 5-9). Performance on the fourth item in *limits of functions* was 37 percent correct, the lower value of which may have been because it required students to evaluate the limit of a given function, where the input value (x) approaches a variable rather than a numerical value. In the other three items, the input value approaches infinity. On the three items in the *using derivatives to solve problems* topic, performance tended to be on the lower end of the range (16 to 31 percent correct). Of the high-coverage *calculus* topics, item performance varied most widely on topics related to *using first and second derivatives to determine slope and local extrema* (21 to 62 percent correct) and *using first and second derivatives to sketch and interpret graphs of functions* (35 to 65 percent), by 41 and 30 percentage points, respectively. Looking across the *derivatives* topics, it seems that many students were able to answer straightforward procedural items correctly but fewer were able to apply their knowledge of derivatives in contextualized problem situations.

Among the high-coverage *geometry* topics, all five are considered foundational knowledge for AP calculus and would be expected to have been covered in prerequisite courses. For IB mathematics, the two topics involving trigonometric functions are covered in the course curricula, and the other three topics are considered foundational knowledge (table A-3c). In general, these high-coverage topics were reported by teachers to be taught to all or nearly all U.S. students.[45] High-coverage topics in *geometry,* however, tended to have the widest variation in performance among the content domains. For example, performance on items in the topic of *trigonometric properties of triangles* (13 to 58 percent correct) and *solving problems involving trigonometric functions* (1 to 44 percent correct) both ranged at least 43 percentage points (figure 5-9).[46] Item performance on the *properties of geometric figures in two and three dimensions* topic also ranged widely (21 to 74 percent, or 53 percentage points), though excluding the highest-performing item shows a tighter overall range for the remaining five items (21 to 45 percent, or 24 percentage points). Item performance on the topic of *trigonometric functions and their graphs* (37 to 61 percent) tended to be at the higher end of the range, as did item performance on the topic of *using coordinate geometry to solve problems in two dimensions* (43 to 60 percent) if excluding the one lower-performing item (23 percent). This lower-performing item required students to identify the equation of a circle representing a set of points.

Looking across the high-coverage topics, about half of the items requiring an extended constructed-response were at the lower end of item performance within their topics (ranging from 13 to 36 percent correct), with several more not exceeding 50 percent correct. (For example, see [1] *linear and quadratic equations and inequalities; exponential, logarithmic, polynomial, rational, and radical equations;* and *using equations and inequalities to solve contextual problems in algebra;* [2] *using derivatives to solve problems; using first and second derivatives to determine slope and local extrema;* and *using first and second derivatives to sketch and interpret graphs of functions in calculus;* and [3] *properties of geometric figures in two and three dimensions* and *trigonometric*

[44] The one exception is *using first and second derivatives to sketch and interpret graphs of functions,* which was not asked on the teacher questionnaire and thus there are no data on the percentage of students taught the topic. This topic was classified at the same level of topic coverage as the closely related topic of *using first and second derivatives to determine slope, local extrema, and points of inflection.*

[45] The one exception is *solving problems involving trigonometric functions,* which was not asked on the teacher questionnaire and thus there are no data on the percentage of students taught the topic. This topic was classified at the same level of topic coverage as the related topic of *trigonometric functions and their graphs.*

[46] The item with 1 percent correct was a short constructed-response *reasoning* item that required students to prove a property of the sines of supplementary angles.

properties of triangles in geometry). However, there were some exceptions of relatively higher-performing extended constructed-response items in *algebra* and *calculus* topics. There were no other clear patterns in item performance by item type among those in the high-coverage topics.

Moderate level of topic coverage

The moderate coverage category included one *algebra* and two *calculus* topics. The moderate-coverage *algebra* topic—the *nth term of arithmetic and geometric sequences and the sums of finite and infinite series*—was covered in both IB mathematics courses and as foundational knowledge for AP calculus covered in prerequisite courses, but mathematics teachers reported that not all U.S. students (93 percent) were taught these topics by the time of the assessment (table A-3a). Performance on the items in this topic ranged from 35 to 61 percent correct (figure 5-9).

The two moderate-coverage *calculus* topics—both related to *integrals*—were covered across the TIMSS Advanced-eligible mathematics courses, including AP calculus and IB mathematics courses. However, as with the prior *algebra* topic, mathematics teachers reported that not all U.S. students (95 percent) had been taught these *calculus* topics by the time of the assessment (table A-3b). It may be that, because these topics are generally covered toward the end of the year, they may have been introduced but not covered in depth by the time of the assessment. For the topic of *evaluating and defining integrals,* item performance for its six items was in the mid-performance range (30 to 65 percent) (figure 5-9). The topic of *integrating functions,* on the other hand, had only two items—one of which reflected one of the lowest item performances (12 percent correct) across all TIMSS Advanced mathematics items and the other relatively high item performance (66 percent correct). The former was a short constructed-response *reasoning* item that required students to find the antiderivative of a sinusoidal function, whereas the latter was a multiple-choice item that required students to simply recall the integral of an exponential function.

Low level of topic coverage

The topics in the low coverage category include one topic from each of the three content domains. The *algebra* and *calculus* topics in this category were both covered in AP calculus courses and the higher-level IB mathematics course but were not covered in the standard-level IB course—although teachers did report that the topics were taught to all or nearly all U.S. students (tables A-3a and A-3b).[47] Performance on the two items in the *algebra* topic, *operations with complex numbers,* were both less than 50 percent correct (34 and 41 percent correct) (figure 5-9). Performance on the four items in the *calculus* topic, *conditions for continuity and differentiability of functions,* ranged from 21 to 62 percent correct and all were multiple-choice items.

The low-coverage *geometry* topic, while covered in both IB mathematics courses and as foundational knowledge for AP calculus, was reported to be taught to just 81 percent of U.S. students (reflecting all students taking IB Mathematics, between 80 and 90 percent of students taking AP Calculus, and 66 percent of those taking other non-AP, non-IB calculus courses) (table A-3c). While this topic (*properties of vectors and their sums and differences*) is expected to have been covered prior to calculus, some calculus teachers may not have considered this to be prerequisite knowledge for their course or did not know what students had been taught in their prior courses. Item performance on this topic ranged from 21 to 38 percent correct (figure 5-9).

[47] The teacher data primarily reflect the majority of students taking AP calculus courses (76 percent) as well as those taking other non-AP, non-IB calculus courses (23 percent), whose teachers reported that all or nearly all students had been taught these topics. The very low percentage of students taking IB mathematics (1 percent), has little effect on the overall percentage of students taught. Also, as noted in section 3, IB mathematics teachers often teach both the standard-level and higher-level courses in their schools, and their responses are likely based on the full set of topics covered in the higher-level course. Thus, these topics were reported as nearly universally taught overall.

Figure 5-9. U.S. performance on TIMSS Advanced mathematics items, by level of topic coverage: 2015

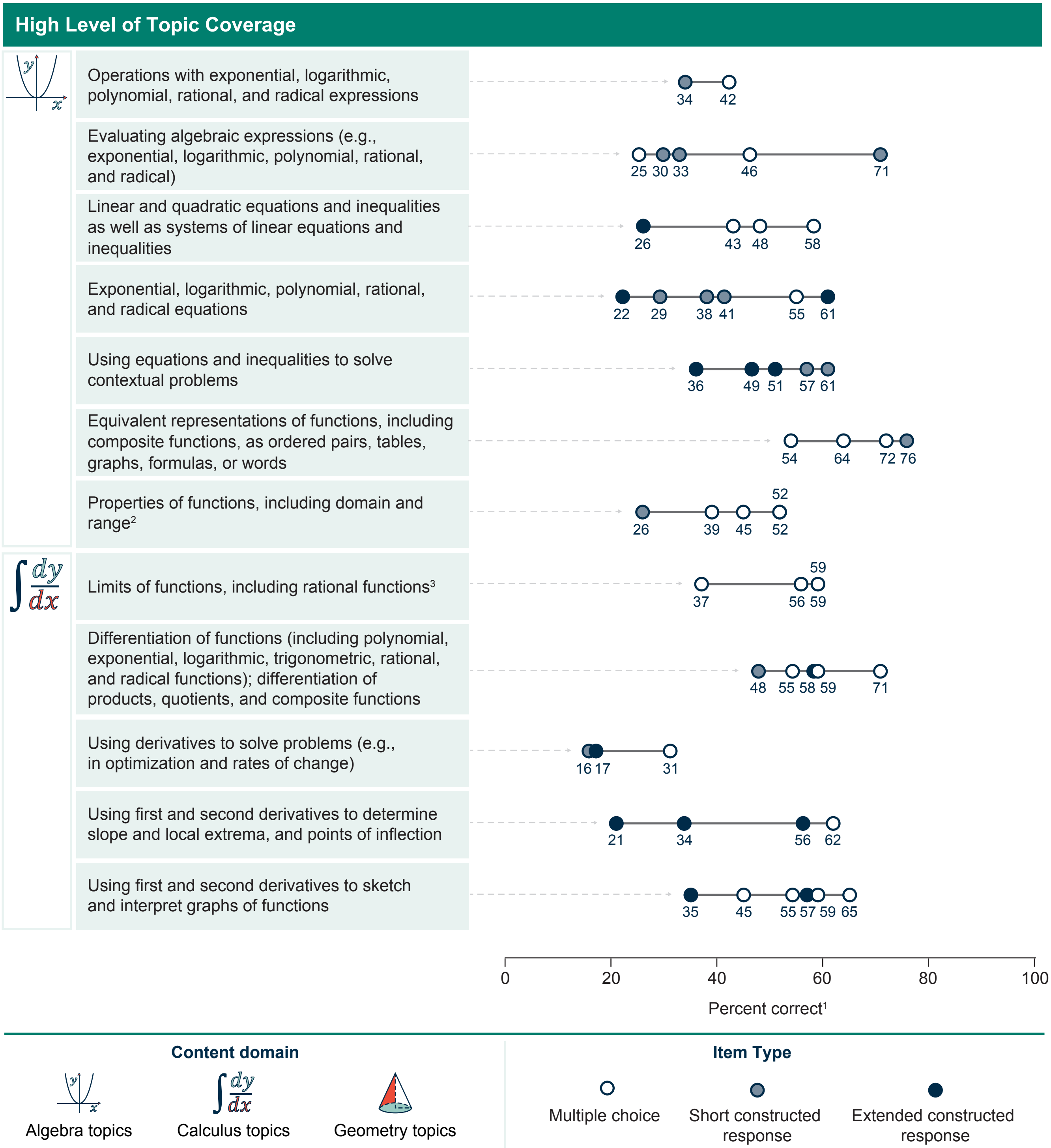
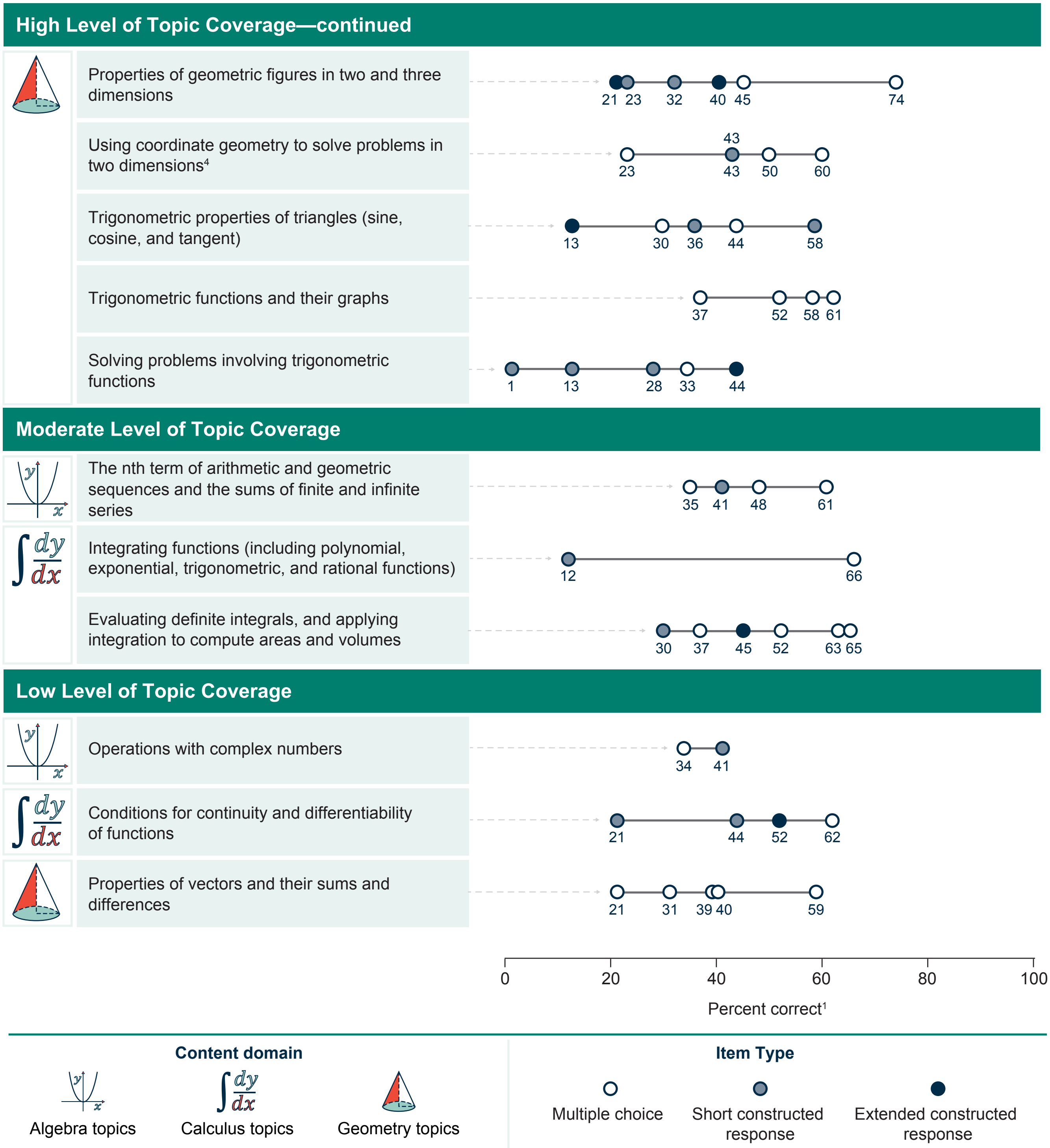


Figure continues on next page.
See notes at end of figure.

Figure 5-9. U.S. performance on TIMSS Advanced mathematics items, by level of topic coverage: 2015—Continued



[1] Percent correct is the percentage of students receiving credit on each item. For multiple-choice and short constructed-response items (each worth one score point), this reflects the percentage of students who provided a correct answer. For extended constructed-response items, this reflects the weighted percentage of students receiving full credit (2 points) or partial credit (1 point).
[2] In this topic, there are two items that round to the same percent correct (52 percent).
[3] In this topic, there are two items that round to the same percent correct (59 percent).
[4] In this topic, there are two items that round to the same percent correct (43 percent).
NOTE: Level of topic coverage is based on (a) topic coverage in the intended curricula for the TIMSS Advanced-eligible Advanced Placement (AP) calculus and International Baccalaureate (IB) mathematics courses; and (b) the percentage of students who had been taught the topic at the time of the assessment (either in the current year or a prior year) as reported by their mathematics teachers. High indicates that the topic (a) is covered in the intended curriculum for AP calculus and IB mathematics courses (or reflects foundational knowledge expected to be have been covered in a prior mathematics course); and (b) has been taught to all or nearly all students (99.0–100 percent). Moderate indicates that the topic (a) is at least partially covered in the intended curriculum for all AP calculus and IB mathematics courses (or reflects foundational knowledge expected to be have been covered in a prior mathematics course); and (b) has been taught to at least 85 percent of students (85.0–98.9 percent). Low indicates that the topic (a) is not covered in the intended curriculum for at least one of the AP calculus or IB mathematics courses; or (b) has been taught to less than 85 percent of students (0–84.9 percent). Detail is provided in supplemental tables A-3a, 3b, and 3c.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

5.3 What are some common approaches, misconceptions, and errors in advanced mathematics demonstrated by U.S. TIMSS Advanced students?

This subsection uses six example items and their associated student performance data to illustrate some common approaches, misconceptions, and errors in advanced mathematics demonstrated by U.S. TIMSS Advanced students. The example items

- cover a range of topics across the content domains and reflect varying item performance and coverage levels; and
- illustrate important misconceptions or errors in advanced mathematics made by U.S. students, especially those for which there are differences by course type.

The textbox below lists the advanced mathematics example items and summarizes the information provided in the accompanying exhibits.

Summary of Advanced Mathematics Example Items

Example 1 ("Graph of a function"): a multiple-choice *calculus* item from the *knowing* cognitive domain (exhibit 5-2)

Example 2 ("Second derivative of a rational function"): a constructed-response *calculus* item from the *knowing* cognitive domain (exhibit 5-3)

Example 3 ("Maximizing profit"): a constructed-response calculus item from the *applying* cognitive domain (exhibit 5-4)

Example 4 ("Comparing rental plans"): a constructed-response *algebra* item from the *applying* cognitive domain (exhibit 5-5)

Example 5 ("Properties of vectors"): a multiple-choice *geometry* item from the *knowing* cognitive domain (exhibit 5-6)

Example 6 ("Changes in animal population"): a constructed-response *geometry* item from the *reasoning* cognitive domain (exhibit 5-7)

Each exhibit shows the item; a sample student response (for constructed-response items); data on how the item was classified in terms of the content domain, cognitive domain, and international benchmark level;[48] patterns in the U.S. percent correct (for multiple-choice and short constructed-response items) or percent full credit (for extended constructed-response items) by course type; the scoring guide, detailing correct, partial, and incorrect responses (including specific misconceptions and errors which were tracked using special diagnostic codes);[49] and data on the percentage distribution of U.S. students across response categories by course type. (See section 4 for a description of the item-level statistics reported in this section.)

For additional advanced mathematics example items that show U.S. student performance in an international context, see the TIMSS Advanced 2015 international report.

[48] Item classification data were provided by the TIMSS International Study Center. An item maps to an international benchmark level on the TIMSS Advanced mathematics scale if students at or above that level are likely to answer the item correctly, while students below that benchmark are not.

[49] See appendix C for more information about diagnostic codes and the two-digit scoring system used in TIMSS Advanced.

Example 1—"Graph of a function"

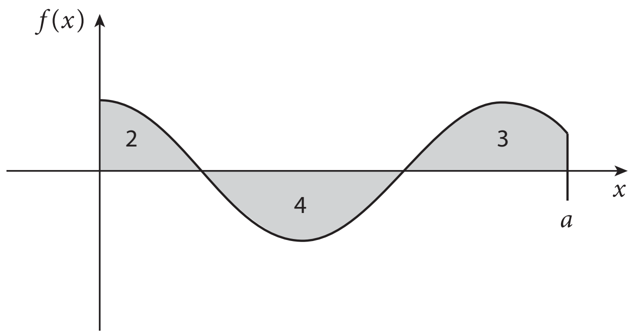
Exhibit 5-2 shows this multiple-choice advanced mathematics item from the topic area of *integrals* in the *calculus* content domain. This item assesses student understanding of the topic *evaluating definite integrals, and applying integration to compute areas and volumes,* which is covered in all the TIMSS Advanced-eligible AP calculus and IB mathematics courses and was reported by teachers to have been taught to 95 percent of U.S. TIMSS Advanced students overall (table A-3b). The topic was categorized as moderate coverage (figure 5-9). The item requires students to use the known area of regions under a curve to determine the value of a definite integral, and maps to the *Advanced* international benchmark level. Overall, 65 percent of U.S. TIMSS Advanced students answered this item correctly, which was higher than the international average (48 percent correct) (see the first page of exhibit 5-2 and table B-12).

The correct response (option B) provides the value of the definite integral by summing the areas of the shaded regions above the *x*-axis minus the area of the shaded region below the *x*-axis. U.S. performance varied by highest mathematics course taken, ranging from 47 percent correct for students who had taken other (non-AP, non-IB) calculus courses (lower than the U.S. total) to 81 percent correct for those who had taken AP Calculus BC (higher than the U.S. total).

The most common incorrect answer was option D, chosen by 23 percent of U.S. students (see the second page of exhibit 5-2). Students who selected this option calculated the value of the definite integral by adding all three shaded areas and ignoring the fact that the area under the middle curve is below the *x*-axis and thus the value of the integral is negative for that portion of the curve. Compared to the U.S. total, this misconception was less frequent among students who had taken AP Calculus BC (14 percent) and more frequent among students who had taken other (non-AP, non-IB) calculus courses (34 percent). The percentages of students choosing either of the other incorrect response options (A or C) did not exceed 9 percent overall or in any of the courses.

Exhibit 5-2. TIMSS Advanced mathematics example item 1 with student performance data, by course type: 2015

The graph of a function f is shown here. The areas of three regions are given.



What is the value of $\int_0^a f(x)\,dx$?

- Ⓐ −4
- ● 1
- Ⓒ 5
- Ⓓ 9

Item classification and description	
Content domain:	Calculus
Topic area:	Integrals
Topic:	Evaluating definite integrals, and applying integration to compute areas and volumes
Cognitive domain:	Knowing
International benchmark:	Advanced
Description:	Identify the value of a definite integral from areas shown on a graph

Student performance data

Course type[1]	Percent correct[2]
International average	**48** ▼
U.S. total	**65**
AP calculus courses	**70** ▲
AP Calculus BC	81 ▲
AP Calculus AB	66
Non-AP mathematics courses	**48** ▼
IB Mathematics[3]	‡
Other calculus courses[4]	47 ▼

▼ Subgroup/international average percent correct is significantly lower than U.S. total percent correct ($p < .05$).

▲ Subgroup/international average percent correct is significantly higher than U.S. total percent correct ($p < .05$).

Exhibit continues on next page.
See notes at end of exhibit.

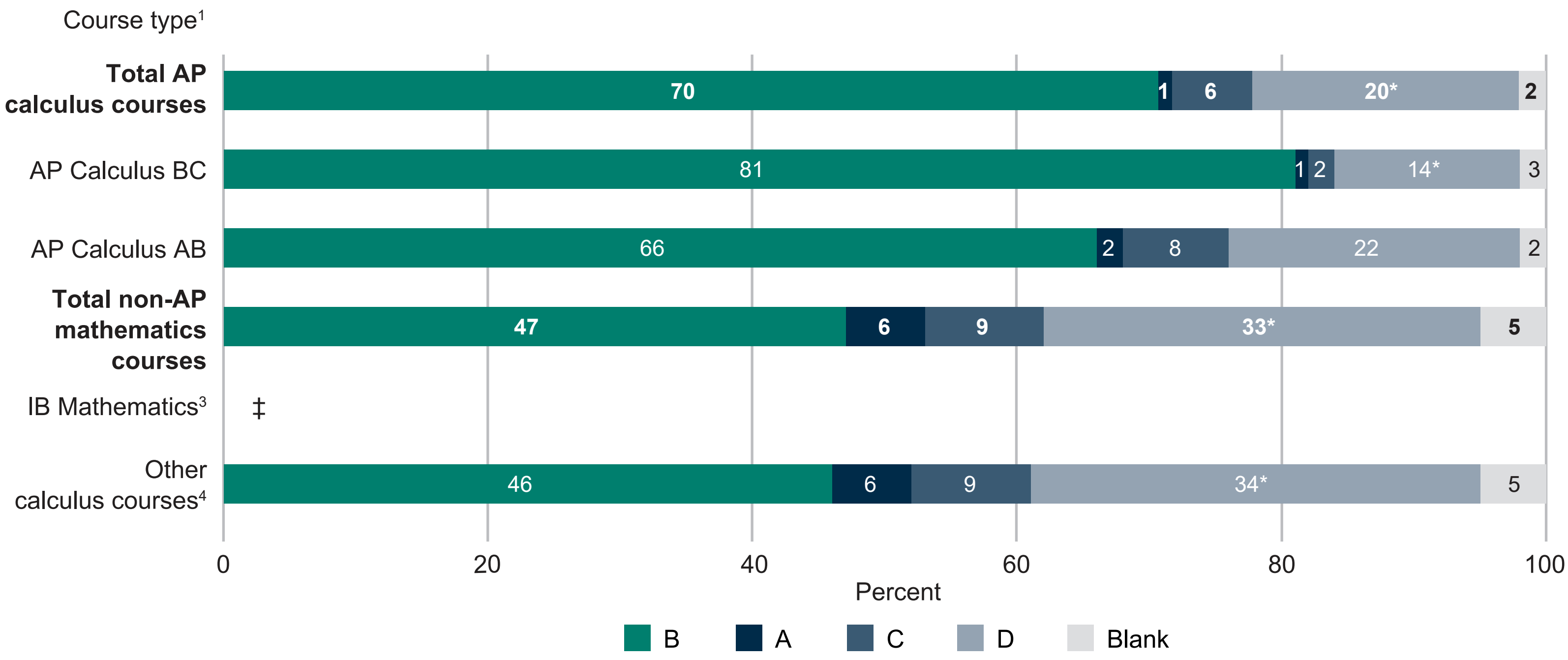
Exhibit 5-2. TIMSS Advanced mathematics example item 1 with student performance data, by course type: 2015—Continued

Scoring guide for multiple choice

Option	Response type	U.S. total percentage[5]
Correct		
B	Correctly provides the value of the definite integral by summing the areas of the shaded regions above the *x*-axis minus the area of the shaded region below the *x*-axis	65
Incorrect		
A	Incorrectly assumes that the value of the integral is equal to the area of the shaded region below the *x*-axis	2
C	Incorrectly assumes that the value of the integral is equal to the sum of the areas of the shaded regions above the *x*-axis	7
D	Incorrectly calculates the value of the definite integral by adding all three shaded areas and ignoring the fact that the area under the middle curve is below the *x*-axis and thus the value of the integral is negative for that portion of the curve	23
	Blank[6]	3

Misconception of interest (option D).

Percentage distribution of U.S. TIMSS Advanced students across response types, by course type: 2015



‡ Reporting standards not met (sample size < 30).
* $p < .05$. Subgroup percentage of students in option D is significantly different from the U.S. total percentage in option D.
[1] Course type reflects the highest level mathematics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the “other calculus courses” category only if they have not taken an AP or IB course.
[2] Percent correct is the weighted percentage of students receiving full credit or partial credit on an assessment item. For multiple choice items, there is only one correct response option, worth 1 point for full credit. Thus, the percent correct is the percentage of students who chose the correct response (option B in the scoring guide). Students who did not reach the item were not included in the calculation of this percentage.
[3] Includes both higher-level and standard-level IB mathematics courses.
[4] Includes other calculus courses (including “honors” or “regents” courses).
[5] Percentages are based on the full set of responses, including students who did not reach the item. Thus, the percentage in the correct category (option B in the scoring guide) may be slightly different from what is shown for percent correct on the first page of the exhibit (which excludes students in the not-reached category).
[6] Blank includes both students who omitted the item and those who did not reach the item.
NOTE: International average is the item percent correct, averaged across the countries participating in the TIMSS Advanced physics assessment. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Example 2—“Second derivative of a rational function”

Exhibit 5-3 shows this extended constructed-response advanced mathematics item from the topic area of *derivatives* in the *calculus* content domain. It assesses the topic *differentiation of functions; differentiation of products, quotients, and composite functions,* which is covered in all of the TIMSS Advanced-eligible AP calculus and IB mathematics courses and was reported by teachers to have been taught to virtually all U.S. TIMSS Advanced students (table A-3b). The topic was categorized as high coverage (figure 5-9). The item requires students to determine the second derivative of a rational function, and maps to the *High* international benchmark level. Overall, 52 percent of U.S. TIMSS Advanced students received full credit on this item, which was higher than the international average (43 percent full credit) (see the first page of exhibit 5-3 and table B-13).

A fully correct response (code 20 in the scoring guide) provided the second derivative of the function as shown in the example correct student response. U.S. performance ranged from 45 percent full credit for students who had taken non-AP mathematics courses (not measurably different from the U.S. total of 52 percent) to 69 percent full credit for those who had taken AP Calculus BC (higher than the U.S. total).

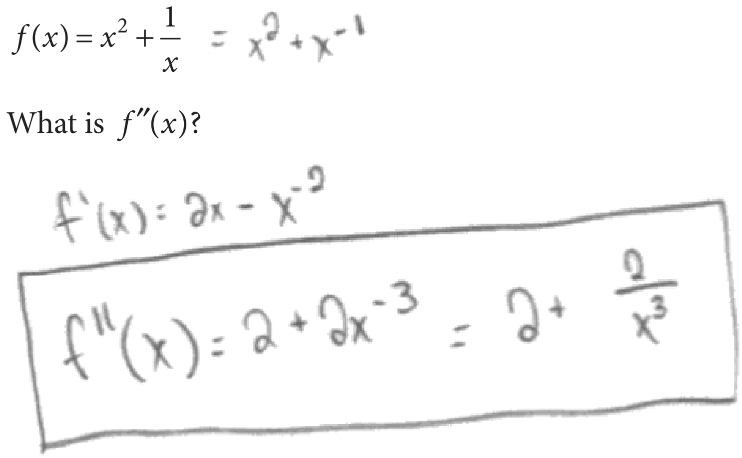
Students received partial credit for determining the first derivative but making a mistake calculating the second derivative (code 10, 11 percent of U.S. students), while responses with a correct first derivative that either did not attempt the second derivative or made a major error were scored as incorrect (code 77, 6 percent of U.S. students) (see second page of exhibit 5-3). Seventy percent of U.S. students overall were able to at least determine the first derivative correctly (codes 20, 10, and 77 combined). The percentage of students demonstrating this ability ranged from 65 and 68 percent of those who had taken non-AP mathematics courses and AP Calculus AB, respectively, to 82 percent of those who had taken AP Calculus BC.

Overall, 27 percent of U.S. students attempted this problem but did not correctly determine the first derivative (code 79). This type of incorrect response was less common among students who had taken AP Calculus BC (13 percent) than among U.S. students overall. Another 3 percent of U.S. students left the item blank.

Exhibit 5-3. TIMSS Advanced mathematics example item 2 with student performance data, by course type: 2015

$f(x) = x^2 + \frac{1}{x}$

What is $f''(x)$?



Item classification and description	
Content domain:	Calculus
Topic area:	Derivatives
Topic:	Differentiation of functions; differentiation of products, quotients, and composite functions
Cognitive domain:	Knowing
International benchmark:	High
Description:	Find the second derivative of a rational function

Student performance data	
Course type[1]	Percent full credit[2]
International average	**43** ▼
U.S. total	**52**
AP calculus courses	**54**
AP Calculus BC	69 ▲
AP Calculus AB	50
Non-AP mathematics courses	**45**
IB Mathematics[3]	‡
Other calculus courses[4]	47

▼ Subgroup/international average percent correct is significantly lower than U.S. total percent correct (p < .05).

▲ Subgroup/international average percent correct is significantly higher than U.S. total percent correct (p < .05).

Exhibit continues on next page.
See notes at end of exhibit.

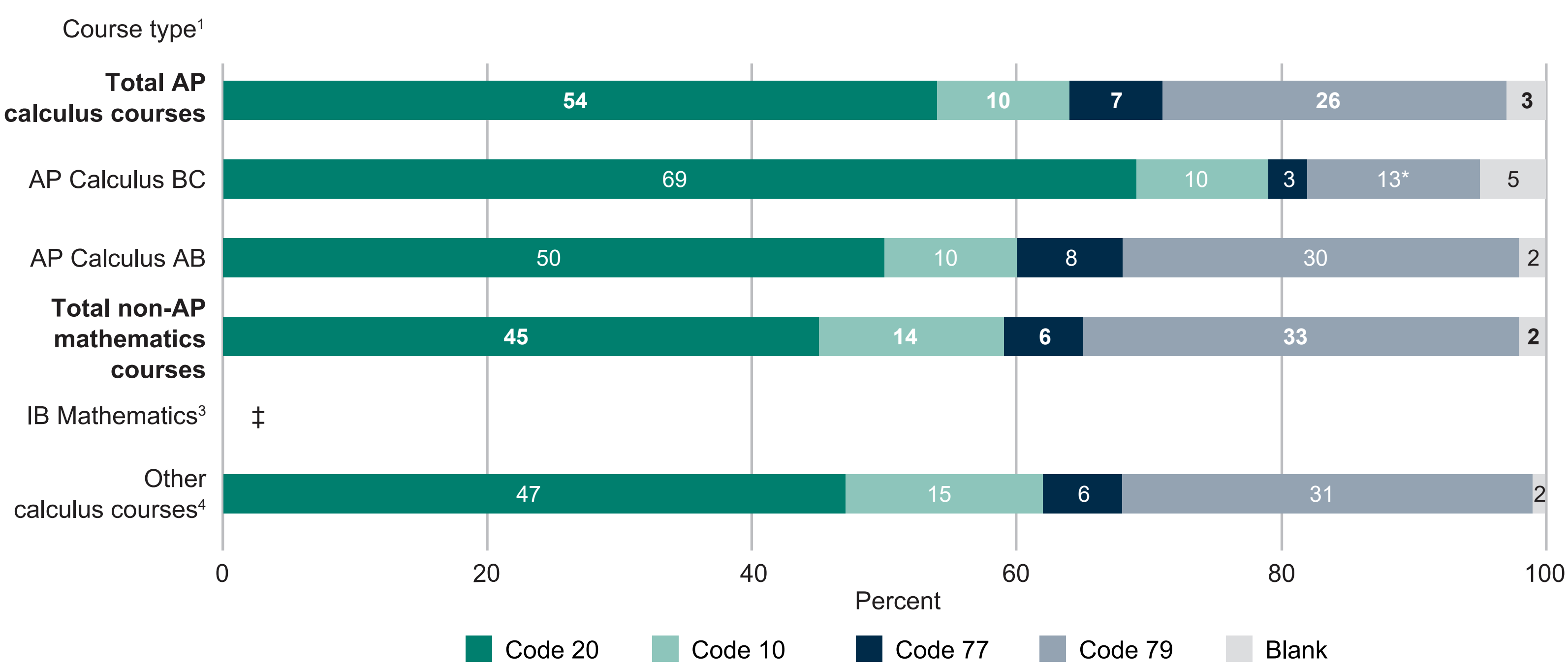
Exhibit 5-3. TIMSS Advanced mathematics example item 2 with student performance data, by course type: 2015—Continued

Scoring guide for contructed response

Code	Response type	U.S. total percentage[5]
Correct		
20	Correctly provides the second derivative of the function	52
Partial		
10	Correctly determines the first derivative and makes a minor error in calculation of the second derivative	11
Incorrect		
77	Correctly determines the first derivative but either does not attempt the second derivative or makes a major error in calculation of the second derviative	6
79	Other incorrect response (including crossed out, erased, stray marks, illegible, or off task)	27
	Blank[6]	3

Incorrect response of interest (scoring code 79).

Percentage distribution of U.S. TIMSS Advanced students across response types, by course type: 2015



‡ Reporting standards not met (sample size < 30).
* *p* < .05. Subgroup percentage of students in score code 79 is significantly different from the U.S. total percentage in score code 79.
[1] Course type reflects the highest level mathematics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other calculus courses" category only if they have not taken an AP or IB course.
[2] Percent full credit is the percentage of students receiving full credit on an assessment item. For extended constructed-response items, there is both a correct level (worth 2 score points) and a partial level (worth 1 score point). Thus, percent full credit reflects the percentage of students who provided a fully correct response (code 20 in the scoring guide). Students who provided a partial response are not reflected in the percent full credit, but are shown in the distributions across the correct and partial response categories on page 2 of the exhibit. Students who did not reach the item were not included in the calculation of this percentage.
[3] Includes both higher-level and standard-level IB mathematics courses.
[4] Includes other calculus courses (including "honors" or "regents" courses).
[5] Percentages are based on the full set of responses, including students who did not reach the item. Thus, the percentage in the correct category (code 20 in the scoring guide) may be slightly different from what is shown for percent full credit on the first page of the exhibit (which excludes students in the not-reached category).
[6] Blank includes both students who omitted the item and those who did not reach the item.
NOTE: International average is the item percent correct, averaged across the countries participating in the TIMSS Advanced physics assessment. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Example 3—"Maximizing profit"

Exhibit 5-4 shows this constructed-response advanced mathematics item from the topic area of *derivatives* in the *calculus* content domain. This item assesses the topic *using derivatives to solve problems in optimization and rates of change,* which is covered in all of the TIMSS Advanced-eligible AP calculus and IB mathematics courses and was reported by teachers to have been taught to virtually all U.S. TIMSS Advanced students (table A-3b). As such, the topic was categorized as high coverage (figure 5-9). In contrast to example 2, this item requires students to use differential calculus to solve a contextualized problem to determine the number of commodity units that should be produced and sold to maximize the profit. This item maps above the *Advanced* international benchmark level. Overall, 16 percent of U.S. TIMSS Advanced students answered this item correctly, which was higher than the international average (9 percent correct) (see the first page of exhibit 5-4 and table B-14).

The correct response of 200 units (as shown in the student example and code 10 in the scoring guide) is based on correctly differentiating the profit function to find the point at which the first derivative is zero and then verifying that profit is maximum at that point using the second derivative. Students also received full credit by using a scientific/graphic calculator to arrive at the correct answer and explaining how the calculator was used to determine the answer (code 11). U.S. performance ranged from 8 percent correct for students who had taken non-AP mathematics courses (lower than the U.S. total) to 28 percent correct for those who had taken AP Calculus BC (higher than the U.S. total).[50]

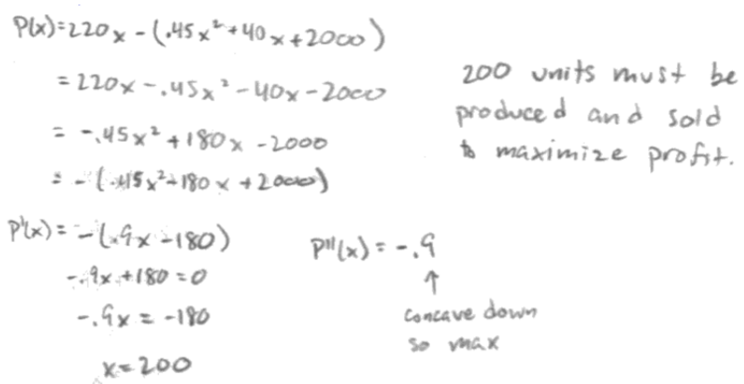
Of the 71 percent of U.S. students who provided an incorrect response (codes 70, 71, 72, and 79 combined), 7 percent made a computational error in determining the derivative of the profit function itself (code 70) (see the second page of exhibit 5-4). This diagnostic code was used to determine how many students knew how to set up the profit function for differentiation to start the multistep problem but made a computational error in the first step and thus could not go further. There were no measurable differences by highest course taken for this type of error. Another 9 percent of the students used the calculator but arrived at an incorrect answer with or without an adequate explanation of how they used the calculator (code 71 or 72). The majority of the students provided other types of completely incorrect responses (code 79, 55 percent) or left the item blank (13 percent).

[50] Percent correct is based on codes 10 and 11 combined.

Exhibit 5-4. TIMSS Advanced mathematics example item 3 with student performance data, by course type: 2015

A company produces x units of a commodity per day. The cost in zeds of producing x units is given by $C(x) = 0.45x^2 + 40x + 2000$. The commodity is sold for 220 zeds per unit. How many units must be produced and sold daily in order to maximize the profit?

Show your work.



Item classification and description	
Content domain:	Calculus
Topic area:	Derivatives
Topic:	Using derivatives to solve problems (e.g., in optimization and rates of change)
Cognitive domain:	Applying
International benchmark:	Above Advanced
Description:	Solve a problem by maximizing the profit function

Student performance data

Course type[1]	Percent correct[2]
International average	**9** ▼
U.S. total	**16**
AP calculus courses	**19** ▲
AP Calculus BC	28 ▲
AP Calculus AB	15
Non-AP mathematics courses	**8** ▼
IB Mathematics[3]	‡
Other calculus courses[4]	8 ▼

▼ Subgroup/international average percent correct is significantly lower than U.S. total percent correct (p < .05).

▲ Subgroup/international average percent correct is significantly higher than U.S. total percent correct (p < .05).

Exhibit continues on next page.
See notes at end of exhibit.

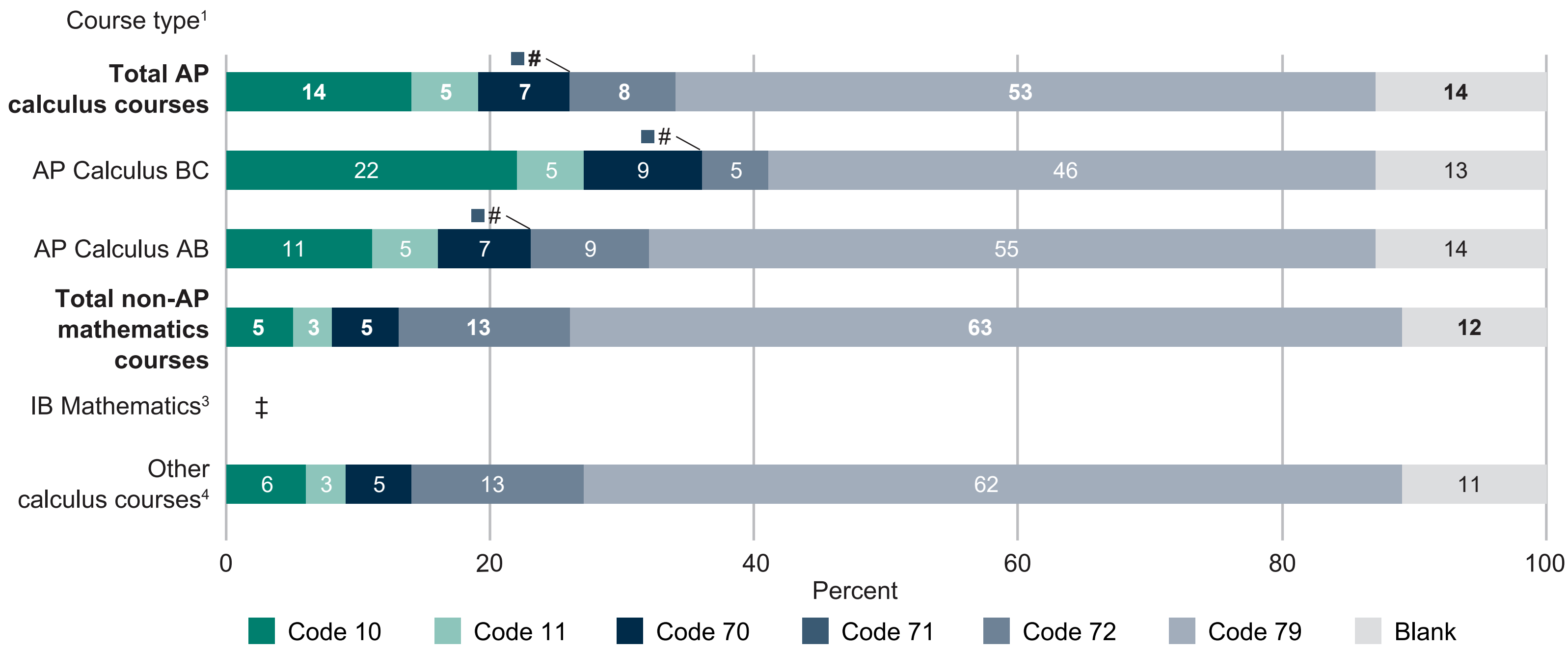
Exhibit 5-4. TIMSS Advanced mathematics example item 3 with student performance data, by course type: 2015—Continued

Scoring guide for contructed response

Code	Response type	U.S. total percentage[5]
Correct		
10	Correctly calculates the derivative of the profit function and gives the answer 200 units with correct work shown. Verifies that the profit is maximum for 200 units.	11
11	Gives correct response of 200 units using calculator with steps explaining how the calculator was used.	4
Incorrect		
70	Uses correct method to calculate the derivative of the profit function but makes a computational error and finds an incorrect answer.	7
71	Gives incorrect response using calculator with correct steps explaining how the calculator was used.	#
72	Uses calculator but gives incorrect answer and/or explanation is inadequate.	9
79	Other incorrect response (including crossed out, erased, stray marks, illegible, or off task)	55
	Blank[6]	13

Error of interest (scoring code 70).

Percentage distribution of U.S. TIMSS Advanced students across response types, by course type: 2015



Rounds to zero.
‡ Reporting standards not met (sample size < 30).
* $p < .05$. Subgroup percentage of students in score code 70 is significantly different from the U.S. total percentage in score code 70.
[1] Course type reflects the highest level mathematics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other calculus courses" category only if they have not taken an AP or IB course.
[2] Percent correct is the weighted percentage of students receiving full credit or partial credit on an assessment item. For short constructed-response items, there is only one correct response type, worth 1 point for full credit. Thus, the percent correct is the percentage of students who provided the correct response (codes 10 and 11 in the scoring guide). Students who did not reach the item were not included in the calculation of this percentage.
[3] Includes both higher-level and standard-level IB mathematics courses.
[4] Includes other calculus courses (including "honors" or "regents" courses).
[5] Percentages are based on the full set of responses, including students who did not reach the item. Thus, the percentage in the correct category (codes 10 and 11 in the scoring guide) may be slightly different from what is shown for percent correct on the first page of the exhibit (which excludes students in the not-reached category).
[6] Blank includes both students who omitted the item and those who did not reach the item.
NOTE: International average is the item percent correct, averaged across the countries participating in the TIMSS Advanced physics assessment. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Example 4—“Comparing rental plans”

Exhibit 5-5 shows this extended constructed-response advanced mathematics item from the topic area of *equations and inequalities* in the *algebra* content domain. This item assesses the topic *using equations and inequalities to solve contextual problems.* This topic is covered in both IB mathematics courses and considered foundational knowledge for both AP calculus courses (table A-3a), and was categorized as high coverage (figure 5-9). The item requires students to apply their understanding to solve a problem within a real-world context (comparing two car rental plans) and answer two questions (parts A and B). It maps to the *Advanced* international benchmark level. Overall, 35 percent of U.S. TIMSS Advanced students received full credit on this item, which was not measurably different from the international average (30 percent full credit) (see the first page of exhibit 5-5 and table B-15).

A fully correct score (code 20 in the scoring guide) requires students to correctly answer both parts A and B. A correct response to part A must provide the correct answer with adequate work shown (algebraically or graphically). The algebraic solution (shown in the example student response) includes writing equations for the two different car rental plans (X and Y) and then solving the simultaneous equations to find the point at which they intersect (3000 km), or, have the same cost. In part B, a correct response requires students to understand and explain that if the same increase in initial cost is applied to both plans with no other change, the difference between the two *y*-intercepts remain the same. Therefore, the distance on the *x*-axis at which the two equations intersect will not change. U.S. performance ranged from 31 percent full credit for students who had taken AP Calculus AB (not measurably different from the U.S. total of 30 percent) to 45 percent full credit for those who had taken AP Calculus BC (higher than the U.S. total).

Students received partial credit for providing a correct response to either part A (code 10, 22 percent) or part B (code 11, 5 percent) (see the second page of exhibit 5-5). A correct response to part A only (code 10) demonstrated that students could write the equations for the situation described and apply the correct procedure to find the intersection point, but they did not understand the equations well enough to explain the impact of the *y*-intercept change on both equations. That is, they did not demonstrate a deeper understanding beyond applying procedures to solve a pair of linear equations. The percentage of students receiving partial credit for part A only was higher for students taking AP calculus courses (24 percent) and lower for students taking non-AP mathematics courses (15 percent) than U.S. students overall.

The percentage of U.S. students who demonstrated at least the level of understanding required for a correct score on part A (codes 20 and 10 combined) was 57 percent overall and ranged from 52 percent of those who had taken other (non-AP, non-IB) calculus courses to 71 percent who had taken AP Calculus BC. In contrast, 31 percent of U.S. students overall provided a completely incorrect response (code 79) on both parts A and B, and 7 percent left the item blank.

Exhibit 5-5. TIMSS Advanced mathematics example item 4 with student performance data, by course type: 2015

Two different plans for renting a car are given in the table below.

Rental Plan	Initial Cost	Cost per kilometer
X	100 zeds	.07 zeds
Y	250 zeds	.02 zeds

A. After how many kilometers does Plan Y become the cheaper plan?

Show your work.

k = number of kilometers
Px = price with plan X, Py = price with plan Y

$P_x = 100 + 0.07k$
$P_y = 250 + 0.02k$

$P_x = P_y$
$100 + 0.07k = 250 + 0.02k$
$0.05k = 150$
$k = 3000$

Answer: Plan Y becomes the cheaper plan after 3000 km

B. If an extra insurance charge of 100 zeds is charged by both plans, does this change the number of kilometers when Plan Y becomes cheaper?

Explain your answer. This does not change the number of kilometers when Plan Y becomes cheaper as we add the same amount to both plans, and thus to to righthand-side and the lefthand side of the equation in A. This amount will therefore cancel eachother.

Item classification and description	
Content domain:	Algebra
Topic area:	Equations and inequalities
Topic:	Using equations and inequalities to solve contextual problems
Cognitive domain:	Applying
International benchmark:	Advanced
Description:	Compare two car rental plans X and Y and find which one will be economical

Student performance data

Course type[1]	Percent full credit[2]
International average	**30**
U.S. total	**35**
AP calculus courses	**34**
AP Calculus BC	45 ▲
AP Calculus AB	31
Non-AP mathematics courses	**38**
IB Mathematics[3]	‡
Other calculus courses[4]	37

▼ Subgroup/international average percent correct is significantly lower than U.S. total percent correct ($p < .05$).

▲ Subgroup/international average percent correct is significantly higher than U.S. total percent correct ($p < .05$).

Exhibit continues on next page.
See notes at end of exhibit.

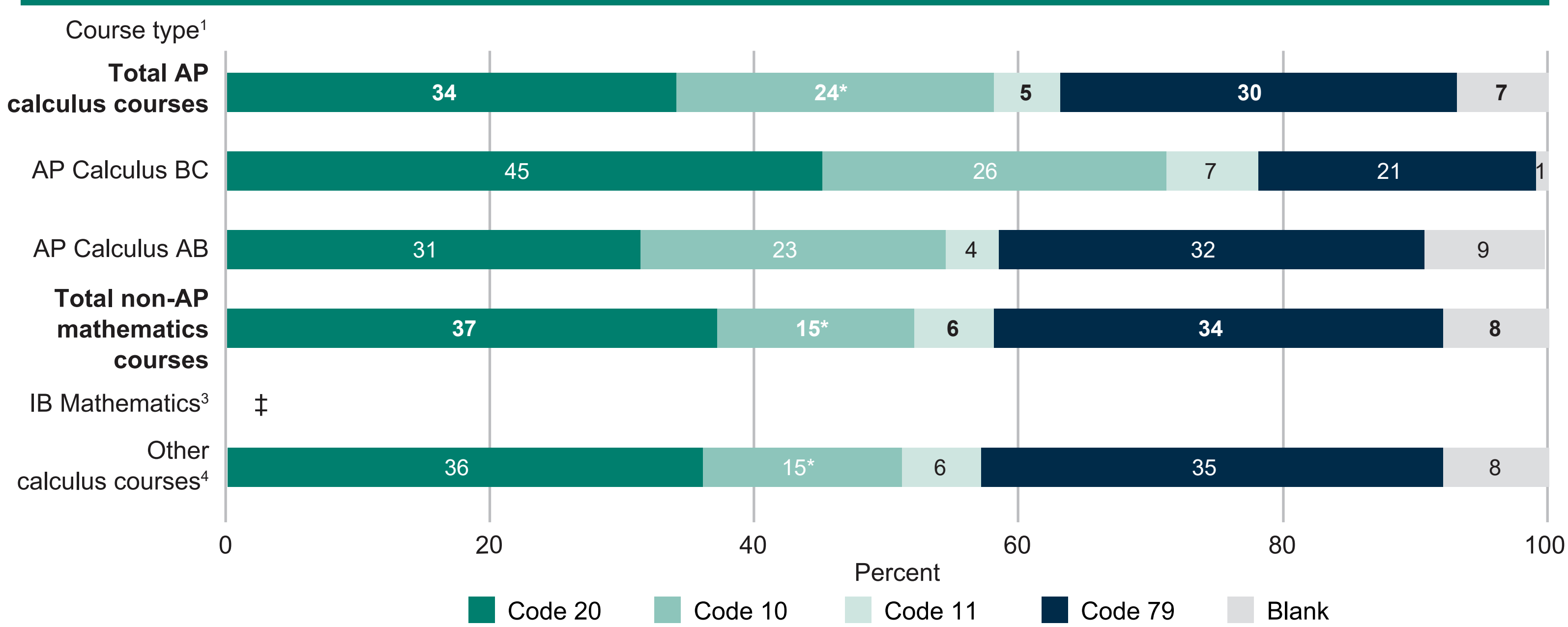
Exhibit 5-5. TIMSS Advanced mathematics example item 4 with student performance data, by course type: 2015—Continued

Scoring guide for contructed response

Code	Response type	U.S. total percentage[5]
Correct		
20	**Completes both part A and part B correctly.** A. Compares the two car rental plans and finds that plan Y will be cheaper after 3000 Km with adequate work shown (algebraically or graphically). B. Answers "no" and explains that the impact of adding the extra insurance charge (effect of intercept) will be the same on both plans; therefore, there is no change to the number of kilometers when plan Y becomes cheaper.	35
Partial		
10	Provides a correct response for part A and an incorrect or missing response for part B	22
11	Provides a correct response for part B and an incorrect or missing response for part A	5
Incorrect		
79	Incorrect for both part A and part B (including crossed out, erased, stray marks, illegible, or off task)	31
	Blank[6]	7

Partial response of interest (scoring code 10).

Percentage distribution of U.S. TIMSS Advanced students across response types, by course type: 2015



‡ Reporting standards not met (sample size < 30).
* $p < .05$. Subgroup percentage of students in score code 10 is significantly different from the U.S. total percentage in score code 10.
[1] Course type reflects the highest level mathematics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other calculus courses" category only if they have not taken an AP or IB course.
[2] Percent full credit is the percentage of students receiving full credit on an assessment item. For extended constructed-response items, there is both a correct level (worth 2 score points) and a partial level (worth 1 score point). Thus, percent full credit reflects the percentage of students who provided a fully correct response (code 20 in the scoring guide). Students who provided a partial response are not reflected in the percent full credit, but are shown in the distributions across the correct and partial response categories on page 2 of the exhibit. Students who did not reach the item were not included in the calculation of this percentage.
[3] Includes both higher-level and standard-level IB mathematics courses.
[4] Includes other calculus courses (including "honors" or "regents" courses).
[5] Percentages are based on the full set of responses, including students who did not reach the item. Thus, the percentage in the correct category (code 20 in the scoring guide) may be slightly different from what is shown for percent full credit on the first page of the exhibit (which excludes students in the not-reached category).
[6] Blank includes both students who omitted the item and those who did not reach the item.
NOTE: International average is the item percent correct, averaged across the countries participating in the TIMSS Advanced physics assessment. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Example 5—"Properties of vectors"

Exhibit 5-6 shows this multiple-choice advanced mathematics item from the topic area of *noncoordinate and coordinate geometry* in the *geometry* content domain. It assesses the topic *properties of vectors and their sums and differences,* which is covered in both IB mathematics courses and is generally considered prerequisite knowledge for both AP calculus courses (table A-3c). However, it was reported by teachers to have been taught to only 81 percent of U.S. TIMSS Advanced students overall at the time of the assessment, and thus is categorized as low coverage (figure 5-9). The item requires students to use their knowledge of properties of vectors to analyze and identify equivalent statements involving two vectors, and maps to the *Advanced* international benchmark level. Overall, 39 percent of U.S. TIMSS Advanced students answered this item correctly, which was the same as the international average (39 percent correct) (see the first page of exhibit 5-6 and table B-16).

The correct response (option C) states that vector *a* is perpendicular to vector *b.* This response indicates that students understand that if the sum and difference of the two vectors are equal, the cosine of the angle between them is zero, which occurs when the angle is 90 degrees. U.S. performance ranged from 30 percent correct for students who had taken other (non-AP, non-IB) calculus courses (lower than the U.S. total) to 50 percent correct for those who had taken AP Calculus BC (higher than the U.S. total).

The most common incorrect answer was option B, chosen by 36 percent of U.S. advanced students (see the second page of exhibit 5-6). Students who selected this option either confused the angle at which cos Θ is zero or erroneously used sin Θ instead of cos Θ in the formula for dot product of vectors. There were no measurable differences in the prevalence of this mistake by the highest course taken. Other incorrect responses (options A and D, each 9 percent of U.S. students overall) included that the two vectors are equal or their sum is zero, both of which are indicative of a minimal of understanding of vectors.

Exhibit 5-6. TIMSS Advanced mathematics example item 5 with student performance data, by course type: 2015

If $\vec{a} \neq \vec{0}$ and $\vec{b} \neq \vec{0}$, which of the following is equivalent to the equation $\left|\vec{a}+\vec{b}\right| = \left|\vec{a}-\vec{b}\right|$?

Ⓐ $\left|\vec{a}\right| = \left|\vec{b}\right|$

Ⓑ $\vec{a}$ and $\vec{b}$ are parallel vectors.

● $\vec{a}$ and $\vec{b}$ are perpendicular vectors.

Ⓓ $\vec{a}+\vec{b}=\vec{0}$

Item classification and description	
Content domain:	Geometry
Topic area:	Noncoordinate and coordinate geometry
Topic:	Properties of vectors and their sums and differences
Cognitive domain:	Knowing
International benchmark:	Advanced
Description:	Using properties of vectors to analyze equivalence of the sum and difference of two vectors

Student performance data

Course type[1]	Percent correct[2]
International average	**39**
U.S. total	**39**
AP calculus courses	**41** ▲
AP Calculus BC	50 ▲
AP Calculus AB	38
Non-AP mathematics courses	**32** ▼
IB Mathematics[3]	‡
Other calculus courses[4]	30 ▼

▼ Subgroup/international average percent correct is significantly lower than U.S. total percent correct ($p < .05$).

▲ Subgroup/international average percent correct is significantly higher than U.S. total percent correct ($p < .05$).

Exhibit continues on next page.
See notes at end of exhibit.

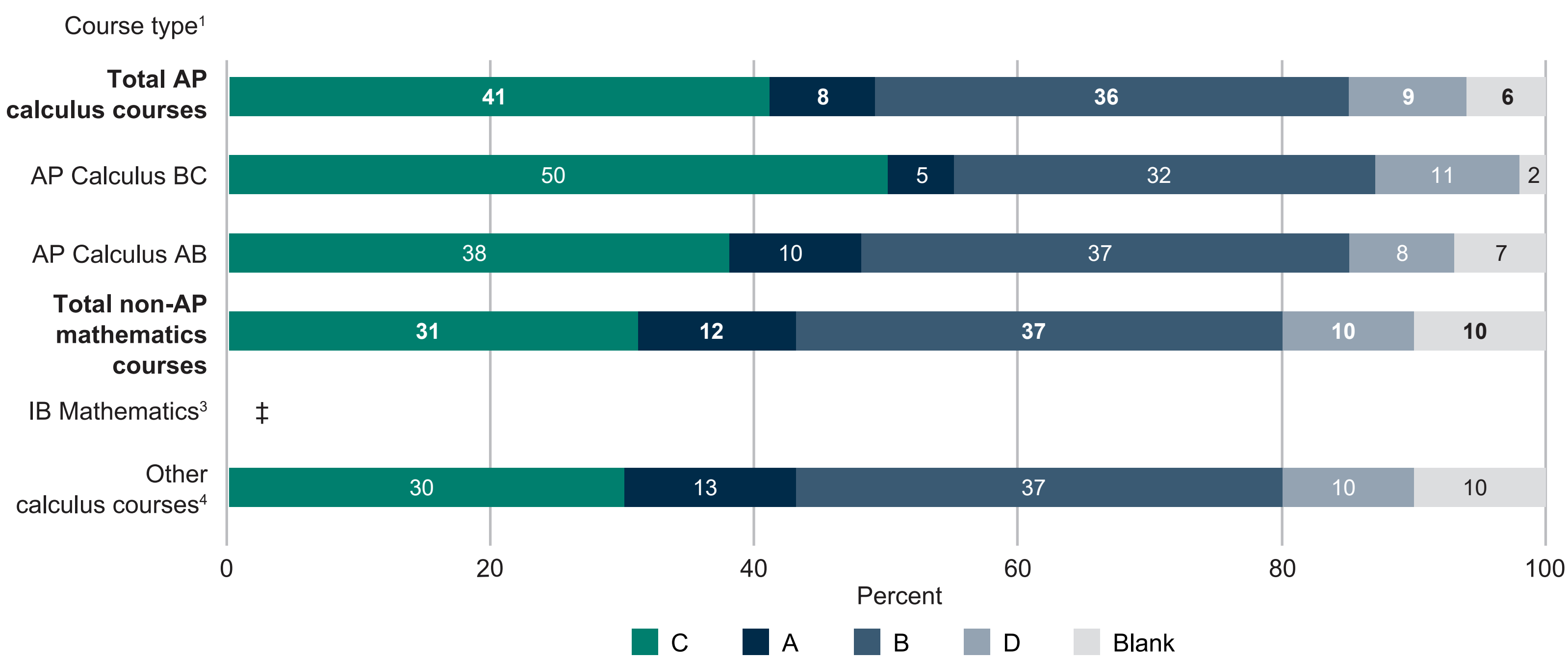
Exhibit 5-6. TIMSS Advanced mathematics example item 5 with student performance data, by course type: 2015—Continued

Scoring guide for multiple choice

Option	Response type	U.S. total percentage[5]
Correct		
C	Correctly selects that $\vec{a}$ is perpendicular to $\vec{b}$.	39
Incorrect		
A	Incorrectly selects that $\left\|\vec{a}+\vec{b}\right\| = \left\|\vec{a}-\vec{b}\right\|$ implies that $\left\|\vec{a}\right\| = \left\|\vec{b}\right\|$.	9
B	Confuses between cos θ and sin θ and incorrectly selects that $\vec{a}$ is parallel to $\vec{b}$.	36
D	Incorrectly selects that $\left\|\vec{a}+\vec{b}\right\| = \left\|\vec{a}-\vec{b}\right\|$ implies that $\vec{a}+\vec{b}=0$.	9
	Blank[6]	7

Misconception of interest (option B).

Percentage distribution of U.S. TIMSS Advanced students across response types, by course type: 2015



‡ Reporting standards not met (sample size < 30).
[1] Course type reflects the highest level mathematics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the “other calculus courses” category only if they have not taken an AP or IB course.
[2] Percent correct is the weighted percentage of students receiving full credit or partial credit on an assessment item. For multiple choice items, there is only one correct response option, worth 1 point for full credit. Thus, the percent correct is the percentage of students who chose the correct response (option C in the scoring guide). Students who did not reach the item were not included in the calculation of this percentage.
[3] Includes both higher-level and standard-level IB mathematics courses.
[4] Includes other calculus courses (including “honors” or “regents” courses).
[5] Percentages are based on the full set of responses, including students who did not reach the item. Thus, the percentage in the correct category (option C in the scoring guide) may be slightly different from what is shown for percent correct on the first page of the exhibit (which excludes students in the not-reached category).
[6] Blank includes both students who omitted the item and those who did not reach the item.
NOTE: International average is the item percent correct, averaged across the countries participating in the TIMSS Advanced physics assessment. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Example 6—"Changes in animal population"

Exhibit 5-7 shows this extended constructed-response advanced mathematics item from the topic area of *trigonometry* in the *geometry* content domain. It assesses the topic *solving problems involving trigonometric functions,* which is covered in both IB mathematics courses and is considered foundational knowledge for both AP calculus courses (table A-3c).[51] The topic was categorized as high coverage (figure 5-9). The item requires students to apply their understanding of trigonometric functions to solve a contextualized problem involving changes in animal population, and maps to the *High* international benchmark level. Overall, 36 percent of U.S. TIMSS Advanced students received full credit for this item, which was higher than the international average (28 percent full credit) (see the first page of exhibit 5-7 and table B-17).

As shown in the example student response, solving the problem involves (a) finding the maximum of the sinusoidal function and the value of the dependent variable (number of animals) at the time it occurs, and (b) determining one of the times at which the maximum occurs. The example student response provides work showing how the answers were obtained, but this was not required for full credit. Students may also use another method (not shown) based on differentiating the given function to find the time at which the maximum occurs and the number of animals at that point. A fully correct response (code 20 shown in the scoring guide) requires students to provide a correct answer for both the maximum number of animals (1500) and a correct time at which the maximum occurs ($t = \pi/6$ or one of its multiples). U.S. performance ranged from 24 percent full credit for students who had taken other (non-AP, non-IB) calculus courses (lower than the U.S. total of 36 percent) to 59 percent full credit for those who had taken AP Calculus BC (higher than the U.S. total).

Students received partial credit for providing a correct answer for either the maximum value of the function (code 10, 10 percent of U.S. students) or the time at which the maximum value occurs (code 11, 6 percent of U.S. students) (see the second page of exhibit 5-7). For these types of responses, students either made an error or did not provide an answer for one part of the question.

Overall, about half (51 percent) of U.S. students demonstrated conceptual understanding of trigonometric functions by providing a complete or partial response (codes 20, 10, and 11 combined). These percentages varied by mathematics course, ranging from 40 percent of students who had taken other (non-AP, non-IB) calculus courses to 77 percent of students who had taken AP Calculus BC, despite the fact that mathematics teachers reported that trigonometric functions had been taught to virtually all U.S. TIMSS Advanced students by the time of the assessment (table A-3c).[52]

Thirty-two percent of U.S. students overall attempted the problem but provided completely incorrect responses (code 79), while 16 percent left the item blank. There were no measurable differences in the percentage of students providing an incorrect response across the advanced mathematics courses except AP Calculus BC (18 percent, lower than the 32 percent of U.S. students overall). In comparison, the percentage of students who did not attempt the problem (left the item blank) ranged from 4 percent of students taking AP Calculus BC to 29 percent of those taking other (non-AP, non-IB) calculus courses.

[51] The topic on *solving problems involving trigonometric functions* was not included as a separate question in the teacher questionnaire and so an estimate of the percentage of students taught the topic is not available. However, teachers did answer a question about the related topic of *trigonometric functions and their graphs.*

[52] The specific trigonometry topic of *solving problems involving trigonometric functions,* however, was not included in the teacher questionnaire.

Exhibit 5-7. TIMSS Advanced mathematics example item 6 with student performance data, by course type: 2015

The number of animals in a certain population $P(t)$ varies periodically with time t. This can be modeled by

$$P(t) = 900 + 600\sin(t + \frac{\pi}{3})$$

What is the maximum number of animals?

Indicate one of the times at which the maximum occurs.

Maximum number of animals:

$P(t) =$ 1500

One time at which maximum occurs:

$t =$ $\frac{\pi}{6}$

Sin f^n varies from to 1

P(t) is maximum when,

$\sin(t + \frac{\pi}{3})$ is 1

$P(t) = 900 + 600 = 1500$

$\sin(t + \frac{\pi}{3}) = \sin\frac{\pi}{2}$

$t = \frac{\pi}{2} - \frac{\pi}{3}$

$t = \frac{\pi}{6}$

Item classification and description	
Content domain:	Geometry
Topic area:	Trigonometry
Topic:	Solving problems involving trigonometric functions
Cognitive domain:	Reasoning
International benchmark:	High
Description:	Find the maximum value of a trigonometric function and a value of the independent variable at which it occurs

Student performance data

Course type[1]	Percent full credit[2]
International average	**28** ▼
U.S. total	**36**
AP calculus courses	**40** ▲
AP Calculus BC	59 ▲
AP Calculus AB	34
Non-AP mathematics courses	**25** ▼
IB Mathematics[3]	‡
Other calculus courses[4]	24 ▼

▼ Subgroup/international average percent correct is significantly lower than U.S. total percent correct ($p < .05$).

▲ Subgroup/international average percent correct is significantly higher than U.S. total percent correct ($p < .05$).

Exhibit continues on next page.
See notes at end of exhibit.

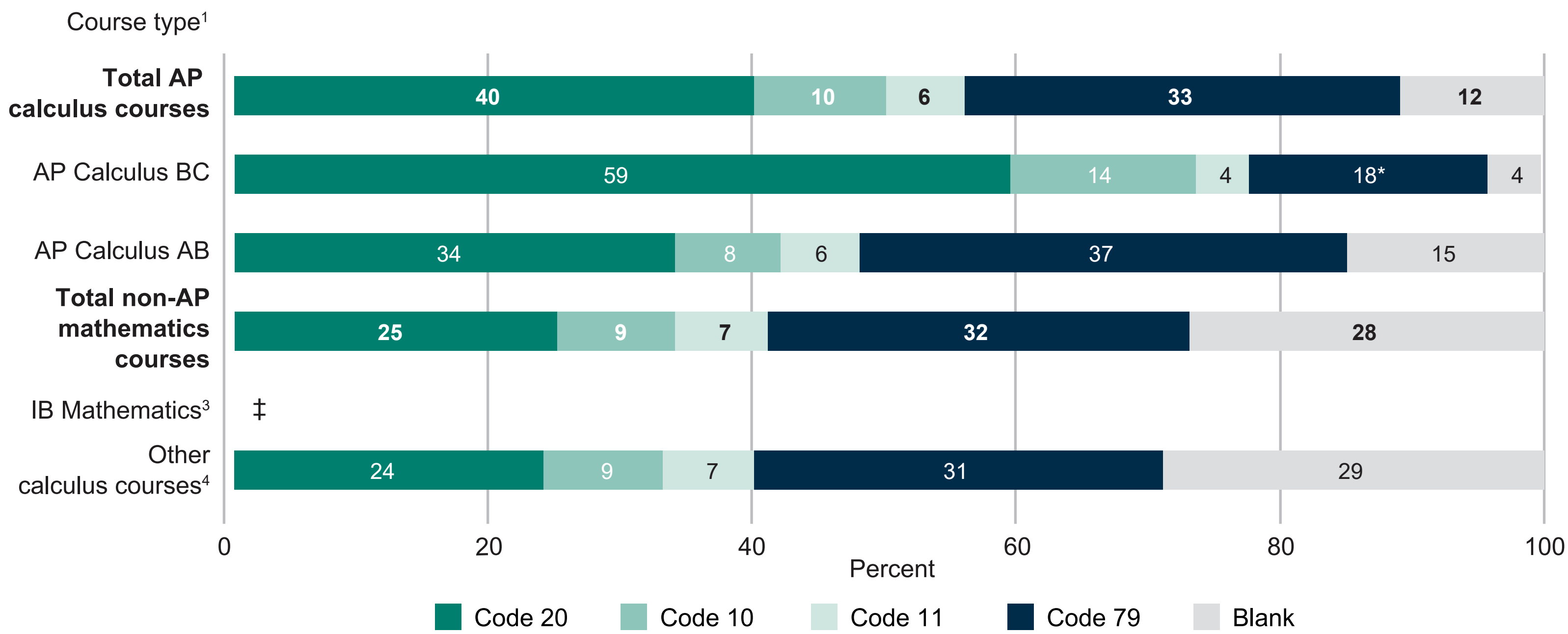
Exhibit 5-7. TIMSS Advanced mathematics example item 6 with student performance data, by course type: 2015—Continued

Scoring guide for contructed response

Code	Response type	U.S. total percentage[5]
Correct		
20	Calculates maximum number of animals $P(t)$ is equal to 1500. The time at which this can happen is at $t = \frac{\pi}{6}$ (or any other value of the type $\frac{\pi}{6} + 2k\pi$)	36
Partial		
10	Only calculates maximum number of animals $P(t)$ is equal to 1500.	10
11	Only gives the correct value of the time at which population is maximum at $t = \frac{\pi}{6}$	6
Incorrect		
79	Incorrect (including crossed out, erased, stray marks, illegible, or off task)	32
	Blank[6]	16

Incorrect response of interest (scoring code 79).

Percentage distribution of U.S. TIMSS Advanced students across response types, by course type: 2015



‡ Reporting standards not met (sample size < 30).
* *p* < .05. Subgroup percentage of students in score code 79 is significantly different from the U.S. total percentage in score code 79.
[1] Course type reflects the highest level mathematics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other calculus courses" category only if they have not taken an AP or IB course.
[2] Percent full credit is the percentage of students receiving full credit on an assessment item. For extended constructed-response items, there is both a correct level (worth 2 score points) and a partial level (worth 1 score point). Thus, percent full credit reflects the percentage of students who provided a fully correct response (code 20 in the scoring guide). Students who provided a partial response are not reflected in the percent full credit, but are shown in the distributions across the correct and partial response categories on page 2 of the exhibit. Students who did not reach the item were not included in the calculation of this percentage.
[3] Includes both higher-level and standard-level IB mathematics courses.
[4] Includes other calculus courses (including "honors" or "regents" courses).
[5] Percentages are based on the full set of responses, including students who did not reach the item. Thus, the percentage in the correct category (code 20 in the scoring guide) may be slightly different from what is shown for percent full credit on the first page of the exhibit (which excludes students in the not-reached category).
[6] Blank includes both students who omitted the item and those who did not reach the item.
NOTE: International average is the item percent correct, averaged across the countries participating in the TIMSS Advanced physics assessment. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Section 6: Physics Results

This section answers the following questions: (1) how did U.S. TIMSS Advanced students, who were described in section 2, perform in physics; (2) how did their performance on TIMSS Advanced physics items vary across the content domains and relate to the level of topic coverage that was described in section 3; and (3) what are their common approaches, misconceptions, and errors in physics?

6.1 How did U.S. TIMSS Advanced students perform in physics?

U.S. performance in physics is described in terms of the average scores and percentages of students reaching the international benchmarks overall and by course type, as well as by student and school characteristics.

Average physics performance overall

U.S. students taking the TIMSS Advanced physics assessment, who represented 5.3 percent of twelfth-graders overall, had an average score of 437, which was 63 points below the TIMSS Advanced scale centerpoint of 500 points (table 6-1). In *mechanics and thermodynamics,* U.S. students' average score was 462 points; in *electricity and magnetism,* it was 380 points; and in *wave phenomena and atomic/nuclear physics,* it was 431 points—all of which were below the scale centerpoint (table 6-2). In particular, U.S. average performance in *electricity and magnetism* was 120 points—or, more than one standard deviation—below the scale centerpoint.

U.S. TIMSS Advanced students' average scores on the cognitive subscales ranged from 420 in *applying* to 455 in *reasoning,* and on all three subscales (including *knowing*), U.S. average scores were below the scale centerpoint (table 6-3). The remainder of the physics results explore the physics performance of U.S. students in more detail.

Average physics performance by course type

In the United States, the average scores of students who had taken one of the AP physics courses (445 for AP total) and those whose highest level course was a non-AP course (402) were not measurably different from the U.S. average overall (437) (table 6-1).[53] The same generalization holds for all three content subscales (table 6-2). However, there were notable differences based on the specific course type, particularly across the range of AP physics courses taken. (See table 6.1 for the percentage of TIMSS Advanced students in each course type as discussed in section 2.)

U.S. TIMSS Advanced students who had taken an AP Physics C course (25 percent), as well as those who had taken AP Physics 2 (4 percent), scored, on average, above the U.S. average overall and on all three content subscales. On the overall scale, the average score differences between U.S. students who had taken these courses and U.S. students overall were at least 45 points (table 6-1). Nearly all of the TIMSS Advanced physics topics were covered either in these courses or in a previous course (table 3-2).

[53] In tables 6-1, 6-2, and 6-3, performance of students who had taken an AP physics course (1, 2, B, or C) is indicated as "Total AP physics courses."

Performance was especially high for students who had taken the AP Physics C course in electricity and magnetism (537), the majority of whom had also taken the C-level course in mechanics (table 6-1). On average, these students performed higher than the U.S. average in all three content domains, ranging from 85 points higher in *wave phenomena and atomic/nuclear physics* to 138 points higher in *electricity and magnetism* (table 6-2). They also were the only specific course group whose average score was either higher than, or not measurably different from, the TIMSS Advanced scale centerpoint, overall and on each content subscale.[54]

Students who had taken AP Physics B as their highest level physics course (in their junior year or prior), on average, did not have measurably different scores from U.S. students overall (442 vs. 437), while students who had taken AP Physics 1 as their highest physics course (in their senior year) scored, on average, below the U.S. average overall (407 vs. 437) and the TIMSS Advanced scale centerpoint (table 6-1). These students also scored below the U.S. averages on all three content subscales (table 6-2). The average score differences for students who had taken AP Physics 1 ranged from 25 points below the U.S. average in *mechanics and thermodynamics* to 41 points below the average in *electricity and magnetism.* Less than half of TIMSS Advanced topics are explicitly covered in the AP Physics 1 curriculum, and for students taking AP Physics 1 in the 2014–15 school year (42 percent of U.S. TIMSS Advanced physics students), this may be their first high school physics course. None of the TIMSS Advanced topics in *electricity and magnetism* are fully covered in AP Physics 1 (table 3-2).

Students who had taken non-AP physics courses (402 points), including those who had taken IB Physics (360 points), had average scores that were not measurably different from the U.S. average (437 points) but were lower than the scale centerpoint (table 6-1). The same pattern holds true for content subscales (table 6-2). Most students who had taken IB Physics were in the standard-level course, which, in contrast to the higher-level IB physics course, does not cover all of the TIMSS Advanced physics topics (table 3-2).

In terms of the cognitive domains (i.e., *knowing, applying,* and *reasoning*), the patterns of performance by course type were generally similar to the content domain results. Relative to the U.S. average, U.S. student performance on the three cognitive subscales did not vary significantly by whether or not students had taken an AP physics course, but did vary by AP course level (table 6-3). Students who had taken an AP Physics C course, as well as those who had taken AP Physics 2, had average scores above the U.S. average on all three cognitive subscales, while those who had taken AP Physics 1 had average scores below the U.S. overall on all three. Students who had taken an AP Physics C course or AP Physics 2 had average scores that were not measurably different from the scale centerpoint in all three cognitive domains, except for those whose highest course was AP Physics C-Electricity and Magnetism, who performed on average above the scale centerpoint on the *knowing* and *reasoning* subscales, and those whose highest course was AP Physics C-Mechanics, who performed on average below the scale centerpoint on the *applying* subscale. Students whose highest course was AP Physics 1 or IB Physics had average scores lower than the scale centerpoints on all three subscales.

[54] These students were higher than the scale centerpoint on the overall physics scale and the *mechanics and thermodynamics* subscale and not measurably different on the *electricity and magnetism* and the *wave phenomena and atomic/nuclear physics* subscales.

Table 6-1. Percentage distribution and average physics scores of U.S. TIMSS Advanced students, by course type: 2015

Course type[1]	Percentage of students in the U.S. TIMSS Advanced physics population[2]	Average score	Score difference from U.S. average[3]	Score difference from TIMSS Advanced scale centerpoint[4]
All course types (U.S. average)	100	437	†	-63*
TIMSS Advanced scale centerpoint	100	500	63*	†
Total AP physics courses[5]	**83**	**445**	**7**	**-55***
AP Physics C-E/M[6]	10	537	100*	37*
AP Physics C-M	15	482	45*	-18
AP Physics B	12	442	4	-58*
AP Physics 2	4	486	48*	-14
AP Physics 1	42	407	-31*	-93*
Total non-AP physics courses	**17**	**402**	**-35**	**-98***
IB Physics[7]	6	360	-77	-140*
Other physics courses[8]	12	423	-14	-77

† Not applicable.
* *p* < .05. Subgroup average score is significantly different from U.S. average score/TIMSS Advanced scale centerpoint.
[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other physics courses" category only if they have not taken an AP or IB course.
[2] The percentage of students in the physics population is based on the weighted counts of students in the sample.
[3] The score difference is calculated by subtracting the subgroup average score from the U.S. average score.
[4] The score difference is calculated by subtracting the subgroup average score from the TIMSS Advanced scale centerpoint. The TIMSS Advanced scale centerpoint represents the international mean of the overall achievement distribution in the first assessment year (1995).
[5] AP Physics C includes courses in mechanics (C-M) and electricity and magnetism (C-E/M). AP Physics B was a 1-year physics course that was discontinued after the 2013–14 school year and was replaced with the two-year course sequence AP Physics 1 and 2 beginning in 2014–15. Data in the AP Physics B category are generally for students who took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering a 1-year course identified as AP Physics B in 2014–15.
[6] A large majority of the students whose highest course was AP Physics C-E/M had also taken AP Physics C-M (89 percent), whether sequentially or in a combined course.
[7] Includes both higher-level and standard-level IB physics courses.
[8] Includes other types of second-year physics courses (including "honors" and "regents" courses).
NOTE: Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table 6-2. Average physics scores of U.S. TIMSS Advanced students, by content domain and course type: 2015

Course type[1]	Mechanics and thermodynamics			Electricity and magnetism			Wave phenomena and atomic/nuclear physics		
	Average score	Score difference from U.S. average[2]	Score difference from TIMSS Advanced scale centerpoint[3]	Average score	Score difference from U.S. average[2]	Score difference from TIMSS Advanced scale centerpoint[3]	Average score	Score difference from U.S. average[2]	Score difference from TIMSS Advanced scale centerpoint[3]
All course types (U.S. average)	462	†	-38*	380	†	-120*	431	†	-69*
TIMSS Advanced scale centerpoint	500	38*	†	500	120*	†	500	69*	†
Total AP physics courses[4]	**471**	**8**	**-29***	**389**	**9**	**-111***	**436**	**5**	**-64***
AP Physics C-E/M[5]	556	94*	56*	517	138*	17	516	85*	16
AP Physics C-M	509	47*	9	435	55*	-65*	466	36*	-34*
AP Physics B	460	-2	-40	382	2	-118*	432	1	-68*
AP Physics 2	504	41*	4	451	72*	-49*	478	47*	-22*
AP Physics 1	437	-25*	-63*	339	-41*	-161*	404	-27*	-96*
Total non-AP physics courses	**423**	**-39**	**-77***	**335**	**-45**	**-165***	**405**	**-26**	**-95***
IB Physics[6]	381	-81	-119*	286	-94	-214*	372	-58	-128*
Other physics courses[7]	443	-19	-57	359	-21	-141*	421	-9	-79

† Not applicable.
* *p* < .05. Subgroup average score is significantly different from U.S. average score/TIMSS Advanced scale centerpoint.
[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the “other physics courses” category only if they have not taken an AP or IB course.
[2] The score difference is calculated by subtracting the subgroup average score from the U.S. average score.
[3] The score difference is calculated by subtracting the subgroup average score from the TIMSS Advanced scale centerpoint. The TIMSS Advanced scale centerpoint represents the international mean of the overall achievement distribution in the first assessment year (1995).
[4] AP Physics C includes courses in mechanics (C-M) and electricity and magnetism (C-E/M). AP Physics B was a 1-year physics course that was discontinued after the 2013–14 school year and was replaced with the 2-year course sequence AP Physics 1 and 2 beginning in 2014–15. Data in the AP Physics B category are generally for students who took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering a 1-year course identified as AP Physics B in 2014–15.
[5] A large majority of the students whose highest course was AP Physics C-E/M had also taken AP Physics C-M (89 percent), whether sequentially or in a combined course.
[6] Includes both higher-level and standard-level IB physics courses.
[7] Includes other types of second-year physics courses (including “honors” and “regents” courses).
NOTE: TIMSS Advanced produces separate subscales for each content domain. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table 6-3. Average physics scores of U.S. TIMSS Advanced students, by cognitive domain and course type: 2015

Course type[1]	Knowing			Applying			Reasoning		
	Average score	Score difference from U.S. average[2]	Score difference from TIMSS Advanced scale centerpoint[3]	Average score	Score difference from U.S. average[2]	Score difference from TIMSS Advanced scale centerpoint[3]	Average score	Score difference from U.S. average[2]	Score difference from TIMSS Advanced scale centerpoint[3]
All course types (U.S. average)	**444**	**†**	**-56***	**420**	**†**	**-80***	**455**	**†**	**-45***
TIMSS Advanced scale centerpoint	**500**	**56***	**†**	**500**	**80***	**†**	**500**	**45***	**†**
Total AP physics courses[4]	**452**	**8**	**-48***	**428**	**7**	**-72***	**461**	**6**	**-39***
AP Physics C-E/M[5]	539	95*	39*	523	102*	23	542	87*	42*
AP Physics C-M	490	46*	-10	466	45*	-34*	497	42*	-3
AP Physics B	451	7	-49*	429	9	-71*	449	-6	-51*
AP Physics 2	490	46*	-10	477	56*	-23	492	38*	-8
AP Physics 1	415	-29*	-85*	387	-33*	-113*	429	-26*	-71*
Total non-AP physics courses	**407**	**-37**	**-93***	**385**	**-36**	**-115***	**427**	**-27**	**-73***
IB Physics[6]	372	-72	-128*	348	-73	-152*	387	-68	-113*
Other physics courses[7]	424	-20	-76	403	-17	-97	447	-7	-53

† Not applicable.
* $p < .05$. Subgroup average score is significantly different from U.S. average score/TIMSS Advanced scale centerpoint.
[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other physics courses" category only if they have not taken an AP or IB course.
[2] The score difference is calculated by subtracting the subgroup average score from the U.S. average score.
[3] The score difference is calculated by subtracting the subgroup average score from the TIMSS Advanced scale centerpoint. The TIMSS Advanced scale centerpoint represents the international mean of the overall achievement distribution in the first assessment year (1995).
[4] AP Physics C includes courses in mechanics (C-M) and electricity and magnetism (C-E/M). AP Physics B was a 1-year physics course that was discontinued after the 2013–14 school year and was replaced with the 2-year course sequence AP Physics 1 and 2 beginning in 2014–15. Data in the AP Physics B category are generally for students who took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering a 1-year course identified as AP Physics B in 2014–15.
[5] A large majority of the students whose highest course was AP Physics C-E/M had also taken AP Physics C-M (89 percent), whether sequentially or in a combined course.
[6] Includes both higher-level and standard-level IB physics courses.
[7] Includes other types of second-year physics courses (including "honors" and "regents" courses).
NOTE: TIMSS produces separate subscales for each cognitive domain. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

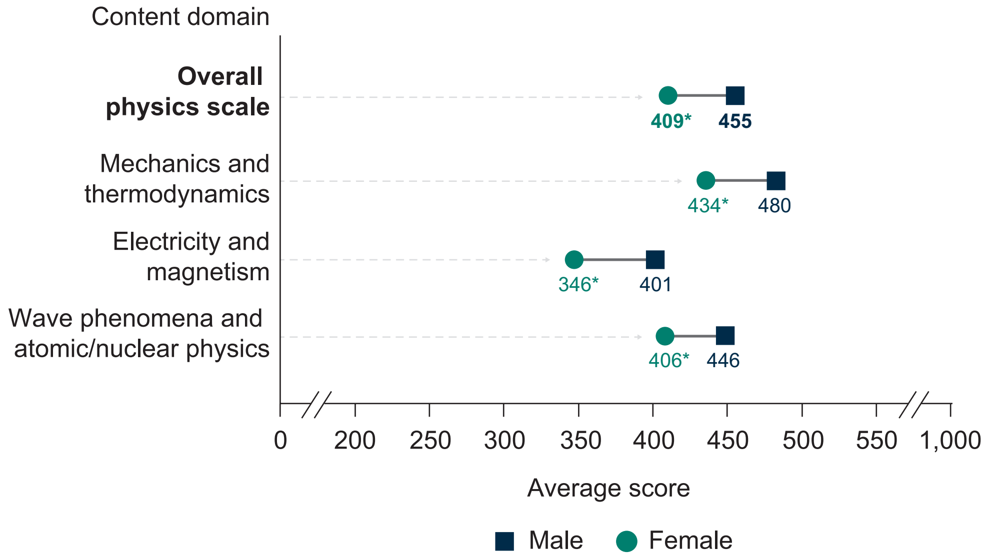
Average physics performance by student and school characteristics

Sex

In the United States, male students outperformed female students in physics overall and on all three content subscales (figure 6-1 and table B-18). Males' average physics score was 455, 46 points higher than females' average score of 409. On the content subscales, male-female differences were 46 points in *mechanics and thermodynamics,* 54 points in *electricity and magnetism,* and 40 points in *wave phenomena and atomic/nuclear physics.*

These differences may be related to the coursetaking patterns observed in section 3, which showed that nearly twice the percentage of males (30 percent) as females (16 percent) had taken AP Physics C as their highest physics course (figure 3-4).

Figure 6-1. Average physics scores of U.S. TIMSS Advanced students, by content domain and sex: 2015



* *p* < .05. Female average score is significantly different from the male average score.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

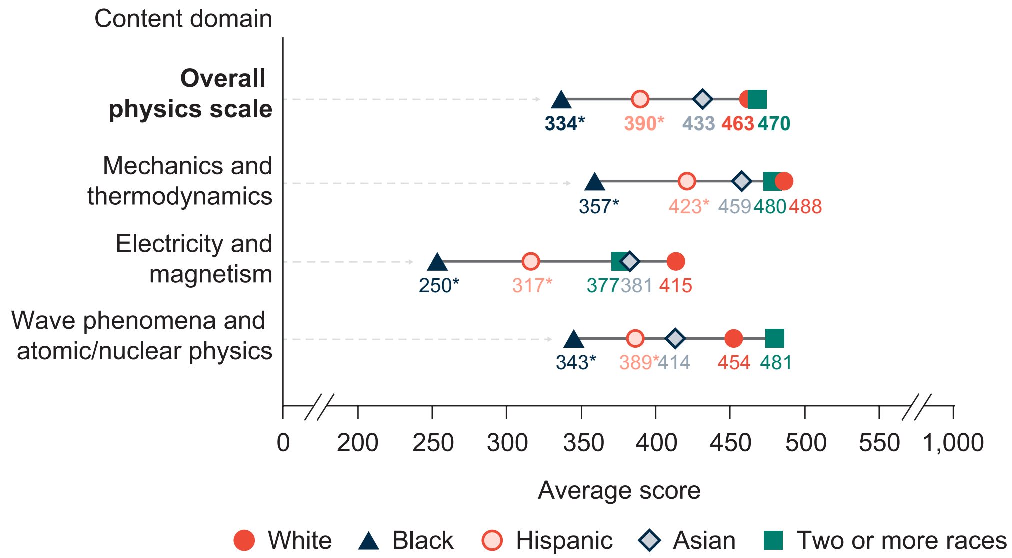
Race/ethnicity

In the United States, average performance in physics differed across racial/ethnic groups overall and on all three content subscales (figure 6-2 and table B-19). The average physics score of White students was higher than the average scores of Black and Hispanic students (463 compared to 334 and 390, respectively) and not measurably different than the average scores of Asian students (433) and students of Two or more races (470).[55] White students' average scores were also higher than Black and Hispanic students' average scores on all three content subscales.

The difference in Black students' average score and the U.S. average was at least 100 points (or about one standard deviation) overall and on all the content subscales except *wave phenomena and atomic/nuclear physics* (where the score difference was 88 points). Average differences for Hispanic students ranged from 39 points (in *mechanics and thermodynamics*) to 63 points (in *electricity and magnetism*) lower than the U.S. average.

As with performance differences by sex, the differences across racial/ethnic groups may be related to the coursetaking patterns observed in section 3, which showed that a lower percentage of Black students than White students had taken one or both of the highest level AP Physics C courses (by 14 percentage points) and a higher percentage of Hispanic students than White students had taken the lowest level AP Physics 1 (by 18 percentage points) (figure 3-5). Additionally, lower percentages of Black, Hispanic, and Asian students than White students had taken AP Physics 2 (by 4, 4, and 3 percent, respectively).

Figure 6-2. Average physics scores of U.S. TIMSS Advanced students, by content domain and race/ethnicity: 2015



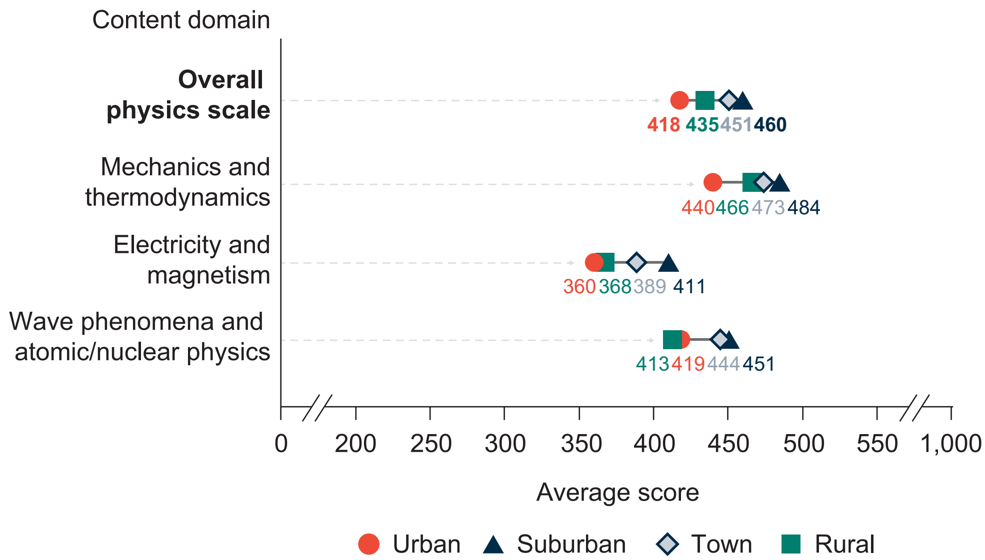
* $p < .05$. Subgroup average score is significantly different from the average score of White students.
NOTE: Reporting standards were not met for Native Hawaiian/Pacific Islander and Native American/Alaska Native students (sample size < 62). Data for these students are not shown separately in the figure, but are included in the U.S. average. Black includes African American and Hispanic includes Latino. Racial categories exclude Hispanic origin.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

[55] Data for American Indian/Alaska Native and Native Hawaiian/Pacific Islander are not reported separately because reporting standards were not met for these groups (i.e., the sample sizes were less than 62 students each). However, these students are included in the U.S. average.

School locale

There were no measurable differences in average scores among U.S. TIMSS Advanced students from different locales—on the overall physics scale or the three content subscales (figure 6-3 and table B-20). This is in contrast to advanced mathematics where the average performance of students in rural schools was lower than those in suburban schools overall and on all three subscales.

Figure 6-3. Average physics scores of U.S. TIMSS Advanced students, by content domain and school locale: 2015



NOTE: Urban is defined as territories inside an urbanized area and inside a principal city. Suburban is defined as territories inside an urbanized area but outside a principal city. Town is defined as territories inside an urban cluster but outside an urbanized area. Rural is defined as territories that are not in an urbanized area or urban cluster. For additional information, see https://nces.ed.gov/programs/edge/docs/NCES_LOCALE_USERSMANUAL_2016012.pdf. There are no measurable differences between the average physics scores of U.S. TIMSS Advanced suburban students and students from other school locales.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Percentage of students reaching TIMSS Advanced international benchmarks in physics

This section describes the percentage of students reaching each of the three TIMSS Advanced international benchmarks: *Advanced, High,* and *Low* (exhibit 6-1). The percentage reaching each benchmark includes the students who reached any higher benchmarks.

Overall, 5 percent of U.S. students reached the *Advanced* international benchmark in physics, 18 percent reached the *High* benchmark, and 39 percent reached the *Intermediate* benchmark (figure 6-4). The international medians for these benchmarks were 5, 18, and 46 percent, respectively.[56] The percentage of U.S. students reaching the Intermediate benchmark was lower than the international median.

[56] The international median represents the middle percentage reaching each benchmark among the countries participating in TIMSS Advanced 2015.

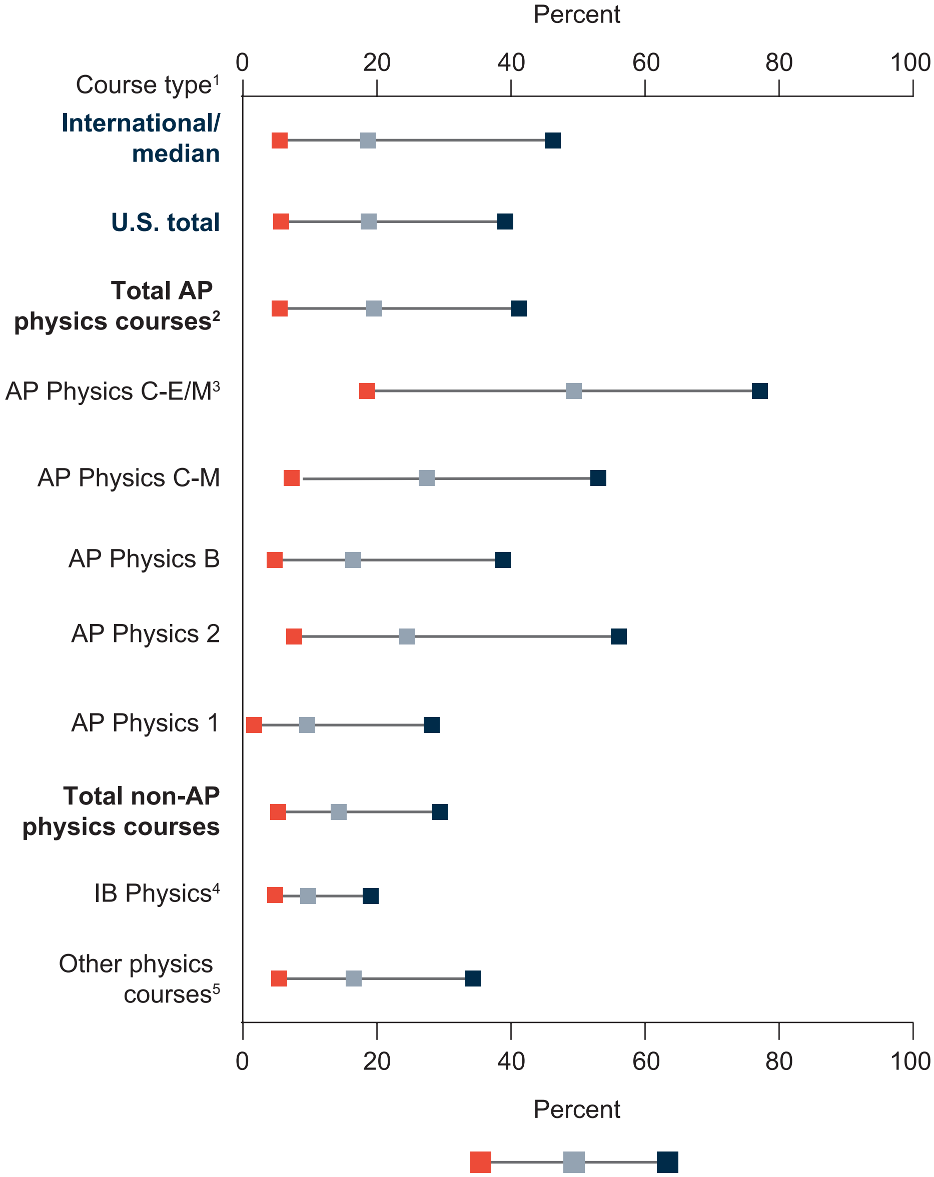
Exhibit 6-1. Description of TIMSS Advanced international benchmarks in physics: 2015

Benchmarks	Descriptions
Advanced (625)	Students communicate their understanding of laws of physics to solve problems in practical and abstract contexts. • In *mechanics and thermodynamics,* students apply knowledge of the motion of objects in freefall, and apply knowledge of heat and temperature to practical problems. • In *electricity and magnetism,* students apply knowledge of electrical circuits and electric fields, and communicate their understanding of magnetic fields. • In *wave phenomena and atomic/nuclear physics,* students communicate their understanding of phenomena related to mechanical and electromagnetic waves, and demonstrate understanding of atomic and nuclear physics. • Across content domains, students design experimental procedures and interpret results; synthesize information in complex diagrams and graphs depicting abstract physics concepts to solve problems; provide multistep calculations of a variety of physical quantities in a range of contexts; draw conclusions about physical phenomena; and provide explanations to communicate scientific knowledge.
High (550)	Students apply basic laws of physics to solve problems in a variety of situations. • In *mechanics and thermodynamics,* students apply knowledge of forces and motion, communicate understanding of the laws of conservation of energy and momentum, and apply knowledge of heat and temperature to solve problems. • In *electricity and magnetism,* students apply knowledge of Ohm's Law and Joule's Law to electric circuits, solve problems involving charged particles in magnetic fields, and apply knowledge of magnetic fields and electromagnetic induction to solve problems. • In *wave phenomena and atomic/nuclear physics,* students show an understanding of phenomena related to electromagnetic waves and knowledge of nuclear reactions. • Across content domains, students interpret information in complex diagrams and graphs depicting abstract concepts; derive formulas and provide calculations of a variety of physical quantities in a range of contexts; evaluate explanations for physical phenomena; and provide brief explanations to communicate scientific knowledge.
Intermediate (475)	Students demonstrate some basic knowledge of the physics underlying a range of phenomena. • In *mechanics and thermodynamics,* students use their knowledge of forces and motion to solve problems, apply knowledge of heat and temperature to energy transfers, and apply knowledge of conservation laws to everyday and abstract contexts. • In *electricity and magnetism,* students show knowledge of electric fields, point charges, and electromagnetic induction. • In *wave phenomena and atomic/nuclear physics,* students apply their knowledge of phenomena related to mechanical and electromagnetic waves and of atomic and nuclear physics to solve problems. • Across content domains, students interpret information in diagrams and graphs to solve problems; calculate a variety of physical quantities in a range of contexts; and evaluate statements to identify explanations for physical phenomena.

Course type

As with the average scores, the percentages of U.S. TIMSS Advanced students reaching the international benchmarks in physics varied by course type (figure 6-4). Among students who had taken the AP Physics C course in electricity and magnetism, 18 percent of students reached the *Advanced* benchmark, 49 percent reached the *High* benchmark, and 77 percent reached the Intermediate benchmark. These percentages were higher than the U.S. total (5, 18, and 39 percent, respectively) and the international medians (5, 18, and 46 percent, respectively).

Figure 6-4. Percentage of U.S. TIMSS Advanced students reaching TIMSS Advanced international benchmarks in physics, by course type: 2015



Course type[1]	Advanced (625)	High (550)	Intermediate (475)
International/ median	**5**	**18**	**46***
U.S. total	**5**	**18**	**39**
Total AP physics courses[2]	**5**	**19**	**41**
AP Physics C-E/M[3]	18*	49*	77*
AP Physics C-M	7	27	53*
AP Physics B	4	16	39
AP Physics 2	7	24	56*
AP Physics 1	1*	9*	28*
Total non-AP physics courses	**5**	**14**	**29**
IB Physics[4]	4	9	19
Other physics courses[5]	5	16	34

* $p < .05$. Percentage is statistically different from the U.S. total percentage at the same benchmark.
[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other physics courses" category only if they have not taken an AP or IB course.
[2] AP Physics C includes courses in mechanics (C-M) and electricity and magnetism (C-E/M). AP Physics B was a 1-year physics course that was discontinued after the 2013–14 school year and was replaced with the 2-year course sequence AP Physics 1 and 2 beginning in 2014–15. Data in the AP Physics B category are generally for students who took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering a 1-year course identified as AP Physics B in 2014–15.
[3] A large majority of the students whose highest physics course was AP Physics C-E/M had also taken AP Physics C-M (89 percent), whether sequentially or in a combined course.
[4] Includes both higher-level and standard-level IB physics courses.
[5] Includes other types of second-year physics courses (including "honors" and "regents" courses).
NOTE: The percentages of U.S. TIMSS Advanced students whose highest course was AP Physics C-E/M who reached each benchmark were higher than the international medians. The percentages of U.S. TIMSS Advanced students whose highest course was AP Physics 1 who reached each of the benchmarks were lower than the international medians.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

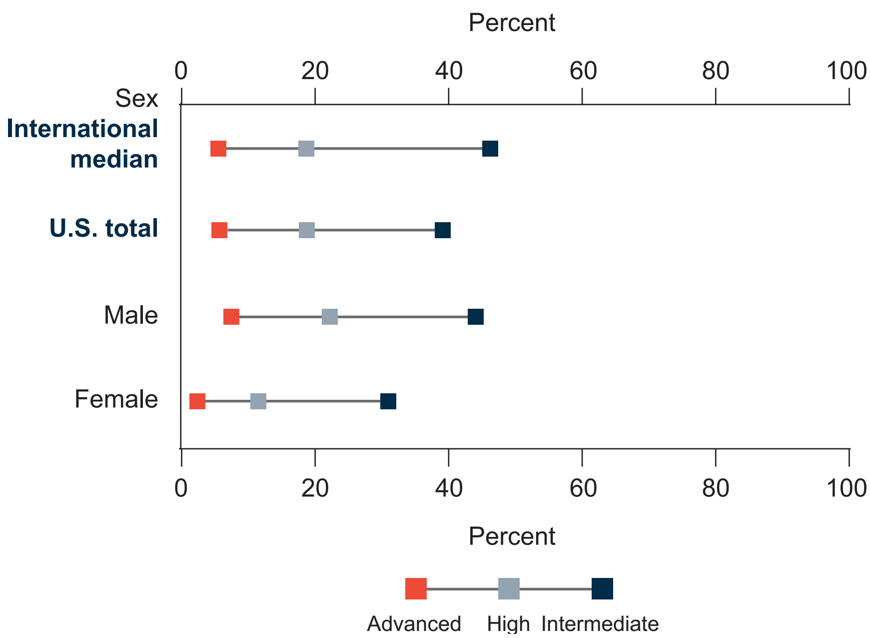
In contrast, among U.S. students who had taken the AP Physics C course in mechanics as their highest physics course, 7 percent of students reached the *Advanced* benchmark, 27 percent reached the *High* benchmark, and 53 percent reached the *Intermediate* benchmark. Only the 53 percent of Physics C mechanics students at the *Intermediate* benchmark was higher than the U.S. total, and there were no measurable differences between the percentage of AP Physics C-Mechanics students and the international medians for any of the benchmarks.

Among U.S. students whose highest course was AP Physics B or 2, there were no measurable differences between the percentages of students reaching any of the benchmarks and the U.S. total or the international medians—except for the 56 percent of AP Physics 2 students who reached the *Intermediate* benchmark, which was higher than the U.S. total. Among students whose highest course was AP Physics 1, some 1 percent reached the *Advanced* benchmark, 9 percent reached the *High* benchmark, and 28 percent reached the *Intermediate* benchmark; these percentages were lower than the U.S. total (and international medians) for all benchmarks.

Sex

In the United States, the percentages of U.S. TIMSS Advanced students reaching the international benchmarks in physics varied by sex (figure 6-5). Seven percent of male students reached the *Advanced* benchmark, 22 percent reached the *High* benchmark, and 44 percent reached the *Intermediate* benchmark; these percentages were higher than the U.S. total (5, 18, and 39 percent, respectively). In contrast, 2 percent of female students reached the *Advanced* benchmark, 11 percent reached the *High* benchmark, and 31 percent reached the *Intermediate* benchmark; these percentages were lower than the U.S. total. For males, the percentage reaching the *High* benchmark was also higher than the international median for that benchmark. The percentages of females reaching each benchmark, however, were lower than the international medians.

Figure 6-5. Percentage of U.S. TIMSS Advanced students reaching TIMSS Advanced international benchmarks in physics, by sex: 2015



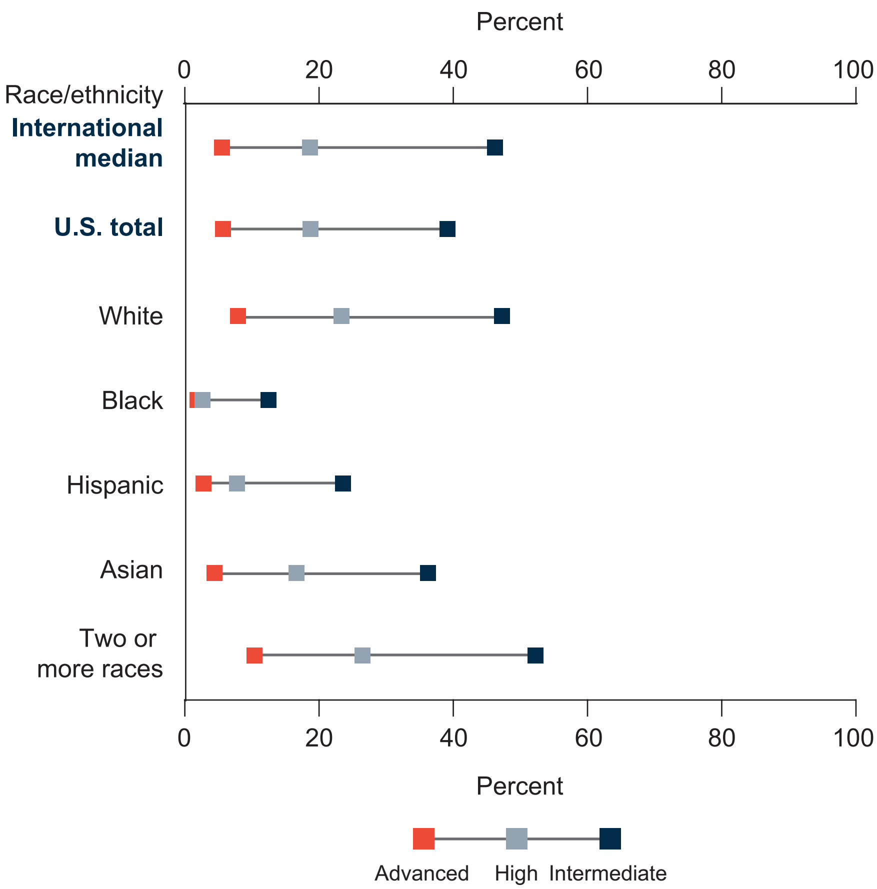
Sex	Advanced (625)	High (550)	Intermediate (475)
International median	5	18	46*
U.S. total	5	18	39
Male	7*	22*	44*
Female	2*	11*	31*

* $p < .05$. Percentage is statistically different from the U.S. total percentage at the same benchmark.
NOTE: The percentage of male U.S. TIMSS Advanced students reaching the *High* benchmark was higher than the international median. The percentages of female U.S. TIMSS Advanced students reaching each of the benchmarks were lower than the international medians.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Race/ethnicity

Differences were also seen across racial/ethnic groups in the United States (figure 6-6). Higher percentages of White students reached the *High* (23 percent) and *Intermediate* (47 percent) international benchmarks in physics than the U.S. total (18 and 39 percent, respectively). The percentage of White students reaching the *High* benchmark was also higher than the international median. In contrast, lower percentages of Black students reached each of the three benchmarks (1, 2, and 12 percent at *Advanced, High,* and *Intermediate,* respectively) than the U.S. total, as did lower percentages of Hispanic students (2, 7, and 23 percent, respectively). Both of these racial/ethnic groups' percentages were also lower than the international medians for each of the benchmarks.

Figure 6-6. Percentage of U.S. TIMSS Advanced students reaching TIMSS Advanced international benchmarks in physics, by race/ethnicity: 2015



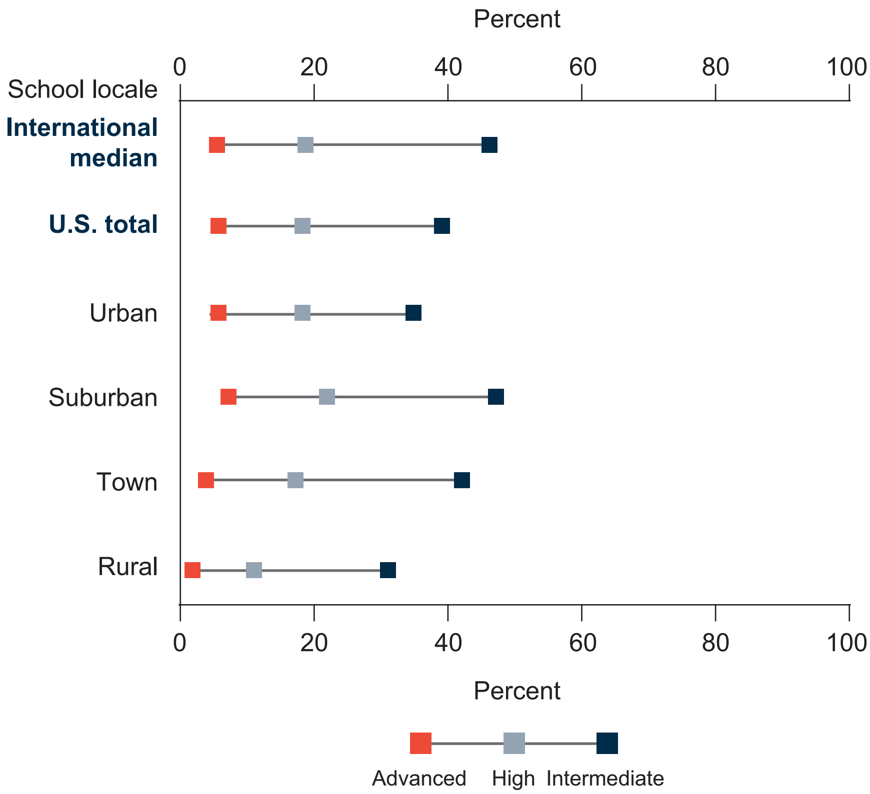
Race/ethnicity	Advanced (625)	High (550)	Intermediate (475)
International median	**5**	**18**	**46***
U.S. total	**5**	**18**	**39**
White	7	23*	47*
Black	1*	2*	12*
Hispanic	2*	7*	23*
Asian	4	16	36
Two or more races	10	26	52

* $p < .05$. Percentage is statistically different from the U.S. total percentage at the same benchmark.
NOTE: Reporting standards were not met for Native Hawaiian/Pacific Islander and Native American/Alaska Native students (sample size < 62). Data for these students are not shown separately in the figure, but are included in the U.S. average. Black includes African American and Hispanic includes Latino. Racial categories exclude Hispanic origin. The percentages of White U.S. TIMSS Advanced students reaching the *High* benchmark was higher than the international median. The percentages of Black and Hispanic students reaching each of the benchmarks were lower than the international medians.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

School locale

Unlike advanced mathematics (where lower percentages of U.S. TIMSS Advanced students in rural schools reached the *Advanced, High* and *Intermediate* benchmarks), there were no measurable differences between U.S. students from different school locales and the U.S. total at any of the benchmarks in physics (figure 6-7). Nor were there any measurable differences from the international medians at any of the benchmarks in physics.

Figure 6-7. Percentage of U.S. TIMSS Advanced students reaching TIMSS Advanced international benchmarks in physics, by school locale: 2015



School locale	Advanced (625)	High (550)	Intermediate (475)
International median	**5**	**18**	**46***
U.S. total	**5**	**18**	**39**
Urban	5	18	35
Suburban	7	22	47
Town	4	17	42
Rural	2	11	31

* $p < .05$. Percentage is statistically different from the U.S. total percentage at the same benchmark.
NOTE: Urban is defined as territories inside an urbanized area and inside a principal city. Suburban is defined as territories inside an urbanized area but outside a principal city. Town is defined as territories inside an urban cluster but outside an urbanized area. Rural is defined as territories that are not in an urbanized area or urban cluster. For additional information, see https://nces.ed.gov/programs/edge/docs/NCES_LOCALE_USERSMANUAL_2016012.pdf. There were no measurable differences in the percentages of U.S. TIMSS Advanced students from any school locale from the international medians reaching any of the benchmarks.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Coursetaking patterns of students reaching each international benchmark in physics

To further understand U.S. performance in physics, this subsection compares the coursetaking patterns of students at different achievement levels to the coursetaking patterns of the U.S. TIMSS Advanced population overall (i.e., U.S. total) (figure 6-8 and table B-21). In the overall population (table 2-3), the largest percentage of students (42 percent) had taken the first-year AP Physics 1 course as their highest level course compared to 25 percent who had taken one or both of the AP Physics C courses. This was not the case for the subsets of U.S. students who reached each of the international benchmarks in physics.

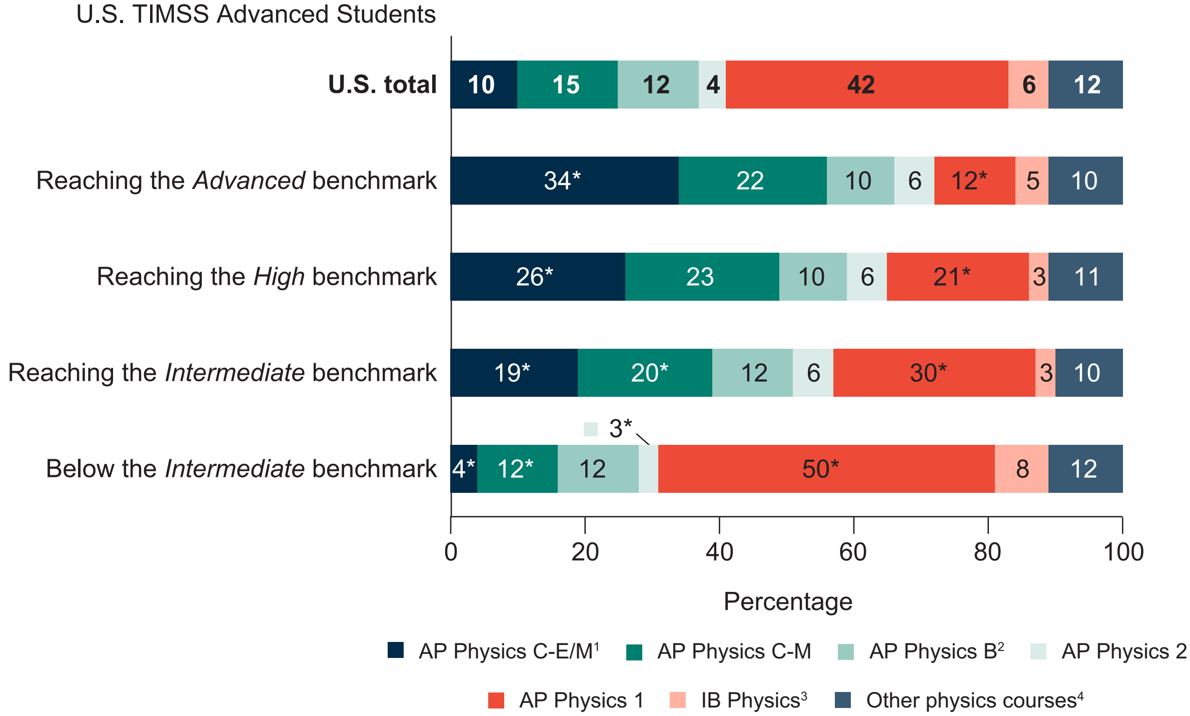
Among the students who reached the *Advanced* benchmark (5 percent of U.S. TIMSS Advanced students overall) (figure 6-4), the majority (56 percent) had taken an AP Physics C course as their highest course—34 percent in electricity and magnetism and 22 percent in mechanics[57]—and only 12 percent had taken AP Physics 1 as their highest course (figure 6-8). This distribution across the different course types differed markedly from the distribution for U.S. TIMSS Advanced students overall. Specifically, the 34 percent of students reaching the *Advanced* benchmark who had taken the highest level AP Physics C course was higher than the U.S. total (10 percent); and the 12 percent taking AP Physics 1 was lower than the U.S. total (42 percent). The percentages of students reaching the *Advanced* benchmark who had taken other AP and non-AP courses as their highest course were also generally low (10 percent or less) and did not differ from the corresponding percentages in the U.S. total.

Similar patterns were observed among students who reached the *High* and *Intermediate* benchmarks (18 percent and 39 percent of U.S. TIMSS Advanced students overall, respectively), with higher percentages taking AP Physics C-Electricity and Magnetism and lower percentages taking AP Physics 1 than the corresponding percentages in the U.S. total.

In contrast, half of the students who did not reach the *Intermediate* benchmark (61 percent of U.S. TIMSS Advanced students overall) had taken AP Physics 1 as their highest course, which was higher than the U.S. total. Lower percentages of these students had taken an AP Physics C course (4 percent in electricity and magnetism and 12 percent in mechanics) compared to the U.S. totals.

[57] A large majority of the students whose highest physics course was the AP Physics C course in electricity and magnetism had also taken the AP Physics C course in mechanics (89 percent), either sequentially or in a combined course.

Figure 6-8. Percentage distribution of U.S. TIMSS Advanced students reaching each TIMSS Advanced international benchmark in physics compared to the U.S. total, by course type: 2015



* *p* < .05. Percentage is statistically different from the U.S. total percentage.
[1] Advanced Placement (AP) Physics C course in electricity and magnetism (C-E/M). A large majority of the students whose highest physics course was AP Physics C-E/M had also taken an AP Physics C course in mechanics (C-M) (89 percent), whether sequentially or in a combined course.
[2] AP Physics B was a 1-year physics course that was discontinued after the 2013–14 school year and was replaced with the 2-year course sequence AP Physics 1 and 2 beginning in 2014–15. Data in the AP Physics B category are generally for students who took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering a 1-year course identified as AP Physics B in 2014–15.
[3] Includes higher-level and standard-level IB physics courses.
[4] Includes other types of second-year physics courses (including "honors" or "regents" courses).
NOTE: Course type reflects the highest level physics course taken. If students took another course in addition to an AP or IB course, they are included in the AP or IB category. Students are included in the "other physics courses" category only if they have not taken an AP or IB course. This figure compares the coursetaking patterns of students reaching each benchmark with the coursetaking patterns of U.S. students overall. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

6.2 How did U.S. students' performance on TIMSS Advanced physics items vary across content domains and relate to the level of topic coverage?

This subsection examines U.S. TIMSS Advanced students' performance on individual physics items. The first part describes the average U.S. performance across items by content domain and broad topic area within each content domain. The second part describes how item performance relates to the level of topic coverage in the intended and implemented curriculum. The textbox below summarizes the item statistics used to produce the results in this section and relates these to the specific tables and figures shown. (See section 4 for a more complete description of the analyses used to produce the results shown in this section.)

Item Statistics Used in Section 6.2

Percent correct is the weighted percentage of students receiving full or partial credit on each item. This report shows item percent correct for each item within the individual TIMSS Advanced physics topics.

Average percent correct is the percent correct averaged across items. In this report, averages are computed for different item sets to provide results for

a. physics overall (across all physics items),
b. each content domain, and
c. each broad topic area within a content domain.

Relative performance by content domain and topic area is determined by comparison to the average percent correct for physics overall.

Average U.S. performance on physics items by content domain and broad topic area within each content domain

In the United States, the average percent correct overall (i.e., across all TIMSS Advanced physics items) was 42 percent (table 6-4). This percentage is used as the reference point for identifying content domains and topics areas in which item performance is relatively higher or lower than physics overall (i.e., those with a higher or lower average percent correct than the overall 42 percent).

Looking across content domains, the U.S. average percent correct was relatively higher on the items in *mechanics and thermodynamics* (44 percent correct) and *wave phenomena and atomic/nuclear physics* (43 percent correct) and was relatively lower on the items in *electricity and magnetism* (36 percent correct), compared to physics items overall (42 percent correct). Within content domains, there were also variations in the average item percent correct across topic areas.

- In *mechanics and thermodynamics,* the U.S. average percent correct for two topic areas—*forces and motion* (48 percent) and *laws of conservation* (49 percent)—was higher than in physics overall. In contrast, in the topic area of *heat and temperature,* U.S. students had a lower average percent correct (36 percent) than in physics overall.
- In *electricity and magnetism,* the average percent correct for both topic areas was relatively lower than physics overall (39 percent correct for *electricity and electric circuits* and 33 percent correct for *magnetism and electromagnetic induction*).

- In *wave phenomena and atomic/nuclear physics,* the average percent correct for one topic area (*wave phenomena*) was relatively higher (44 percent) than physics overall, while that in the other topic area (*atomic and nuclear physics*) was not measurably different than physics overall.

Table 6-4. Average U.S. performance on TIMSS Advanced physics items, by content domain and topic area: 2015

Content domain and topic area	Number of items	U.S. average percent correct[1]
Physics (all content domains and topic areas)	**101**	**42**
Mechanics and thermodynamics	**38**	**44** ▲
Forces and motion	19	48 ▲
Laws of conservation	9	49 ▲
Heat and temperature	10	36 ▼
Electricity and magnetism	**28**	**36** ▼
Electricity and electrical circuits	14	39 ▼
Magnetism and electromagnetic induction	14	33 ▼
Wave phenomena and atomic/nuclear physics	**35**	**43** ▲
Wave phenomena	19	44 ▲
Atomic and nuclear physics	16	41

▲ Content domain/topic area average is higher than physics (all content domains and topic areas) average ($p < .05$).

▼ Content domain/topic area average is lower than physics (all content domains and topic areas) average ($p < .05$).

[1] Average percent correct is the weighted percentage of students receiving full credit (1 point on multiple-choice items, 1 point on short constructed-response items, and 2 points on extended constructed-response items) or partial credit (1 point on extended constructed-response items), averaged across items.

SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.(TIMSS) Advanced, 2015.

How U.S. students' performance on TIMSS Advanced physics items relates to the level of topic coverage

The prior results show how average U.S. student performance on physics items varied across the content domains and broad topic areas within each content domain. The results presented here show how student performance on the set of items in each individual topic relates to the level of topic coverage (from section 3) in their physics courses.

To examine how U.S. student performance relates to students' opportunity to learn, TIMSS Advanced topics are organized into three levels of topic coverage in the United States—high, moderate, and low—based on both the intended and implemented curriculum data (tables A-4a, A-4b and A-4c)[58] For each topic, a graph is included that shows the distribution of percent correct for the set of items in that topic and identifies the item type: multiple-choice, short constructed-response, or extended constructed-response (figure 6-9 and table B-22). These distributions show how tightly clustered or how widely spread items are in terms of percent correct and thus indicate the range of U.S. performance across the specific set of items developed to assess each topic. Even for topics reported as taught to most students, performance across the individual items within and across topics will still vary for a variety of reasons, including how challenging the content covered by the topic is, the breadth of content covered by the topic, and the specific requirements of the items and their contexts.

[58] See definitions of the high, moderate, and low categories for the level of topic coverage in section 4 and in the notes on figure 6-9. Percent correct reflects the weighted percentage of students receiving full or partial credit on each item.

Four physics topics are in the high coverage category, and three are in the moderate category, all but one of which are from the topic areas related to mechanics (*forces and motion* and *laws of conservation*). The majority of physics topics (16 of 23) are in the low coverage category—including 3 of 9 topics from *mechanics and thermodynamics* (all related to thermodynamics) and all topics from *electricity and magnetism* and *wave phenomena and atomic/nuclear physics* (except *mechanical waves*).

The sections below describe the range of U.S. item performance on TIMSS Advanced physics topics in the high, moderate and low coverage categories.

High level of topic coverage

All four of the high coverage topics involve mechanics and are universally taught across the TIMSS Advanced-eligible physics courses (table A-4a). Still, there is a range of performance across the items assessing these topics. In particular, a broad range of item performance was found for *kinetic and potential energy; conservations of mechanical energy* (13 to 84 percent correct) and *forces, including frictional force, acting on a body* (23 to 73 percent correct) (figure 6-9). In comparison, performance on items in *applying Newton's laws and laws of motion* tended to be on the higher end of the performance range (45 to 85 percent), and the three items in *law of conservation of momentum; elastic and inelastic collisions* were clustered in the mid-performance range (44 to 50 percent correct) of U.S. students on TIMSS Advanced physics items.

Moderate level of topic coverage

The three topics in the moderate coverage category include two topics related to mechanics and one topic related to mechanical waves. Like the high coverage topics, the two moderate-coverage mechanics topics are also covered in the intended curriculum across the TIMSS Advanced-eligible physics courses, but physics teachers reported that a slightly lower percentage of students (95 to 97 percent) had been taught these topics by the time of the assessment (table A-4a). With the exception of one extended constructed-response item in the topic of *law of gravitation in relation to movement of celestial objects* requiring students to solve a quantitative problem and show their work (13 percent correct), item performance in both of the mechanics topics was in the mid-performance range (31 to 60 percent correct) (figure 6-9).

The percentage of students taught the third moderate-coverage topic of *mechanical waves; the relationship between speed, frequency, and wavelength* was lower (87 percent of students overall) and varied across the different TIMSS Advanced-eligible courses, ranging from about 80 percent of students in AP Physics 1 and AP Physics C-mechanics to all or nearly all students (99–100 percent) taking AP Physics B, IB Physics, and other non-AP, IB courses (table A-4c). Performance on the items related to mechanical waves ranged from 41 to 69 percent correct, with five of six items with at least 50 percent correct (figure 6-9).

Low level of topic coverage

The low coverage category includes three topics related to thermodynamics (from *mechanics and thermodynamics*), all six topics from *electricity and magnetism,* and seven of eight topics from *wave phenomena and atomic/nuclear physics.* All of the low-coverage topics are included in the intended curriculum of AP Physics B, AP Physics 2 and the higher level IB physics course (except *thermal radiation, temperature, and wavelength*), while only three topics are even partially covered in AP Physics 1 (*electrostatic attraction or repulsion between isolated charged particles; electrical circuits;* and *reflection, refraction, interference, and diffraction*) (tables A-a, A-4b and A-4c). The percentage of U.S. TIMSS Advanced students overall who had been taught the low-coverage topics by the time of the assessment ranged from 45 percent (*mass-energy equivalence in nuclear reactions and particle transformations*) to 82 percent (*electrostatic attraction or repulsion between isolated charged particles-Coulomb's law*), with most topics (13 of 16) taught to between 50 and 75 percent of students.

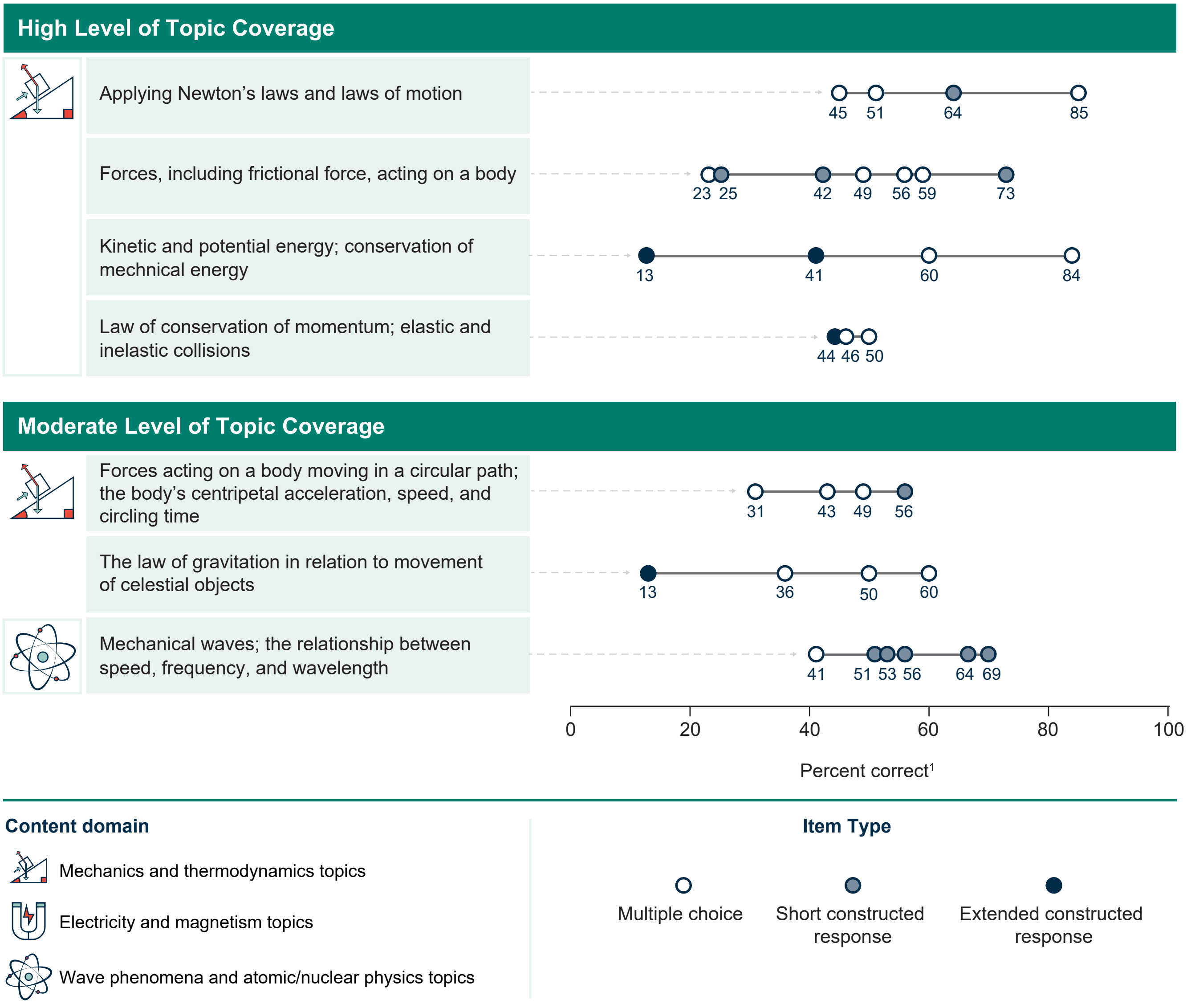
All three topics related to thermodynamics were reported by teachers as taught to between 62 and 64 percent of students overall, which reflects about half of students taking AP Physics 1 and less than 60 percent of students taking the specialized AP Physics C courses in mechanics and electricity and magnetism (table A-4a). These topics are not included in the intended curricula of AP Physics 1 or the specialist AP Physics C courses in mechanics and electricity and magnetism. Although they may have been covered in prior physics courses, coverage may be underestimated if some AP Physics teachers do not know what topics students covered in their prior courses. The performance of U.S. students on the items covering thermodynamics ranged from 13 percent correct (*heat transfer and specific heat capacities*) to 71 percent correct (*first law of thermodynamics*) (figure 6-9). In general, though, performance on these topics was lower than the topics at the high and moderate coverage level, with half of the low-coverage thermodynamics items (6 of 12) at or below 30 percent correct (all of which were constructed-response items). Three multiple-choice items had relatively higher performance, including two items in *heat transfer and specific heat capacities* (53 and 69 percent correct) involving energy transfer in familiar Earth systems (radiation from the Sun and temperature decrease when air rises) and one item in *first law of thermodynamics* (71 percent correct) relating work done on a container of gas to changes in internal energy and temperature.

All of the topics in *electricity and magnetism* are included in the intended curriculum for AP Physics B, AP Physics 2, and the specialized AP Physics C course as well as the higher level IB Physics course (table A-4b). These topics are not covered in AP Physics 1 except for *electrostatic attraction/repulsion* and *electrical circuits,* which are covered at a basic level. Teachers reported that many students had not been taught some of these topics by the time of the assessment. The first three topics from the topic area of *electricity and electric circuits* were reported to have been taught to between 74 and 82 percent of students overall, while the last three topics from the topic area of *magnetism and electromagnetic induction* had been taught to between 49 and 58 percent of students. Overall, U.S. item performance covered a broad range for both the *electricity and electric circuits* topics (14 to 68 percent correct) and especially the *magnetism and electromagnetic induction* topics (5 to 74 percent correct) (figure 6-9). One exception is the topic of *electrical circuits; using Ohm's law and Joule's law* (which is at least partially covered in AP Physics 1) where performance on the five items was more clustered in the mid-performance range (29 to 45 percent correct). The item in the topic of *charged particles in a magnetic field* with extremely low performance (5 percent correct) is an extended constructed-response item that requires students to predict and explain the direction of movement of a foil strip connected to a power source when it is placed next to a bar magnet. In comparison, there are two items from the topics of (a) *relationship between magnetism and electricity; magnetic fields around electric conductors; electromagnetic induction* and (b) *Faraday's and Lenz's laws of induction* with relatively higher performance (74 and 71 percent correct, respectively). These two multiple-choice items involve a magnet moving through a metal coil but, in contrast to other items in these topics, do not require specific knowledge about the direction of induced current.

The low-coverage category includes three topics from the topic area of *wave phenomena* and four topics from the topic area of *atomic and nuclear physics.* All these topics (except *thermal radiation, temperature, and wavelength*) are included in the intended curricula for AP Physics B, AP Physics 2 and the higher level IB Physics course, while none are included for AP Physics 1, except a portion of the topic on *reflection, refraction, interference, and diffraction* (table A-4c). The specialized AP Physics C courses in mechanics and electricity and magnetism also do not include any of these topics. As noted above for thermodynamics topics, students may have covered some of these topics in prior courses, but their AP Physics teachers may not know this. Based on teachers' reports, coverage of these topics in the implemented curriculum varied across topics and courses.

The percentage of students overall who had been taught the topics from the topic area of *wave phenomena* ranged from 53 percent (*thermal radiation, temperature, and wavelength*) to 73 percent (*electromagnetic radiation*), and U.S. item performance ranged from 11 percent correct (*thermal radiation, temperature, and wavelength*) to 65 percent correct (*electromagnetic radiation*) (figure 6-9). Coverage of the topics from the topic area of *atomic and nuclear physics* ranged from 45 percent of students taught overall (*mass-energy equivalence in nuclear reactions and particle transformations*) to 73 percent (*structure of the atom and its nucleus; atomic number and atomic mass; electromagnetic emission and absorption and the behavior of electrons*). As might be expected, the percentage of AP Physics 1 students who had been taught the topics related to nuclear physics was very low (between 20 and 30 percent). Even for AP Physics B and AP Physics 2, where the topics are included in the intended curriculum, many students had not been taught these topics by the time of the assessment (between 75 and 85 percent taught). Only for the IB Physics courses did teachers report nearly universal coverage of these topics. In comparison, about 60 percent of students in other non-AP, non-IB physics courses had been taught these topics. U.S. item performance in the two topics related to nuclear reactions ranged from 9 to 59 percent correct. The two items in this topic with the lowest performance (9 percent correct) were constructed-response items that required detailed or quantitative understanding of the nuclear process. In comparison, the three items on *structure of the atom and its nucleus; atomic number and atomic mass; electromagnetic emission and absorption and the behavior of electrons* were clustered in the mid-performance range (35 to 41 percent correct). Despite the low topic coverage for *wave-particle duality and the photoelectric effect* (50 percent of students taught overall), U.S. performance on the specific set of items that assess this topic was more clustered on the upper half of the performance range (47 to 69 percent correct).

Figure 6-9. U.S. performance on TIMSS Advanced physic items, by level of topic coverage: 2015



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See notes at end of figure.

Figure 6-9. U.S. performance on TIMSS Advanced physic items, by level of topic coverage: 2015—Continued

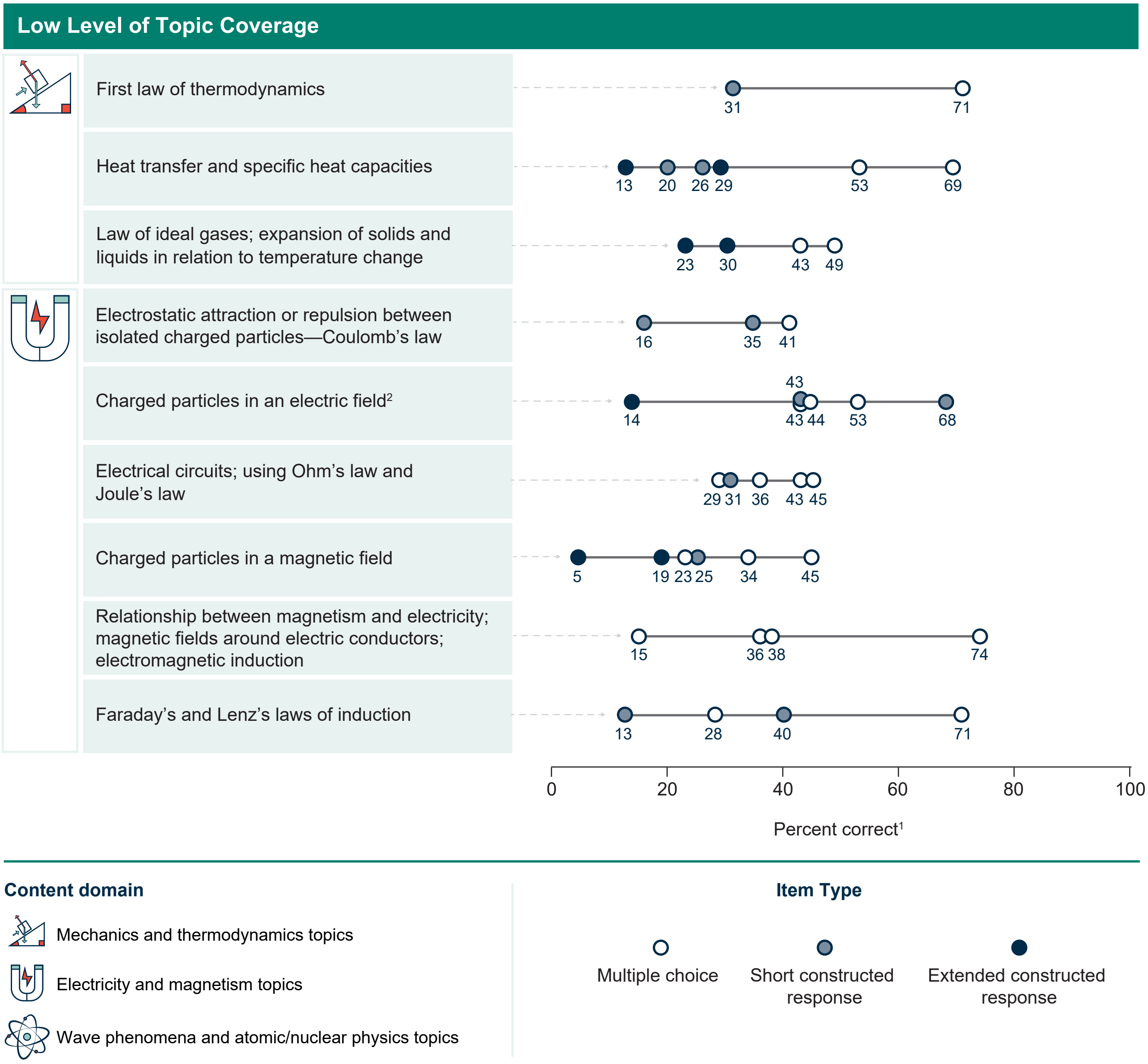
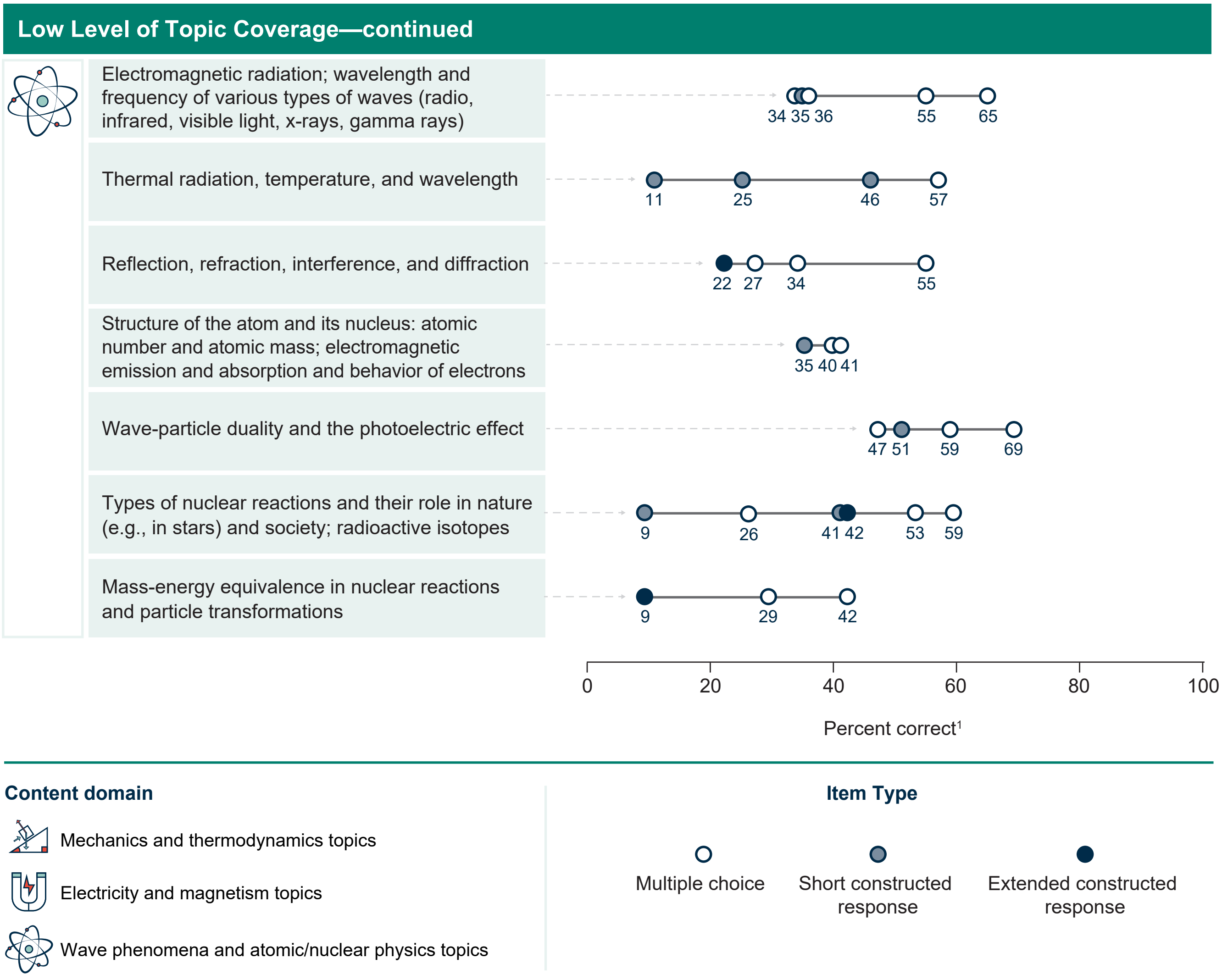


Figure continues on next page.
See notes at end of figure.

Figure 6-9. U.S. performance on TIMSS Advanced physic items, by level of topic coverage: 2015—Continued



[1] Percent correct is the percentage of students receiving credit on each item. For multiple-choice and short constructed-response items (each worth one score point), this reflects the percentage of students who provided a correct answer. For extended constructed-response items, this reflects the weighted percentage of students receiving full credit (2 points) or partial credit (1 point).
[2] In this topic, there are two items that round to the same percent correct (43 percent).
NOTE: Level of topic coverage is based on (a) topic coverage in the intended curricula for the TIMSS Advanced-eligible Advanced Placement (AP) and International Baccalaureate (IB) physics courses; and (b) the percentage of students who had been taught the topic at the time of the assessment (either in the current year or a prior year) as reported by their physics teachers. High indicates that the topic (a) is covered in the intended curriculum for AP and IB physics courses (or reflects foundational knowledge expected to be have been covered in a prior physics course); **and** (b) has been taught to all or nearly all students (99.0–100 percent). Moderate indicates that the topic (a) is at least partially covered in the intended curriculum for all AP and IB physics courses (or reflects foundational knowledge expected to be have been covered in a prior physics course); **and** (b) has been taught to at least 85 percent of students (85.0–98.9 percent). Low indicates that the topic (a) is *not* covered in the intended curriculum for at least one of the AP or IB physics courses; **or** (b) has been taught to less than 85 percent of students (0–84.9 percent). Detail is provided in supplemental tables A-4a, 4b and 4c.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

6.3 What are some common approaches, misconceptions, and errors in physics demonstrated by U.S TIMSS Advanced students?

This subsection uses six example items and their associated student performance data to illustrate some common approaches, misconceptions, and errors in physics demonstrated by U.S. TIMSS Advanced students. The example items

- cover a range of topics from across the physics domain and reflect varying item performance and coverage levels; and
- illustrate important misconceptions or errors made by U.S. students, especially those for which there are differences by course type.

The textbox below identifies the physics example items and summarizes the information provided in the accompanying exhibits.

Summary of Physics Example Items

Example 1 ("Motion of a ball thrown vertically upward"): a two-part (multiple-choice and constructed-response) *mechanics and thermodynamics* (mechanics) item from the *knowing* cognitive domain (exhibit 6-2)

Example 2 ("Skiers collide"): a constructed-response *mechanics and thermodynamics* (mechanics) item from the *applying* cognitive domain (exhibit 6-3)

Example 3 ("Volume of a gas"): a constructed-response *mechanics and thermodynamics* (thermodynamics) item from the *applying* cognitive domain (exhibit 6-4)

Example 4 ("Fish generates an electric field"): a multiple-choice *electricity and magnetism* (electricity) item from the *applying* cognitive domain (exhibit 6-5)

Example 5 ("Electron beam in a magnetic field"): a multiple-choice *electricity and magnetism* (magnetism) item from the *reasoning* cognitive domain (exhibit 6-6)

Example 6 ("Mass change in a nuclear reaction"): a multiple-choice *wave phenomena and atomic/nuclear physics* (atomic/nuclear physics) item from the *knowing* cognitive domain (exhibit 6-7)

Each exhibit shows the item; a sample student response (for constructed-response items); data on how the item was classified in terms of the content domain, cognitive domain, and international benchmark level;[59] patterns in the U.S. percent correct (for multiple-choice and short constructed-response items) or percent full credit (for extended constructed-response items) by course type; the scoring guide, detailing correct, partial, and incorrect responses (including specific misconceptions and errors which were tracked using special diagnostic codes);[60] and data on the percentage distribution of U.S. students across response categories by course type. (See section 4 for a description of the item-level statistics reported in this section.)

For additional physics example items that show U.S. student performance in an international context, see the TIMSS Advanced 2015 international report.

[59] Item classification data were provided by the TIMSS International Study Center. An item maps to an international benchmark level on the TIMSS Advanced physics scale if students at or above that level are likely to answer the item correctly, while students below that benchmark are not.

[60] See appendix C for more information about diagnostic codes and the two-digit scoring system used in TIMSS Advanced.

Example 1—"Motion of a ball thrown vertically upward"

Exhibit 6-2 shows this two-part physics item from the topic area of *forces and motion* in the *mechanics and thermodynamics* content domain. It requires students to apply Newton's laws of motion to answer two questions about the motion of a ball thrown vertically upward. Although the topic of *applying Newton's laws and laws of motion*—classified as high coverage (figure 6-9)—is covered in all of the TIMSS Advanced-eligible physics courses and has been taught to all U.S. TIMSS Advanced students (table A-4a), there are some common misconceptions and differences in performance depending on the specific physics courses taken.

Part A is a multiple-choice item requiring students to identify the acceleration of the ball at its highest position. A correct response to part A (option D) requires students to know that the acceleration due to gravity is constant and applies equally to the ball at all positions. Part A maps to the *High* international benchmark level. Overall, 51 percent of U.S. TIMSS Advanced students answered part A correctly, which was higher than the international average (34 percent correct) (see the first page of exhibit 6-2 and table B-23a).

U.S. performance on part A varied by the highest physics course taken, with 28 percent of students who had taken non-AP physics courses providing a correct response (which was lower than the U.S. total of 51 percent), compared to 55 percent who had taken an AP physics course (which was higher than the U.S. total). The higher-than-average percentage for students who had taken AP physics courses was driven by the students in AP Physics C courses (75 percent correct for those taking the electricity and magnetism course and 77 percent correct for those taking the mechanics course). The percentages correct for students in AP Physics 1, 2, and B were not measurably different from the U.S. total.

Of the 49 percent of U.S. students overall who did not provide a correct response to part A, most (41 percent overall) demonstrated a specific misconception that there is no acceleration when the ball momentarily stops moving as it reverses direction at its maximum height (option A) (see the second page of exhibit 6-2). This misconception was less common among students who had taken an AP Physics C course (21 percent) and more common among students taking non-AP physics courses (65 percent) than U.S. students overall (41 percent). In comparison, the percentage of students who had taken other AP physics courses demonstrating the misconception ranged from 42 to 49 percent.

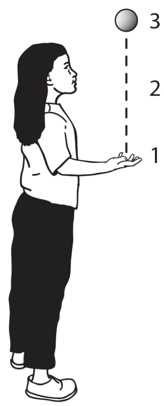
In part B, students were asked to determine the time duration between two points on the path of the ball. A correct response to part B (code 10 in the scoring guide) requires students to indicate that the time traveled by the ball is the same on the way up as it is on the way down, as shown in the example correct student response. This example presents a higher level student response that includes an explanation; however, an explanation is not required and students who just state that the times are the same also received full credit. Part B maps to the *Intermediate* international benchmark level. Overall, 64 percent of U.S. TIMSS Advanced students answered part B correctly, which was higher than the international average (48 percent correct) (see the first page of exhibit 6-2 and table B-23b).

Forty-nine percent of students who had taken non-AP physics courses provided a correct response compared to 67 percent who had taken an AP physics course (although neither of these percentages was measurably different from the U.S. total of 64 percent). Students who had taken AP Physics B or an AP Physics C course, however, performed higher than the U.S. total (75 and 73 percent, respectively).

Of the 36 percent of U.S. students overall who did not provide a correct response to part B, more than half (19 percent overall) demonstrated a specific misconception that the time on the way down is shorter because the ball is accelerating (speeding up) on the way down and decelerating (slowing down) on the way up (code 78) (see the third page of exhibit 6-2). This misconception was more common among students who had taken an IB physics course (30 percent) and less common among students who had taken an AP Physics C course (14 percent) than among U.S. TIMSS Advanced students overall. Another 15 percent of U.S. students overall provided other types of incorrect responses, and 2 percent left part B of the item blank.

Exhibit 6-2. TIMSS Advanced physics example item 1 with student performance data, by course type: 2015

Sally throws a ball vertically upward as shown. The ball moves from her hand at point 1 to a maximum height at point 3. Point 2 is halfway between points 1 and 3. The ball has an acceleration of $-10 m/s^2$ at point 2.



A. What is the acceleration at point 3 at the instant between its upward motion and downward motion? Disregard air resistance.

Ⓐ zero m/s^2

Ⓑ $\frac{-10}{2} m/s^2$

Ⓒ $2(-10) m/s^2$

● $-10 m/s^2$

B. How does the time duration between points 2 and 3 on the way up compare to the time duration between points 3 and 2 on the way down? Disregard air resistance.

the time duration would be the same from 2-3 as 3-2 because the initial velocity of the ball from 2-3 is the same as the final velocity from 3-2 and the acceleration is the same.

Item classification and description	
Content domain:	Mechanics and thermodynamics
Topic area:	Forces and motion
Topic:	Applying Newton's laws and laws of motion
Cognitive domain:	Knowing
International benchmark:	High (part A); Intermediate (part B)
Description:	Determine the acceleration and time duration of vertically thrown ball

Student performance data

Course type[1]	Percent correct[2]	
	Part A	Part B
International average	**34** ▼	**48** ▼
U.S. total	**51**	**64**
AP physics courses[3]	**55** ▲	**67**
AP Physics C courses	76 ▲	73 ▲
AP Physics C-E/M[4]	75 ▲	73
AP Physics C-M	77 ▲	73 ▲
AP Physics B	45	75 ▲
AP Physics 2	53	61
AP Physics 1	46	61
Non-AP physics courses	**28** ▼	**49**
IB Physics[5]	23! ▼	39! ▼
Other physics courses[6]	31!	54!

▼ Subgroup/international average percent correct is significantly lower than U.S. total percent correct ($p < .05$).

▲ Subgroup/international average percent correct is significantly higher than U.S. total percent correct ($p < .05$).

Exhibit continues on next page.
See notes at end of exhibit.

Exhibit 6-2. TIMSS Advanced physics example item 1 with student performance data, by course type: 2015—Continued

Scoring guide for multiple choice (part A)		
Option	Response type	U.S. total percentage[7]
Correct		
D	The acceleration due to gravity (-10 m/s^2) is constant and applies equally to the ball at all positions.	51
Incorrect		
A	*Demonstrates a misconception that there is no acceleration when the ball momentarily stops moving as it reverses direction at its maximum height (velocity is zero, so acceleration must be zero).*	41
B	The acceleration at point 3 is half that at point 2.	3
C	The acceleration at point 3 is twice that at point 2.	4
	Blank[8]	1

Misconception of interest (option A).

Percentage distribution of U.S. TIMSS Advanced students across response types for part A, by course type: 2015

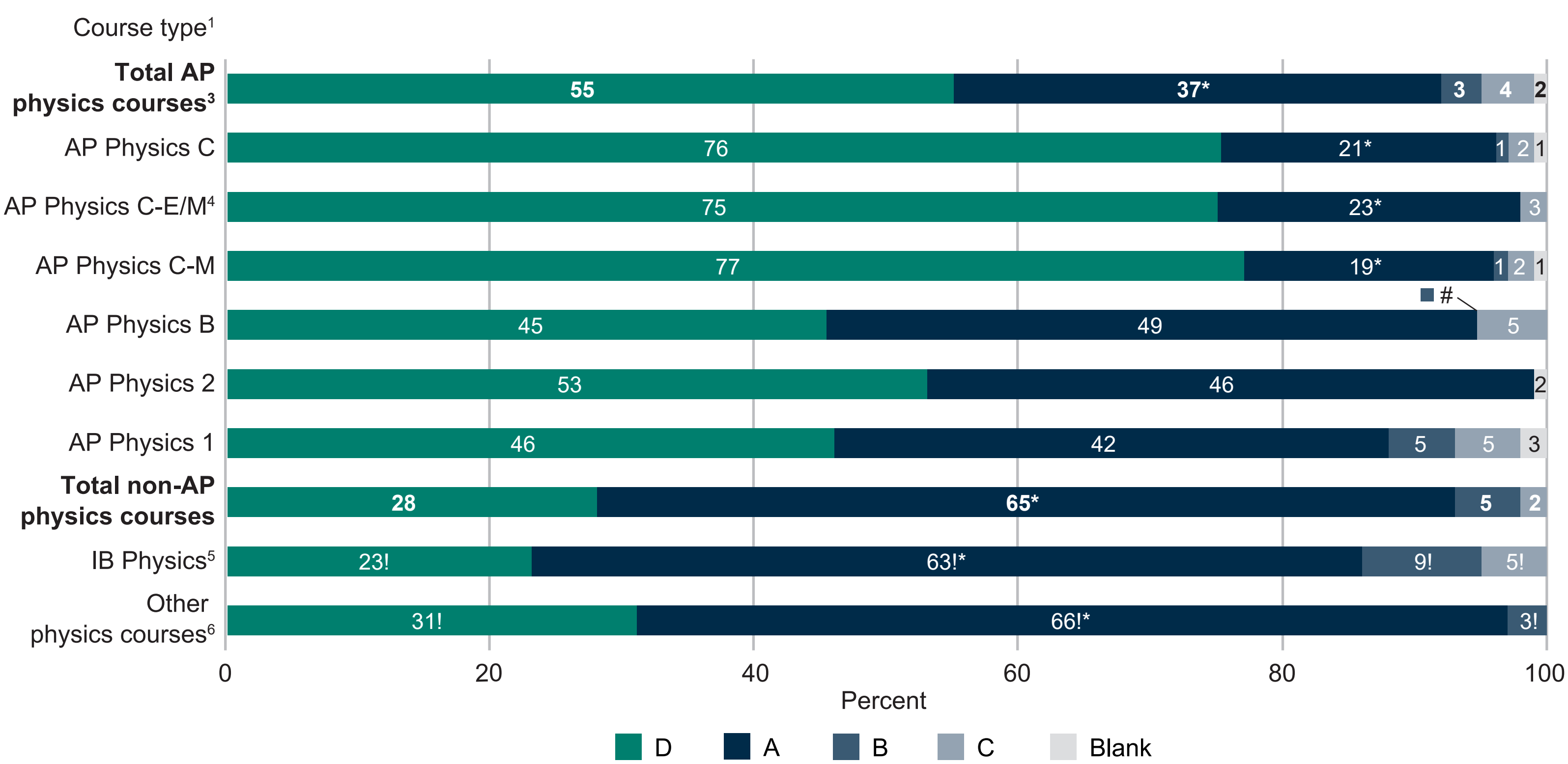


Exhibit continues on next page.
See notes at end of exhibit.

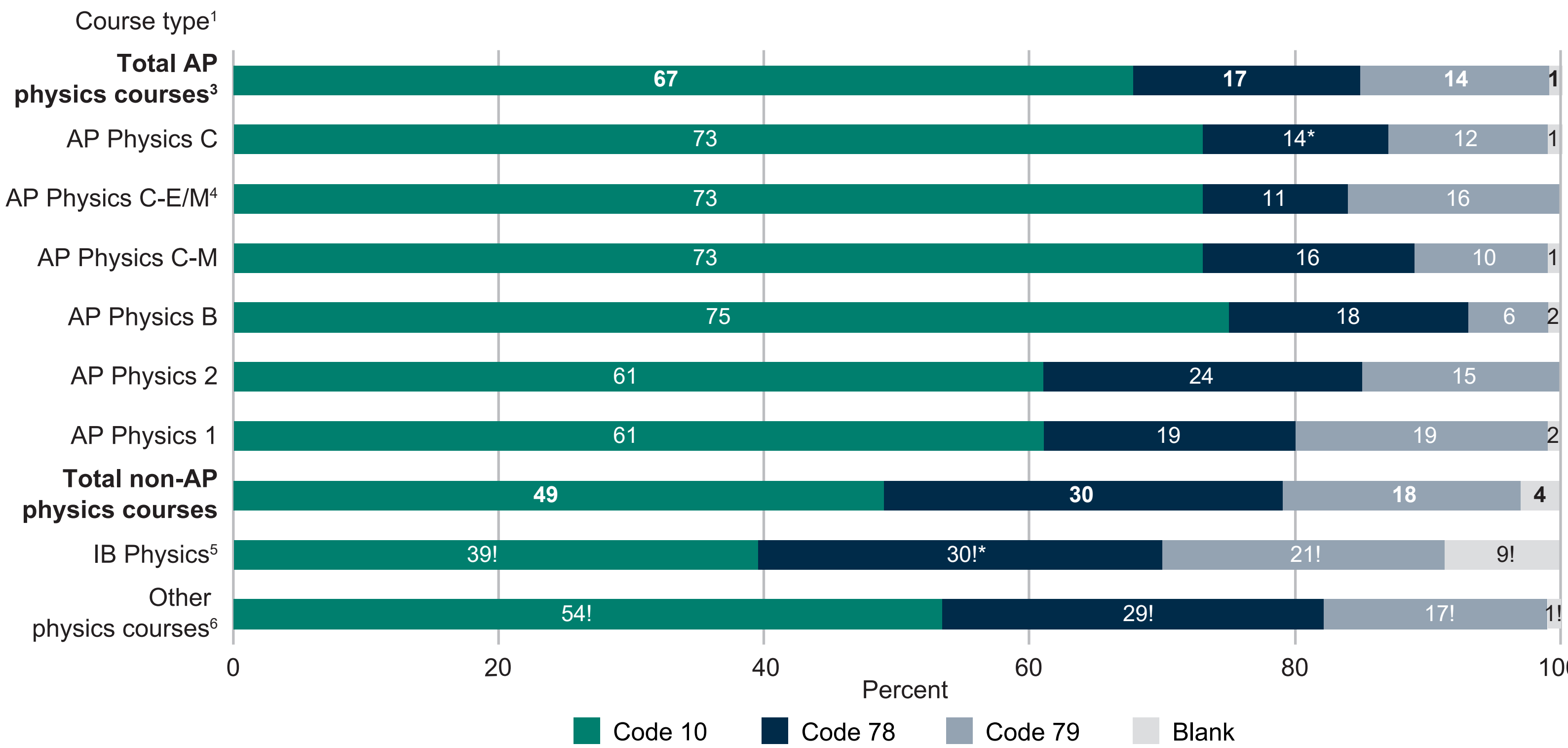
Exhibit 6-2. TIMSS Advanced physics example item 1 with student performance data, by course type: 2015—Continued

Scoring guide for contructed response (part B)

Code	Response type	U.S. total percentage[7]
Correct		
10	Indicates that the times are equal	64
Incorrect		
78	States that the time on the way down is shorter or the time on the way up is longer. *[Demonstrates a misconception that the times are different because the ball is accelerating (speeding up) on the way down and decelerating (slowing down) on the way up.]*	19
79	Other incorrect response (including crossed out, erased, stray marks, illegible, or off task)	15
	Blank[8]	2

Misconception of interest (scoring code 78).

Percentage distribution of U.S. TIMSS Advanced students across response types for part B, by course type: 2015



Rounds to zero.
! Interpret with caution (sample size < 62, but > 30).
* $p < .05$. Subgroup percentage of students in option A/score code 78 is significantly different from the U.S. total percentage in option A/score code 78.
[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other physics courses" category only if they have not taken an AP or IB course
[2] Percent correct is the weighted percentage of students receiving full credit or partial credit on an assessment item. For multiple-choice and short constructed-response items, there is only one correct response option, worth 1 point for full credit. Thus, the percent correct is the percentage of students who chose the correct response for Part A (option D in the scoring guide) and provided the correct response for Part B (code 10 in the scoring guide). Students who did not reach the item were not included in the calculation of these percentages.
[3] AP Physics C includes courses in mechanics (C-M) and electricity and magnetism (C-E/M). AP Physics B was a 1-year physics course that was discontinued after the 2013–14 school year and was replaced with the 2-year course sequence AP Physics 1 and 2 beginning in 2014–15. Data in the AP Physics B category are generally for students who took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering a 1-year course identified as AP Physics B in 2014–15.
[4] A large majority of the students whose highest course was AP Physics C-E/M had also taken AP Physics C-M (89 percent), whether sequentially or in a combined course.
[5] Includes both higher-level and standard-level IB physics courses; nearly all students took the standard-level course.
[6] Includes other second-year physics courses (including "honors" or "regents" courses).
[7] Percentages are based on the full set of responses, including students who did not reach the item. Thus, the percentage in the correct category (option D/code 10 in the scoring guide) may be slightly different from what is shown for percent correct on the first page of the exhibit (which excludes students in the not-reached category).
[8] Blank includes both students who omitted the item and those who did not reach the item.
NOTE: International average is the item percent correct, averaged across the countries participating in the TIMSS Advanced physics assessment. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Example 2—"Skiers collide"

Exhibit 6-3 shows this extended constructed-response item from the topic area of *laws of conservation* in the *mechanics and thermodynamics* content domain. It requires students to apply the law of conservation of momentum to solve a contextualized problem involving two skiers colliding. The topic of *law of conservation of momentum; elastic and inelastic collisions* is covered in all TIMSS Advanced-eligible physics courses (or in a prerequisite course) and has been taught to virtually all U.S. TIMSS Advanced students (table A-4a). The item maps to the *High* international benchmark level, and the topic was categorized as high coverage (figure 6-9). Overall, 35 percent of U.S. TIMSS Advanced students received full credit on this item, which was not measurably different from the international average (34 percent full credit) (see the first page of exhibit 6-3 and table B-24).

To solve the problem, students must first determine that the law of conservation of momentum should be applied and then set up the appropriate equations to calculate the final velocity of the two skiers (after they collide inelastically). To receive full credit (code 20 in the scoring guide), students must get the correct answer (2 m/s) and show their work, including any equations used (as shown in the example student response). Fifty-three percent of students who had taken one or both AP Physics C courses provided a fully correct response (higher than the U.S. total of 35 percent), compared to 18 percent of students who had taken a non-AP physics course (lower than the U.S. total). Performance of students taking other AP physics courses ranged from 29 percent of those who had taken AP Physics B, to 33 and 37 percent of those who had taken AP Physics 1 or 2 as their highest course, respectively.

Students received partial credit either for providing a correct answer without complete work shown (code 10, 16 percent of U.S. students) or for setting up the correct equations but arriving at an incorrect answer due to a substitution or calculation error (code 11, 1 percent of U.S. students) (see the second page of exhibit 6-3). The frequency of providing a correct answer with incomplete work was similar for total AP and total non-AP physics students (and not measurably different from U.S. students overall for any course type).

Overall, 52 percent of U.S. students demonstrated conceptual understanding by providing a complete or partial response (codes 20, 10, and 11 combined). These percentages ranged widely across physics courses, from 34 percent of students who had taken non-AP physics to 75 percent of students who had taken AP Physics C, despite the fact that physics teachers reported that the topic had been taught to virtually all U.S. advanced students by the time of the assessment.

Among the incorrect responses, some students showed an equation related to conservation of momentum but were unable to correctly apply this to the problem (code 77, 3 percent of U.S. students overall), while 40 percent of U.S. students provided other completely incorrect responses not based on the law of conservation of momentum (code 79) and 6 percent left the item blank.

Exhibit 6-3. TIMSS Advanced physics example item 2 with student performance data, by course type: 2015

Robert is skiing down a hill. At the bottom of the hill when his velocity is 5 m/s, he collides with David, who is at rest. They continue together in the same direction. Robert's mass is 60 kg and David's mass is 90 kg. Assume no frictional effects.

What is the common velocity of David and Robert right after the collision?

Show your work, including any equations you use.

Answer: ____2____ m/s

$$(m_{DAVID} + m_{ROBERT})\, v_f = m_{DAVID} v_{DAVID} + m_{ROBERT} v_{ROBERT}$$

$$= 0 + m_{ROBERT} \cdot v_{ROBERT}$$

$$v_f = \frac{m_{ROBERT}\, v_{ROBERT}}{m_{DAVID} + m_{ROBERT}}$$

$$= \frac{(60)(5)}{(60+90)} = 2 \text{ m/s}$$

Item classification and description	
Content domain:	Mechanics and thermodynamics
Topic area:	Laws of conservation
Topic:	Law of conservation of momentum; elastic and inelastic collisions
Cognitive domain:	Applying
International benchmark:	High
Description:	Calculate the final velocity of two skiers after they collide inelastically

Student performance data

Course type[1]	Percent full credit[2]
International average	**34**
U.S. total	**35**
AP physics courses[3]	**39**
AP Physics C courses	53 ▲
AP Physics C-E/M[4]	49
AP Physics C-M	57 ▲
AP Physics B	29
AP Physics 2	37
AP Physics 1	33
Non-AP physics courses	**18** ▼
IB Physics[5]	6! ▼
Other physics courses[6]	24!

▼ Subgroup/international average percent correct is significantly lower than U.S. total percent correct ($p < .05$).

▲ Subgroup/international average percent correct is significantly higher than U.S. total percent correct ($p < .05$).

Exhibit continues on next page.
See notes at end of exhibit.

Exhibit 6-3. TIMSS Advanced physics example item 2 with student performance data, by course type: 2015—Continued

Code	Response type	U.S. total percentage[7]
Correct		
20	**Answer: 2** Student work includes both of these points: • The final momentum is equal to the initial momentum: $(m_D + m_R)v = m_D v_D + m_R v_R = 0 + m_R v_R$ (a mathematical statement of the conservation of momentum) • $v = \frac{m_R v_R}{(m_D + m_R)} = \left(\frac{60 \text{ kg} \cdot 5 \text{ m/s}}{60 \text{ kg} + 90 \text{ kg}}\right) = 2 \text{ m/s}$ (substitution of the relevant values and final answer) Note: It is not necessary for students to show units within their calculations.	35
Partial		
10	**Answer: 2** Student work either does not include both of the points listed for Code 20 above or the work shown for one or both of the points is missing or incomplete.	16
11	Sets up the equations correctly as shown for a Code 20, but arrives at an incorrect answer by making a substitution error or a calculation error.	1
Incorrect		
77	States or shows an equation indicating conservation of momentum, but does not solve the problem. Includes responses that show an incorrect momentum equation that does not consider the combined mass (e.g., $m_1v_1 = m_2v_2$). Responses may also set up the correct equation and not include any further steps to solve the problem.	3
79	Other incorrect response (including crossed out, erased, stray marks, illegible, or off task)	40
	Blank[8]	6

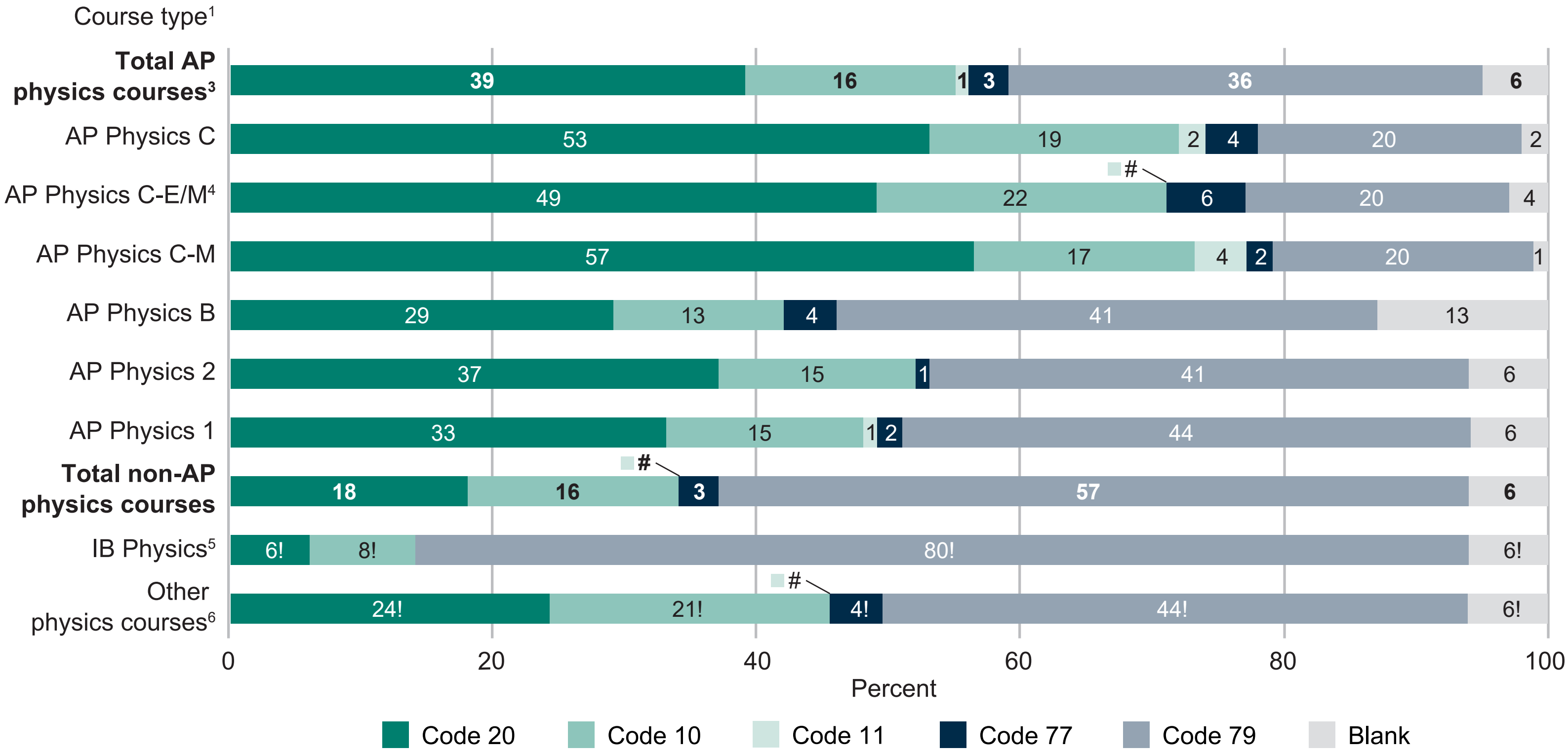
Scoring guide for contructed response

Partial response of interest (scoring code 10).

Exhibit continues on next page.
See notes at end of exhibit.

Exhibit 6-3. TIMSS Advanced physics example item 2 with student performance data, by course type: 2015—Continued

Percentage distribution of U.S. TIMSS Advanced students across response types, by course type: 2015



\# Rounds to zero.
! Interpret with caution (sample size < 62, but > 30).
* $p < .05$. Subgroup percentage of students in score code 10 is significantly different from the U.S. total percentage in score code 10.
[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other physics courses" category only if they have not taken an AP or IB course
[2] Percent full credit is the percentage of students receiving full credit on an assessment item. For extended constructed-response items, there is both a correct level (worth 2 score points) and a partial level (worth 1 score point). Thus, percent full credit reflects the percentage of students who provided a fully correct response (code 20 in the scoring guide). Students who provided a partial response are not reflected in the percent full credit, but are shown in the distributions across the correct and partial response categories on page 2 of the exhibit. Students who did not reach the item were not included in the calculation of this percentage.
[3] AP Physics C includes courses in mechanics (C-M) and electricity and magnetism (C-E/M). AP Physics B was a 1-year physics course that was discontinued after the 2013–14 school year and was replaced with the 2-year course sequence AP Physics 1 and 2 beginning in 2014–15. Data in the AP Physics B category are generally for students who took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering a 1-year course identified as AP Physics B in 2014–15.
[4] A large majority of the students whose highest course was AP Physics C-E/M had also taken AP Physics C-M (89 percent), whether sequentially or in a combined course.
[5] Includes both higher-level and standard-level IB physics courses; nearly all students took the standard-level course.
[6] Includes other second-year physics courses (including "honors" or "regents" courses).
[7] Percentages are based on the full set of responses, including students who did not reach the item. Thus, the percentage in the correct category (code 20 in the scoring guide) may be slightly different from what is shown for percent full credit on the first page of the exhibit (which excludes students in the not-reached category).
[8] Blank includes both students who omitted the item and those who did not reach the item.
NOTE: International average is the item percent correct, averaged across the countries participating in the TIMSS Advanced physics assessment. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Example 3—"Volume of gas"

Exhibit 6-4 shows this extended constructed-response item from the topic area of *heat and temperature* in the *mechanics and thermodynamics* content domain. It requires students to use the law of ideal gases to calculate the new volume of a gas when pressure and temperature change. The first part of the topic (*law of ideal gases*) is covered in AP Physics B, AP Physics 2, and IB Physics (higher level), but is not covered in AP Physics 1 or C or in IB Physics (standard level) (table A-4a). Students whose highest course was an AP physics C course may have covered this topic in a previous first-year physics course, but this likely varies across states, districts, and schools. Students whose highest course was AP Physics 1 are not likely to have covered this topic. However, the topic of ideal gas laws is typically covered in upper-level high school chemistry courses and some U.S. students may have learned this topic in these courses. The topic was reported by physics teachers to have been taught to 62 percent of U.S. TIMSS Advanced students overall at the time of the assessment. This item maps above the *Advanced* international benchmark level, and the topic was categorized as low topic coverage (figure 6-9). Overall, 11 percent of U.S. TIMSS Advanced students received full credit, which was lower than the international average (18 percent full credit) (see the first page of exhibit 6-4 and table B-25).

To solve the problem, students use the ideal gas equation pV/T = constant (or $pV = nRT$), where T is the temperature on the Kelvin scale ($K = {}^{o}C + 273$). To receive full credit (code 20 in the scoring guide), students must get the correct answer (2.0 m^3) and show their work, including any equations used (as shown in the example student response). Accepted work must include applying the initial and final conditions in the ideal gas equation to generate the equation $pV_1/T_1 = 2pV_2/T_2$, and substituting the values for V_1, T_1, and T_2 to calculate the correct answer (including units). Performance on this item was lower than the U.S. overall for students who had taken AP Physics 1 or non-AP physics courses as their highest course (5 percent correct for both). Performance was higher than the U.S. total for students who had taken AP Physics 2 or C (electricity and magnetism) courses, with 25 percent and 26 percent correct, respectively.

About twice as many U.S. students overall received partial credit (23 percent) as full credit (11 percent) on this item (see the second page of exhibit 6-4). Students received partial credit (code 10 or 11) for demonstrating conceptual understanding by setting up the ideal gas equations correctly but making an error. Code 10 was used when students made a calculation error or had missing or incorrect units. Code 11 was used to track a specific type of error in which students show correct work but use temperature in degrees Celsius without first converting to Kelvin. Overall, the latter error was more common for U.S. students (code 11, 17 percent) than the former error (code 10, 6 percent).

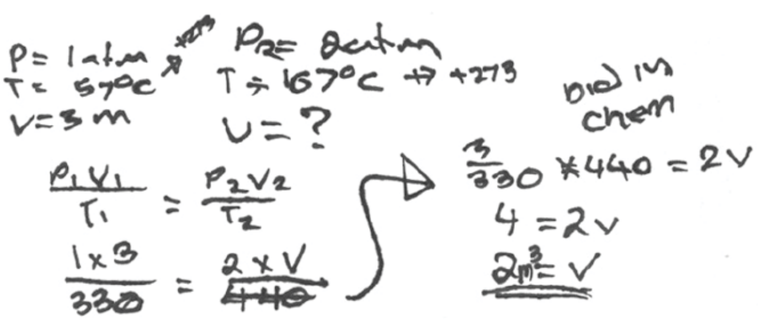
Overall, 34 percent of U.S. students demonstrated conceptual understanding of ideal gas laws by providing a complete or partial response (codes 20, 10, and 11 combined). These percentages ranged widely across physics courses, from 19 percent of students whose highest course was AP Physics 1 to 61 percent of students who had taken AP Physics B and 66 percent of those taking AP Physics 2, which is consistent with expectations based on differences in topic coverage by course.

The prevalence of the error in temperature scale (code 11) varied by highest physics course taken. Twenty-two percent of students who had taken AP Physics C and 32 percent of students who had taken AP Physics 2 received partial credit by making this error only, which were both higher than the U.S. total of 17 percent. In contrast, 11 percent of students who had taken AP Physics 1 and 9 percent who had taken IB Physics made this error only, which was lower than the U.S. total. Rather, 58 percent of students who had taken AP Physics 1 and 62 percent who had taken IB Physics provided incorrect responses reflecting additional types of errors (codes 71 and 79 combined) compared to 30 percent of those who had taken AP Physics 2 and 32 percent of those taking AP Physics C-Electricity and Magnetism). In addition, about one-quarter of students in AP Physics 1 and IB Physics (23 and 24 percent, respectively) left the item blank, compared to 16 percent of U.S. students overall.

Exhibit 6-4. TIMSS Advanced physics example item 3 with student performance data, by course type: 2015

A container with a moveable piston has a volume of 3.0 m^3 and contains an ideal gas at atmospheric pressure and a temperature of 57 °C. The gas is heated to a temperature of 167 °C, and its pressure has doubled. What is the new volume of the gas?

Show your work.



Item classification and description	
Content domain:	Mechanics and thermodynamics
Topic area:	Heat and temperature
Topic:	Law of ideal gases; expansion of solids and liquids in relation to temperature change
Cognitive domain:	Applying
International benchmark:	Above Advanced
Description:	Calculate the new volume of a gas when pressure and temperature change

Student performance data

Course type[1]	Percent full credit[2]
International average	**18** ▲
U.S. total	**11**
AP physics courses[3]	**13** ▲
AP Physics C courses	18 ▲
AP Physics C-E/M[4]	26 ▲
AP Physics C-M	14
AP Physics B	23
AP Physics 2	25 ▲
AP Physics 1	5 ▼
Non-AP physics courses	**5** ▼
IB Physics[5]	3! ▼
Other physics courses[6]	6!

▼ Subgroup/international average percent correct is significantly lower than U.S. total percent correct ($p < .05$).

▲ Subgroup/international average percent correct is significantly higher than U.S. total percent correct ($p < .05$).

Exhibit continues on next page.
See notes at end of exhibit.

Exhibit 6-4. TIMSS Advanced physics example item 3 with student performance data, by course type: 2015—Continued

Scoring guide for contructed response

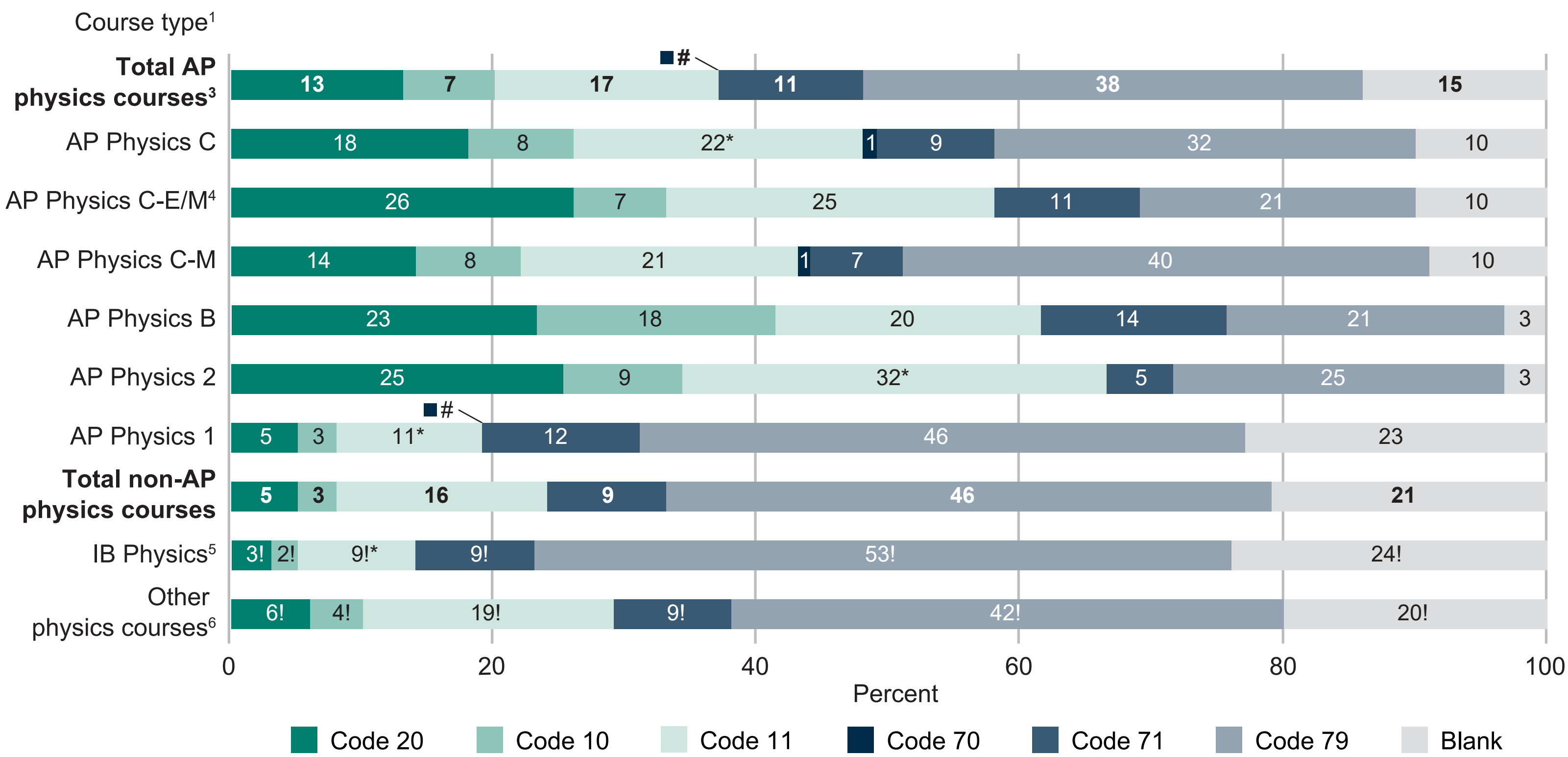
Code	Response type	U.S. total percentage[7]
Correct		
20	**Answer: V_2 = 2.0 m³** Uses the ideal gas equation pV/T = constant (or pV = nRT), where T is the temperature on the Kelvin scale (˚C + 273). Applies the initial and final conditions to generate the equation $pV_1/T_{1-} = 2pV_2/T_2$ and substitutes the values for V_1, T_1, and T_2 to calculate the correct answer (including units).	11
Partial		
10	Correct equation shown but makes a calculation error and/or has missing or incorrect units.	6
11	Correct work shown but uses temperature in Celsius instead of Kelvin (gives 4.4 m³). *[Demonstrate knowledge of concept, but makes a common error.]*	17
Incorrect		
70	Correct answer with no work shown	#
71	Incorrect answer using temperature in Celsius.	11
79	Other incorrect response (including crossed out, erased, stray marks, illegible, or off task)	39
	Blank[8]	16

Error of interest (scoring code 11).

Exhibit continues on next page.
See notes at end of exhibit.

Exhibit 6-4. TIMSS Advanced physics example item 3 with student performance data, by course type: 2015—Continued

Percentage distribution of U.S. TIMSS Advanced students across response types, by course type: 2015



Rounds to zero.
! Interpret with caution (sample size < 62, but > 30).
* $p < .05$. Subgroup percentage of students in score code 11 is significantly different from the U.S. total percentage in score code 11.
[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other physics courses" category only if they have not taken an AP or IB course
[2] Percent full credit is the percentage of students receiving full credit on an assessment item. For extended constructed-response items, there is both a correct level (worth 2 score points) and a partial level (worth 1 score point). Thus, percent full credit reflects the percentage of students who provided a fully correct response (code 20 in the scoring guide). Students who provided a partial response are not reflected in the percent full credit, but are shown in the distributions across the correct and partial response categories on page 2 of the exhibit. Students who did not reach the item were not included in the calculation of this percentage.
[3] AP Physics C includes courses in mechanics (C-M) and electricity and magnetism (C-E/M). AP Physics B was a 1-year physics course that was discontinued after the 2013–14 school year and was replaced with the 2-year course sequence AP Physics 1 and 2 beginning in 2014–15. Data in the AP Physics B category are generally for students who took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering a 1-year course identified as AP Physics B in 2014–15.
[4] A large majority of the students whose highest course was AP Physics C-E/M had also taken AP Physics C-M (89 percent), whether sequentially or in a combined course.
[5] Includes both higher-level and standard-level IB physics courses; nearly all students took the standard-level course.
[6] Includes other second-year physics courses (including "honors" or "regents" courses).
[7] Percentages are based on the full set of responses, including students who did not reach the item. Thus, the percentage in the correct category (code 20 in the scoring guide) may be slightly different from what is shown for percent full credit on the first page of the exhibit (which excludes students in the not-reached category).
[8] Blank includes both students who omitted the item and those who did not reach the item.
NOTE: International average is the item percent correct, averaged across the countries participating in the TIMSS Advanced physics assessment. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

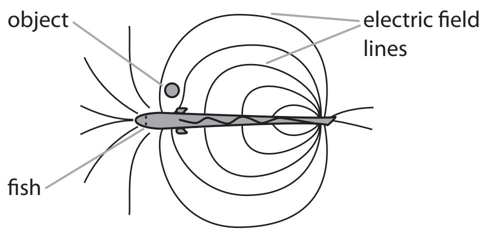
Example 4—"Fish generates an electric field"

Exhibit 6-5 shows this multiple-choice item from the topic area of *electricity and electric circuits* and the topic of *charged particles in an electric field* in the *electricity and magnetism* content domain that requires students to identify the direction of the electric force on a charged object in an electric field. While the topic of *electrostatic attraction or repulsion between isolated charged particles–Coulomb's law* is widely covered in all of the TIMSS Advanced-eligible physics courses (except the AP Physics C course in mechanics), the topic of *charged particles in an electric field* is not covered in the first-year AP Physics 1 course (table A-4b). *Charged particles in an electric field* was reported by physics teachers to have been taught to 74 percent of U.S. TIMSS Advanced students overall. The item maps to the *Intermediate* international benchmark level, and the topic was categorized low coverage (figure 6-9). Overall, 43 percent of U.S. TIMSS Advanced students answered the item correctly, which was lower than the international average (51 percent correct) (see the first page of exhibit 6-5 and table B-26).

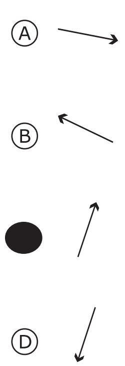
The correct response (option C) identifies the direction of the net electric force on the positively charged object as being parallel to the closest electric field line and pointed away from the positively charged head. U.S. advanced students who had taken AP Physics C (electricity and magnetism) or AP Physics 2 performed above the U.S. total of 43 percent, with 61 and 65 percent, respectively; whereas students who had taken AP Physics 1 performed lower than the U.S. total, with 35 percent correct. These AP Physics 1 students, who have studied electrostatic attraction/repulsion between isolated charges, are not likely to have covered this type of problem involving electric fields due to multiple charges.

The most common incorrect response (option A, 38 percent overall) identifies an incorrect direction that points toward the negatively charged tail (and is essentially perpendicular to the electric field line) (see the second page of exhibit 6-5). This misconception demonstrates a lack of knowledge about electric fields and does not account for the net electric force resulting from the positive and negative charges on the fish based on their relative distances from the positively charged object. Instead, it points in a direction that would occur from only the attraction of the positively charged object toward the negative tail. The prevalence of this misconception varied by course type. The percentage of students whose highest course was AP Physics 1 and who selected option A (45 percent) was higher than the U.S. total, whereas the percentages of students who had taken AP Physics 2 or AP Physics C (electricity and magnetism) and selected option A were lower than the U.S. total (23 percent and 25 percent, respectively). These higher level AP physics courses would likely have covered both electric fields and electromagnetism, and, thus, students selecting option A who had taken these courses may be confusing the behavior of a charged particle in an electric field with the behavior of a moving charged particle in a magnetic field, where the force is perpendicular to both the magnetic field and the velocity of the charged particle.

Exhibit 6-5. TIMSS Advanced physics example item 4 with student performance data, by course type: 2015



Some fish generate an electric field to detect objects in muddy water. The tail of the fish becomes negatively charged, and the head becomes positively charged. If the small object in the position shown above has a positive charge, which arrow BEST shows the correct direction of the electric force on it?



Item classification and description	
Content domain:	Electricity and magnetism
Topic area:	Electricity and electric circuits
Topic:	Charged particles in an electric field
Cognitive domain:	Applying
International benchmark:	Intermediate
Description:	Identify the direction of the electric force on a charged object in an electric field

Student performance data

Course type[1]	Percent correct[2]
International average	**51** ▲
U.S. total	**43**
AP physics courses[3]	**43**
AP Physics C courses	56 ▲
AP Physics C-E/M[4]	61 ▲
AP Physics C-M	52
AP Physics B	35
AP Physics 2	65 ▲
AP Physics 1	35 ▼
Non-AP physics courses	**40**
IB Physics[5]	37!
Other physics courses[6]	42!

▼ Subgroup/international average percent correct is significantly lower than U.S. total percent correct ($p < .05$).

▲ Subgroup/international average percent correct is significantly higher than U.S. total percent correct ($p < .05$).

Exhibit continues on next page.
See notes at end of exhibit.

Exhibit 6-5. TIMSS Advanced physics example item 4 with student performance data, by course type: 2015—Continued

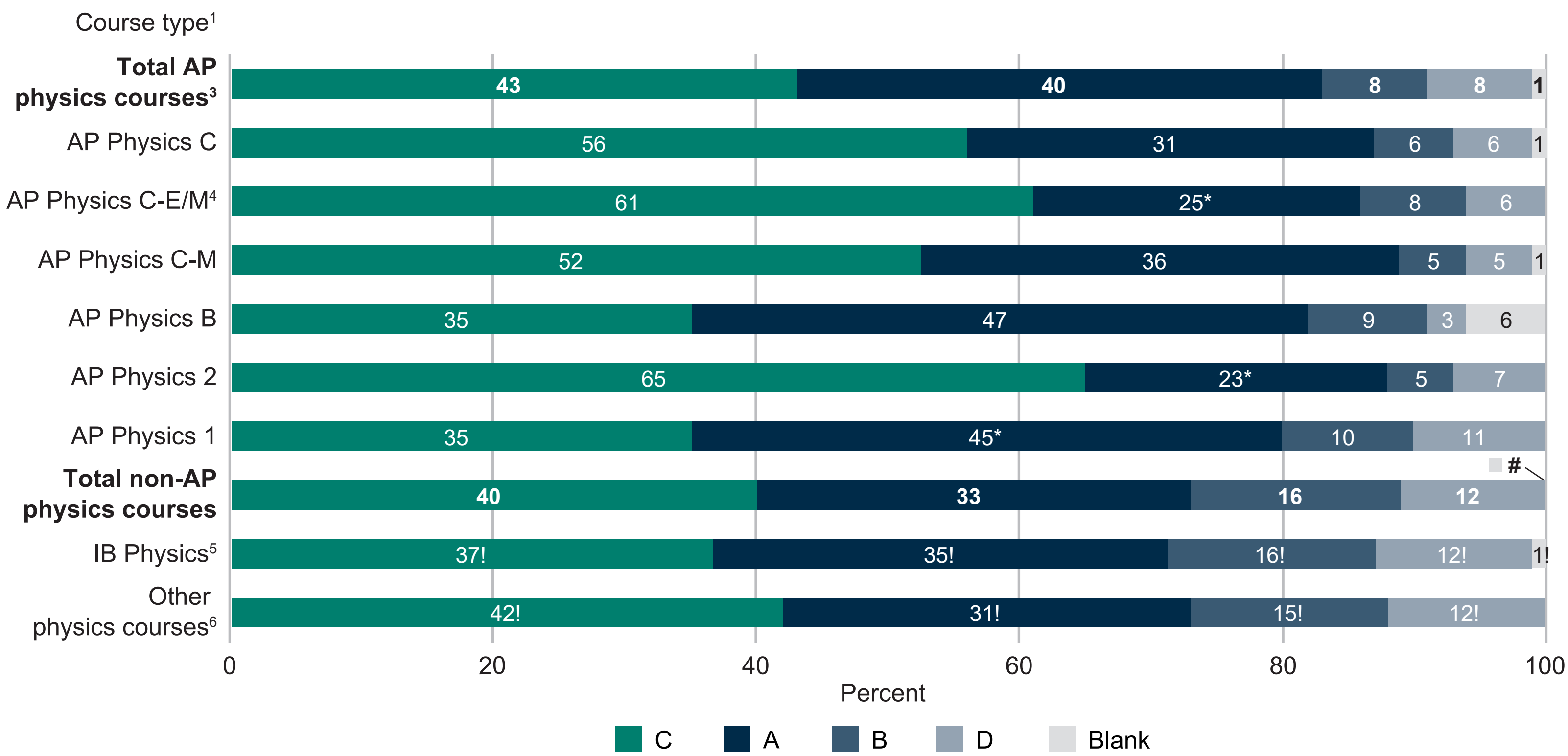
Scoring guide for multiple choice		
Option	Response type	U.S. total percentage[7]
Correct		
C	Identifies the correct direction parallel to the closest electric field line and away from the positively charged head	43
Incorrect		
A	Identifies an incorrect direction toward the negatively charged tail that is essentially perpendicular to the electric field lines *[Demonstrates a misconception about electric fields that does not account for the net electric force resulting from the positive and negative charges on the fish based on their relative distances from the positively charged object. Students may also confuse the behavior of a charged object in an electric field with the behavior of a moving charged particle in a magnetic field, where the force is perpendicular to the magnetic field and the velocity of the charged particle.]*	38
B	Indicates an incorrect direction away from both the head and the tail that is essentially perpendicular to the electric field lines.	10
D	Identifies an incorrect direction parallel to the closest electric field line but toward the positively charged head (opposite direction of the electric field)	9
	Blank[8]	1

Misconception of interest (option A).

Exhibit continues on next page.
See notes at end of exhibit.

Exhibit 6-5. TIMSS Advanced physics example item 4 with student performance data, by course type: 2015—Continued

Percentage distribution of U.S. TIMSS Advanced students across response types, by course type: 2015



Rounds to zero.
! Interpret with caution (sample size < 62, but > 30).
* $p < .05$. Subgroup percentage of students in option A is significantly different from the U.S. total percentage in option A.
[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other physics courses" category only if they have not taken an AP or IB course
[2] Percent correct is the weighted percentage of students receiving full credit or partial credit on an assessment item. For multiple choice items, there is only one correct response option, worth 1 point for full credit. Thus, the percent correct is the percentage of students who chose the correct response (option C in the scoring guide). Students who did not reach the item were not included in the calculation of this percentage.
[3] AP Physics C includes courses in mechanics (C-M) and electricity and magnetism (C-E/M). AP Physics B was a 1-year physics course that was discontinued after the 2013–14 school year and was replaced with the 2-year course sequence AP Physics 1 and 2 beginning in 2014–15. Data in the AP Physics B category are generally for students who took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering a 1-year course identified as AP Physics B in 2014–15.
[4] A large majority of the students whose highest course was AP Physics C-E/M had also taken AP Physics C-M (89 percent), whether sequentially or in a combined course.
[5] Includes both higher-level and standard-level IB physics courses; nearly all students took the standard-level course.
[6] Includes other second-year physics courses (including "honors" or "regents" courses).
[7] Percentages are based on the full set of responses, including students who did not reach the item. Thus, the percentage in the correct category (option C in the scoring guide) may be slightly different from what is shown for percent correct on the first page of the exhibit (which excludes students in the not-reached category).
[8] Blank includes both students who omitted the item and those who did not reach the item.
NOTE: International average is the item percent correct, averaged across the countries participating in the TIMSS Advanced physics assessment. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Example 5—"Electron beam in a magnetic field"

Exhibit 6-6 shows this multiple-choice item from the topic area of *magnetism and electromagnetic induction* and the topic of *charged particles in a magnetic field* in the *electricity and magnetism content* domain that requires students to predict the change in motion of an electron beam moving in an applied magnetic field. Like the topic assessed in the previous example (*charged particles in an electric field*), the topic assessed in example 5 (*charged particles in a magnetic field*) is covered in IB Physics and in the AP Physics B, 2, and C (electricity and magnetism) courses, but is not covered in AP Physics 1 (table A-4b). It was reported by teachers to have been taught to 58 percent of U.S. TIMSS Advanced students overall at the time of the assessment. This item maps to the *Advanced* international benchmark level, and the topic was categorized as low coverage (figure 6-9). Overall, 23 percent of U.S. TIMSS Advanced students answered the item correctly, which was lower than the international average (32 percent correct) (see the first page of exhibit 6-6 and table B-27).

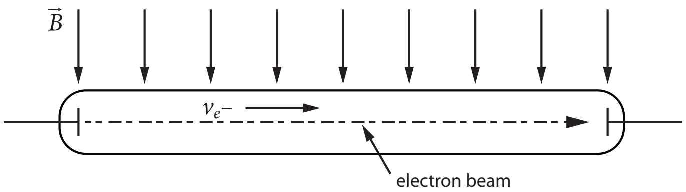
The correct response (option B) identifies that the electron beam will curve up out of the plane of the page. This direction results from a correct application of the "right-hand rule" for an electron (negative charge) moving in a uniform magnetic field (force is perpendicular to both the direction of the magnetic field and the velocity of the electron). As was the case with the previous item involving electric fields, students whose highest course was AP Physics C (electricity and magnetism) or AP Physics 2 (36 percent and 40 percent correct, respectively) performed above the U.S. total of 23 percent, whereas students who had taken AP Physics 1 (14 percent correct) performed below the U.S. total, as they would not likely have covered this advanced topic at the time of the assessment.

The most common incorrect response overall (option C) identifies that the electron beam curves down (in the direction of the magnetic field) (see the second page of exhibit 6-6). This type of response demonstrates the misconception that the behavior of a charged particle moving in a magnetic field is analogous to that in a uniform electric field (force is parallel to the direction of the electric field), and 36 percent of U.S. students chose this option. The prevalence of this misconception was higher among students who had taken AP Physics 1 (46 percent) than among U.S. students overall. In contrast, it was lower among students who had taken AP Physics B, 2, or C (electricity and magnetism)—at 24 percent, 16 percent, and 15 percent, respectively—than among U.S. students overall.

Among students who had taken AP Physics B, AP Physics 2 or AP Physics C (electricity and magnetism) courses, option A (beam curves down into the page) was the most common incorrect response (35 to 39 percent). This response shows that students know that the force on the electron beam will be perpendicular to both the magnetic field and the velocity of the electrons, but they make an error in applying the "right-hand rule."

Exhibit 6-6. TIMSS Advanced physics example item 5 with student performance data, by course type: 2015

A beam of electrons inside an evacuated glass tube is directed from left to right.



A uniform magnetic field is applied to the tube directed down, as shown in the diagram. What will happen to the electrons in the beam?

- Ⓐ The beam curves into the page.
- ● The beam curves out of the page.
- Ⓒ The beam curves down.
- Ⓓ The beam curves up.

Item classification and description	
Content domain:	Electricity and magnetism
Topic area:	Magnetism and electromagnetic induction
Topic:	Charged particles in magnetic field
Cognitive domain:	Reasoning
International benchmark:	Advanced
Description:	Predict the change in motion of an electron beam in an applied magnetic field

Student performance data

Course type[1]	Percent correct[2]
International average	**32** ▲
U.S. total	**23**
AP physics courses[3]	**24**
AP Physics C courses	32 ▲
AP Physics C-E/M[4]	36 ▲
AP Physics C-M	30
AP Physics B	32
AP Physics 2	40 ▲
AP Physics 1	14 ▼
Non-AP physics courses	**18**
IB Physics[5]	24!
Other physics courses[6]	14!

▼ Subgroup/international average percent correct is significantly lower than U.S. total percent correct ($p < .05$).

▲ Subgroup/international average percent correct is significantly higher than U.S. total percent correct ($p < .05$).

Exhibit continues on next page.
See notes at end of exhibit.

Exhibit 6-6. TIMSS Advanced physics example item 5 with student performance data, by course type: 2015—Continued

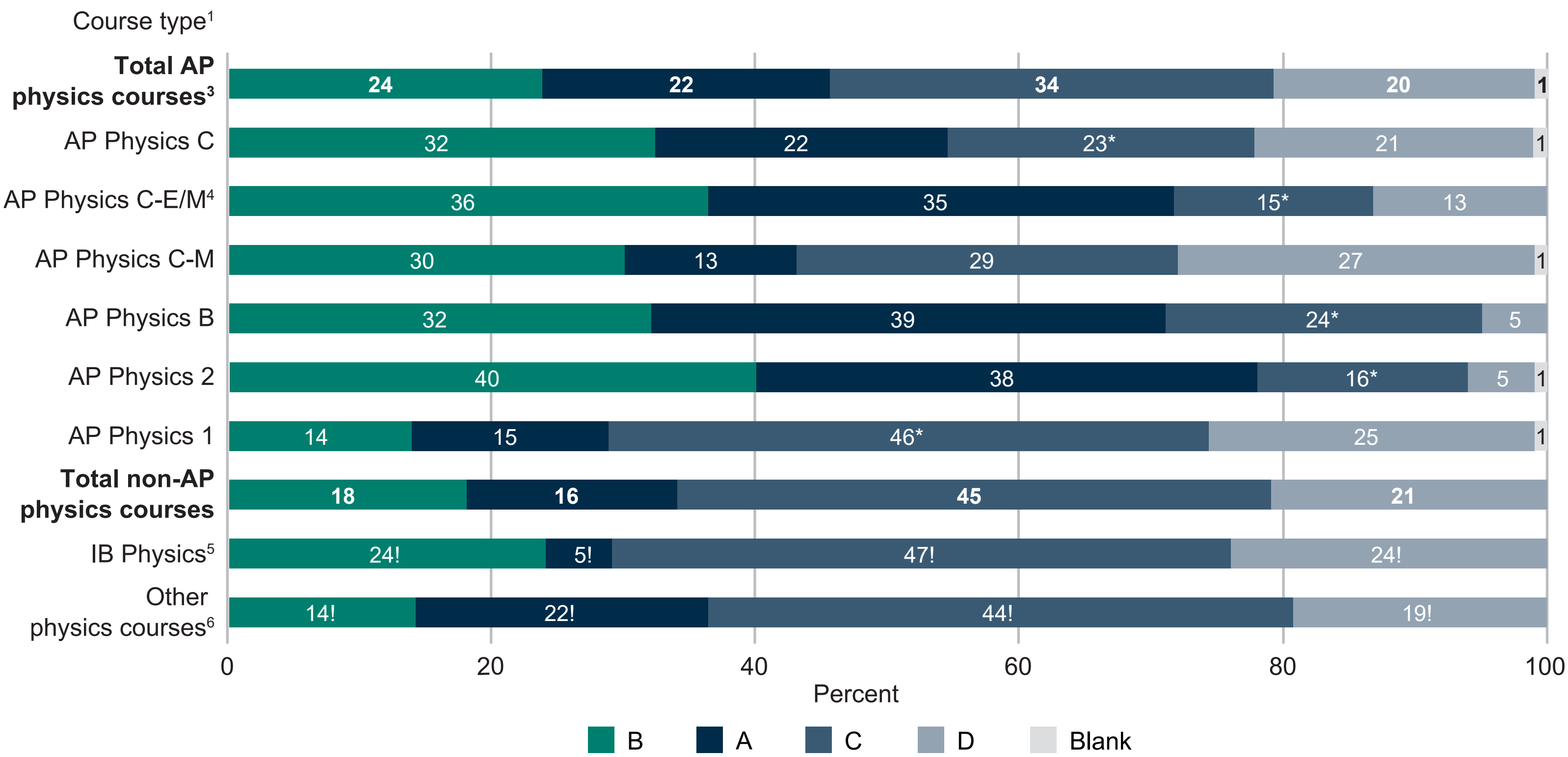
Scoring guide for multiple choice		
Option	Response type	U.S. total percentage[7]
Correct		
B	Correct application of the “right-hand rule” for an electron moving in a uniform magnetic field (force is perpendicular to both the direction of the magnetic field and the velocity of the electron and points out of the page)	23
Incorrect		
A	Misapplication of the right-hand rule (force is perpendicular to the magnetic field and velocity of the electron but in the opposite direction)	21
C	Indicates that the electron moves in the direction of the magnetic field. *[Demonstrates misconception that the behavior of a charged particle moving in a magnetic field is the same as that in a uniform electric field (force parallel to the direction of the electric field).]*	36
D	Indicates that the electron moves in the opposite direction of the magnetic field.	20
	Blank[8]	1

Misconception of interest (option C).

Exhibit continues on next page.
See notes at end of exhibit.

Exhibit 6-6. TIMSS Advanced physics example item 5 with student performance data, by course type: 2015—Continued

Percentage distribution of U.S. TIMSS Advanced students across response types, by course type: 2015



! Interpret with caution (sample size < 62, but > 30).
* *p* < .05. Subgroup percentage of students in option C is significantly different from the U.S. total percentage in option C.
[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other physics courses" category only if they have not taken an AP or IB course
[2] Percent correct is the weighted percentage of students receiving full credit or partial credit on an assessment item. For multiple choice items, there is only one correct response option, worth 1 point for full credit. Thus, the percent correct is the percentage of students who chose the correct response (option B in the scoring guide). Students who did not reach the item were not included in the calculation of this percentage.
[3] AP Physics C includes courses in mechanics (C-M) and electricity and magnetism (C-E/M). AP Physics B was a 1-year physics course that was discontinued after the 2013–14 school year and was replaced with the 2-year course sequence AP Physics 1 and 2 beginning in 2014–15. Data in the AP Physics B category are generally for students who took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering a 1-year course identified as AP Physics B in 2014–15.
[4] A large majority of the students whose highest course was AP Physics C-E/M had also taken AP Physics C-M (89 percent), whether sequentially or in a combined course.
[5] Includes both higher-level and standard-level IB physics courses; nearly all students took the standard-level course.
[6] Includes other second-year physics courses (including "honors" or "regents" courses).
[7] Percentages are based on the full set of responses, including students who did not reach the item. Thus, the percentage in the correct category (option B in the scoring guide) may be slightly different from what is shown for percent correct on the first page of the exhibit (which excludes students in the not-reached category).
[8] Blank includes both students who omitted the item and those who did not reach the item.
NOTE: International average is the item percent correct, averaged across the countries participating in the TIMSS Advanced physics assessment. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Example 6—"Mass change in a nuclear reaction"

Exhibit 6-7 shows this multiple-choice item from the topic area of *atomic and nuclear physics* in the *wave phenomena and atomic/nuclear physics* content domain. It requires knowing the concept of mass-energy equivalence to identify what accounts for the difference in the mass of an atom before and after a nuclear reaction. The topic *mass-energy equivalence in nuclear reactions and particle transformations* is covered in IB Physics, AP Physics B, and AP Physics 2. It is not covered in either AP Physics 1 or AP Physics C (table A-4c). Students in the more advanced AP Physics C courses may have had this topic in a previous first-year physics course, but students whose highest course was AP Physics 1 are not likely to have covered this topic. However, the topic of nuclear reactions may be covered in upper-level high school chemistry courses and some U.S. students may have learned this topic in these courses. Physics teachers reported that this topic had been taught to 45 percent of U.S. TIMSS Advanced students at the time of the assessment. This item maps to the *High* international benchmark level, and the topic was categorized as low coverage (figure 6-9). Overall, 42 percent of U.S. TIMSS Advanced students answered the item correctly, which was not measurably different from the international average (44 percent correct) (see the first page of exhibit 6-7 and table B-28).

The correct response (option A) relates the loss of mass of the atom and neutron to the energy emitted (gamma ray). Compared to the U.S. total of 42 percent correct, performance was higher for students taking AP Physics C (electricity and magnetism) (61 percent correct), while performance was lower for students whose highest course was AP Physics 1 (35 percent correct).

There were two similarly common incorrect responses chosen by U.S. students, option B (23 percent) and option C (26 percent) (see the second page of exhibit 6-7). Students who chose option B incorrectly related the difference in mass to ionization (removal of an electron). This response demonstrates the misconception that the mechanism of neutron capture and gamma emission involves the ionization of electrons in the outer shell of the atom (which are involved in chemical rather than nuclear reactions). This misconception was less common among students who had taken AP Physics B (9 percent) than among U.S. students overall.

Students who chose option C demonstrated the misunderstanding that the addition of a neutron results in an isotope with lower atomic mass. The percentage of students selecting this incorrect response ranged from 15 percent of students who had taken AP Physics C (electricity and magnetism) or AP Physics 2 to 30 percent of students who had taken AP Physics 1 and 37 percent who had taken AP Physics B.

Exhibit 6-7. TIMSS Advanced physics example item 6 with student performance data, by course type: 2015

The nucleus of an atom captures a neutron and produces a gamma ray. The total mass of the atom and the neutron before the reaction is greater than the mass of the combined atom and neutron after the reaction.

Which of the following best accounts for this difference in mass?

● The emitted gamma ray has an energy equivalent to the mass difference.

Ⓑ Electrons were ejected from the outer shell of the atom.

Ⓒ The neutron caused the atom to change to an isotope of lower mass.

Ⓓ The neutron was changed into a gamma ray.

Item classification and description	
Content domain:	Wave phenomena & atomic/ nuclear physics
Topic area:	Atomic and nuclear physics
Topic:	Mass-energy equivalence in nuclear reactions and particle transformations
Cognitive domain:	Knowing
International benchmark:	High
Description:	Recognize what accounts for the difference in the mass of an atom before and after a nuclear reaction

Student performance data

Course type[1]	Percent correct[2]
International average	**44**
U.S. total	**42**
AP physics courses[3]	**42**
AP Physics C courses	53 ▲
AP Physics C-E/M[4]	61 ▲
AP Physics C-M	46
AP Physics B	41
AP Physics 2	48
AP Physics 1	35 ▼
Non-AP physics courses	**44**
IB Physics[5]	42!
Other physics courses[6]	46!

▼ Subgroup/international average percent correct is significantly lower than U.S. total percent correct ($p < .05$).

▲ Subgroup/international average percent correct is significantly higher than U.S. total percent correct ($p < .05$).

Exhibit continues on next page.
See notes at end of exhibit.

Exhibit 6-7. TIMSS Advanced physics example item 6 with student performance data, by course type: 2015—Continued

Option	Response type	U.S. total percentage[7]
Correct		
A	Correctly relates the loss of mass of the atom and neutron to the energy emitted (gamma ray).	42
Incorrect		
B	Incorrectly relates the difference in mass to the ejection of electrons *[Demonstrates misconception that the mechanism of neutron capture and gamma emission involves ionization (removal of electrons) from the outer shell of the atom which is involved in chemical rather than nuclear reactions.]*	23
C	Incorrectly identifies that the addition of a neutron results in an isotope with lower atomic mass	26
D	Incorrectly identifies that neutron capture and gamma emission transforms a neutron into a gamma ray	7
	Blank[8]	1

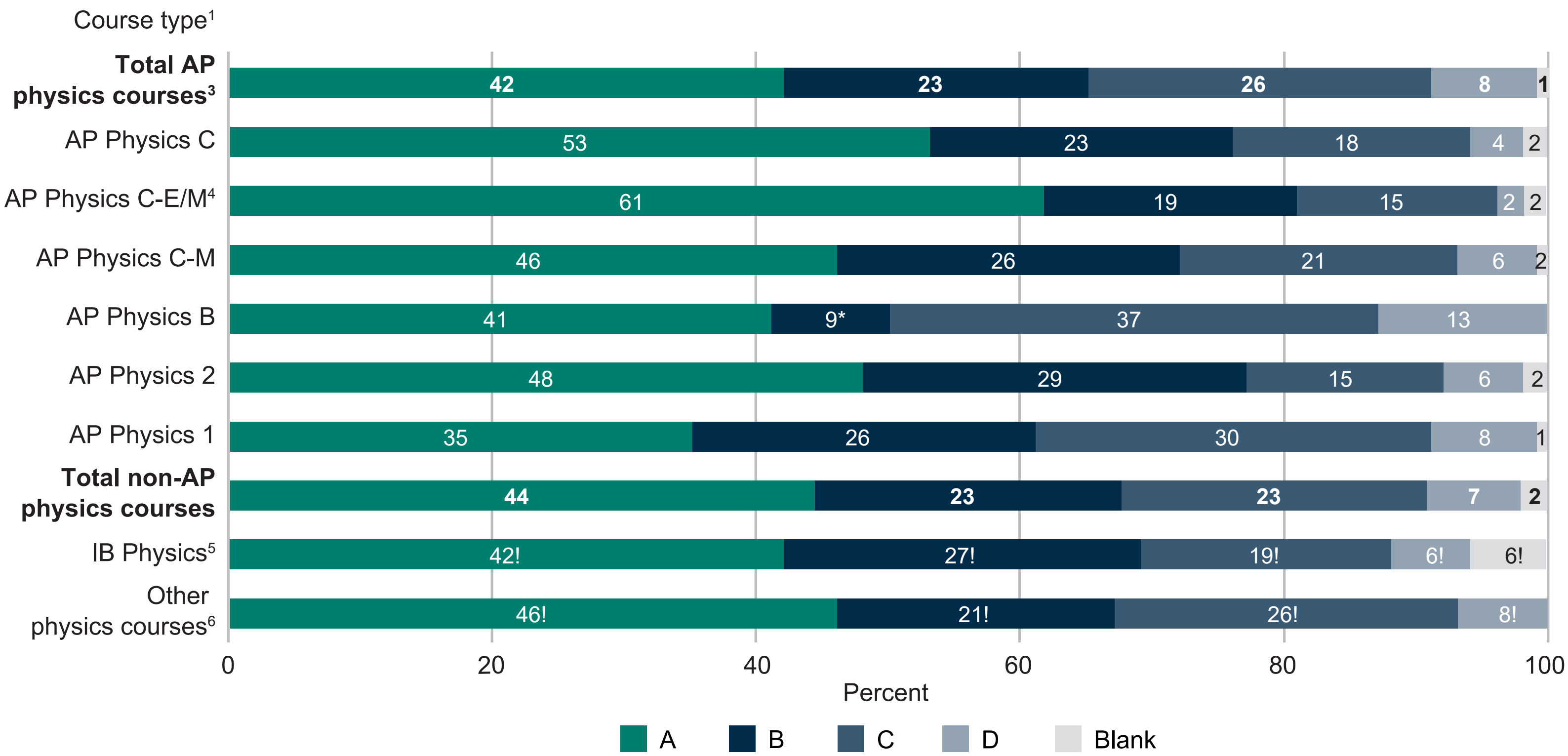
Scoring guide for multiple choice

Misconception of interest (option B).

Exhibit continues on next page.
See notes at end of exhibit.

Exhibit 6-7. TIMSS Advanced physics example item 6 with student performance data, by course type: 2015—Continued

Percentage distribution of U.S. TIMSS Advanced students across response types, by course type: 2015



Rounds to zero.
! Interpret with caution (sample size < 62, but > 30).
* *p* < .05. Subgroup percentage of students in option B is significantly different from the U.S. total percentage in option B.
[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other physics courses" category only if they have not taken an AP or IB course
[2] Percent correct is the weighted percentage of students receiving full credit or partial credit on an assessment item. For multiple choice items, there is only one correct response option, worth 1 point for full credit. Thus, the percent correct is the percentage of students who chose the correct response (option A in the scoring guide). Students who did not reach the item were not included in the calculation of this percentage.
[3] AP Physics C includes courses in mechanics (C-M) and electricity and magnetism (C-E/M). AP Physics B was a 1-year physics course that was discontinued after the 2013–14 school year and was replaced with the 2-year course sequence AP Physics 1 and 2 beginning in 2014–15. Data in the AP Physics B category are generally for students who took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering a 1-year course identified as AP Physics B in 2014–15.
[4] A large majority of the students whose highest course was tAP Physics C-E/M had also taken AP Physics C-M (89 percent), whether sequentially or in a combined course.
[5] Includes both higher-level and standard-level IB physics courses; nearly all students took the standard-level course.
[6] Includes other second-year physics courses (including "honors" or "regents" courses).
[7] Percentages are based on the full set of responses, including students who did not reach the item. Thus, the percentage in the correct category (option A in the scoring guide) may be slightly different from what is shown for percent correct on the first page of the exhibit (which excludes students in the not-reached category).
[8] Blank includes both students who omitted the item and those who did not reach the item.
NOTE: International average is the item percent correct, averaged across the countries participating in the TIMSS Advanced physics assessment. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Section 7: Summary and Future Research

This final section summarizes findings across both advanced mathematics and physics and suggests how future research might build on or supplement these findings.

7.1 Summary of findings

- The U.S. TIMSS Advanced 2015 population is a select group of students. The students taking the advanced mathematics assessment represented 12.5 percent of U.S. twelfth-graders overall, and the students taking the physics assessment represented 5.3 percent.
- Most U.S. TIMSS Advanced students took an Advanced Placement (AP) course: 76 percent of students in the advanced mathematics assessment took AP calculus and 83 percent of students in the physics assessment took AP physics (section 2, tables 2-2 and 2-3).[61]
  - Among those who had taken an AP calculus course, over twice as many had taken the lower level AP Calculus AB as had taken the higher level AP Calculus BC as their highest mathematics course. Overall, 56 percent of TIMSS Advanced students had taken AP Calculus AB as their highest mathematics course.
  - Among those who had taken an AP physics course, over one and a half times as many students had taken the lowest level algebra-based Physics 1 compared with the highest level calculus-based Physics C courses.[62] Overall, 42 percent of TIMSS Advanced students had taken AP Physics I as their highest physics course.
- U.S. students' opportunity to learn the advanced mathematics and physics content assessed in TIMSS Advanced varied by subject and highest level course they took. Generally, coverage of advanced mathematics topics in the intended and implemented curriculum was more comprehensive than the coverage of physics topics (section 3, tables 3-1 and 3-3).[63]
  - AP Calculus AB, AP Calculus BC, and the higher level IB mathematics course curricula generally covered all the TIMSS Advanced mathematics topics (either in the current course or in a prerequisite course). In contrast, the standard-level IB mathematics course covered 91 percent (21 of 23) of the TIMSS Advanced mathematics topics. The topics not covered by the standard-level IB course included one *algebra* topic (*operations with complex numbers*) and one *calculus* topic (*conditions for continuity and differentiability of functions*). Also, topics related to *finite and infinite series* in *algebra* and *vectors* in *geometry* appear to receive more emphasis in the IB courses and AP Calculus BC than in AP Calculus AB. On average,

[61] The rest of the U.S. TIMSS Advanced students (24 percent in advanced mathematics and 17 percent in physics) were in non-AP courses, including International Baccalaureate (IB) mathematics or physics courses (high-level or standard-level) and state-, district-, or school-specific calculus or second-year physics courses.

[62] AP physics courses additionally included AP Physics 2 and AP Physics B (16 percent of students combined). AP Physics C included courses in mechanics, electricity and magnetism, and combination courses.

[63] See also the supplemental tables A-3a, A-3b, and A-3c; and A-4a, A-4b, and A-4c.

across topics in all content domains, advanced mathematics teachers reported that the TIMSS Advanced mathematics topics were taught to 98 percent of all U.S. TIMSS Advanced students by the time of the assessment (either in the current or a prior year), which reflects an average of 99 percent for *algebra,* 98 percent for *calculus* and 96 percent for *geometry.*[64]

- The course curricula for AP Physics B, the higher-level IB physics course, as well as the two-year AP Physics 1 and 2 course sequence covered all or nearly all the TIMSS Advanced physics topics (at least 22 of 23 topics), and the standard-level IB physics course covered most topics (19). In contrast, the first-year AP Physics 1 course curriculum covered less than half of the TIMSS Advanced physics topics (10 of 23), excluding most topics in *electricity and magnetism* and *wave phenomena and atomic/nuclear physics.* The two AP Physics C course curricula covered all the topics in their corresponding content areas (*mechanics and electricity and magnetism*); coverage of other topics in *thermodynamics* and in *wave phenomena and atomic/nuclear physics* depend on which prior physics course(s) students had taken. On average, across topics in all content domains, physics teachers reported that the TIMSS Advanced physics topics were taught to 73 percent of all U.S. TIMSS Advanced students by the time of the assessment (either in the current or a prior year), which reflects an average of 87 percent for *mechanics and thermodynamics,* 66 percent for *electricity and magnetism* and 62 percent for *wave phenomena and atomic/nuclear physics.* The lower coverages in *electricity and magnetism* and *wave phenomena and atomic/nuclear physics* are driven by AP Physics 1 (reflecting 42 percent of students overall); less than half of these students had been taught the topics in these content domains (49 and 46 percent on average, respectively).

- U.S. students were below the TIMSS Advanced scale centerpoints in both advanced mathematics and physics.
  - In advanced mathematics, U.S. students had an average score of 485, which was 15 score points below the scale centerpoint. In physics, U.S. students had an average score of 437, which was 63 score points below the scale centerpoint.
  - U.S. students also performed, on average, below the centerpoints on the content domain subscales, except for the *calculus* subscale in advanced mathematics. In *calculus,* there was no measurable difference between the U.S. average score and the subscale centerpoint. In physics, average U.S. performance was especially low on the *electricity and magnetism* subscale (120 score points below the centerpoint).
- U.S. students who took the more rigorous AP courses in calculus and physics, however, scored on average higher than students who took other courses (sections 5 and 6, tables 5-1, 5-2, 6-1, and 6-2).
  - U.S. students who had taken an AP calculus course scored above the U.S. average in advanced mathematics (497 vs. 485), whereas those taking non-AP courses (448) scored below the U.S. average—with the pattern holding true for all three content domain subscales (*algebra, calculus,* and *geometry*). Students who had taken the higher level AP Calculus BC course tended to pull up the average score for AP calculus students overall (556 vs. 497). The average score for AP Calculus BC students was 71 points higher than the U.S. average, and these students were the only course subgroup to score higher than the international advanced mathematics centerpoint (by 56 points).

[64] The average percentage of students taught the TIMSS Advanced mathematics topics was at least 90 percent in all countries. See TIMSS Advanced 2015 International Results for Advanced Mathematics and Physics (exhibit M9.8).

- U.S. student performance in physics did not vary significantly by whether or not students had taken an AP physics course but did vary by the level of AP physics course taken. U.S. students who had taken an AP Physics C course, as well as those taking AP Physics 2, scored above the U.S. average overall and on all three content domain subscales (*mechanics and thermodynamics, electricity and magnetism,* and *wave phenomena and atomic/nuclear physics*). In particular, students in the highest level AP Physics C-Electricity and Magnetism course had an overall average score (537) that was 100 score points higher than the U.S. average (437), and they were the only course subgroup to score higher than the international physics centerpoint (by 37 points). Students who had taken AP Physics 1 as their highest level course scored below the U.S. average overall (407 vs. 437)—a pattern that held for all three content subscales.

- U.S. TIMSS Advanced students' average performance in advanced mathematics and physics varied by sex and race/ethnicity, and these differences may be related to coursetaking patterns (section 3, figures 3-1, 3-2, 3-4, and 3-5; sections 5 and 6, figures 5-1, 5-2, 6-1, and 6-2).
  - Males outperformed females in advanced mathematics and physics overall and on all three content subscales in each subject. A higher percentage of males (21 percent) than females (17 percent) had taken the higher level AP Calculus BC; and nearly twice the percentage of males (30 percent) as females (16 percent) had taken AP Physics C as their highest physics course.
  - In advanced mathematics, White students scored higher on average than Black and Hispanic students overall and on all three content subscales, but lower than students of Two or more races overall and on the *algebra* and *calculus* subscales. A higher percentage of White students (19 percent) had taken the higher level AP Calculus BC than Black or Hispanic students (6 and 12 percent, respectively), while a higher percentage of Black and Hispanic students had taken the lower level AP Calculus AB (72 and 74 percent, respectively) compared to 52 percent of White students. There were no measurable performance differences between Asian students and White students in advanced mathematics, nor in the percentages taking either AP calculus course.
  - In physics, White students scored higher on average than Black and Hispanic students overall and on all three content subscales. A higher percentage of White students (28 percent) had taken one or both of the highest level AP Physics C courses than Black students (14 percent). A higher percentage of Hispanic students (58 percent) than White students (40 percent) had taken the lowest level AP Physics 1 course. There were no measurable performance differences between Asian students and White students in physics, nor in the percentages taking the highest level AP Physics C courses.

- Average U.S. performance also varied by school locale (urban, suburban, town, or rural), but only for advanced mathematics; average performance did not vary by school control (public or private) for either subject (sections 5 and 6, figures 5-3 and 6-3).
  - Suburban students scored higher on average than rural students in advanced mathematics overall and on all three content subscales. A higher percentage of suburban students took AP Calculus BC than their rural (and town) counterparts.
  - There were no measurable differences in average physics scores among students from different locales on physics overall or on the three content subscales, although a higher percentage of suburban students took an AP Physics C course than students from urban and town locales.

- The percentages of U.S. TIMSS Advanced students reaching each of the three international benchmarks—*Advanced, High,* and *Intermediate*—were higher than the respective international medians in advanced mathematics, but in physics the percentages were lower or not measurably different than the international medians (sections 5 and 6, figures 5-4 and 6-4).
  - In advanced mathematics, 7 percent of U.S. students reached the *Advanced* benchmark, 26 percent reached the *High* benchmark, and 56 percent reached the *Intermediat*e benchmark. These percentages were all higher than the international medians (2, 14, and 43 percent, respectively).
  - In physics, 5 percent of U.S. students reached the *Advanced* benchmark, 18 percent reached the *High* benchmark, and 39 percent reached the *Intermediate* benchmark. These percentages were not measurably different than the international medians for the *Advanced* and *High* benchmarks (5 and 18 percent, respectively). However, the percentage of U.S. students reaching the *Intermediate* benchmark was lower than the international median (46 percent).
- U.S. students who had taken the highest level AP calculus and AP physics courses reached the international benchmarks in higher proportions than U.S. TIMSS Advanced students overall, while the proportion of students who had taken the lower level AP (or non-AP) courses reaching each benchmark were, generally, lower than or not measurably different from the U.S. total (sections 5 and 6, figures 5-4, 5-8, 6-4, and 6-8).
  - Among students who had taken AP Calculus BC, 20 percent reached the *Advanced* benchmark, 56 percent reached the *High* benchmark, and 85 percent reached the *Intermediate* benchmark in advanced mathematics. These percentages were higher than the U.S. total (7, 26, and 56 percent, respectively). In comparison, among students who had taken Calculus AB, 5 percent of students reached the *Advanced* benchmark, 22 percent reached the *High* benchmark, and 54 percent reached the *Intermediate* benchmark. Among students who had taken a non-AP course as their highest mathematics course, lower percentages of students reached the *Advanced* (2 percent), *High* (12 percent) and *Intermediate* benchmarks (39 percent) than the U.S. total.
  - Among students who had taken AP Physics C-Electricity and Magnetism, 18 percent of students reached the *Advanced* benchmark, 49 percent reached the *High* benchmark, and 77 percent reached the *Intermediat*e benchmark in physics. These percentages were higher than the U.S. total (5, 18, and 39 percent, respectively). In contrast, among students whose highest course was AP Physics 1, only 1 percent reached the *Advanced* benchmark, 9 percent reached the *High* benchmark, and 28 percent reached the *Intermediate* benchmark; these percentages were lower than the U.S. total for all benchmarks. For the other AP physics courses (C-Mechanics, B and 2), the percentages reaching the international benchmarks were generally not measurably different from the U.S. total, with the exception of the 53 percent of students who had taken AP Physics C in mechanics and the 56 percent who had taken AP Physics 2 reaching the *Intermediate* benchmark, both of which were higher than the U.S total.
- The coursetaking patterns of U.S. TIMSS Advanced students reaching the different achievement levels in advanced mathematics and physics differed from the coursetaking patterns in the population overall. In both subjects, the percentage of students reaching the *Advanced* and *High* benchmarks who had taken the most advanced AP courses was higher than U.S. students overall (19 percent taking AP Calculus BC and 10 percent taking the AP Physics C course in electricity and

magnetism). In advanced mathematics, 55 percent of students reaching the *Advanced* benchmark and 41 percent of those reaching the *High* benchmark had taken AP Calculus BC. In physics, 34 percent of students reaching the *Advanced* benchmark and 26 percent of those reaching the *High* benchmark had taken AP Physics C-Electricity and Magnetism (the majority of whom had also taken AP Physic C-Mechanics).

- The percentages of U.S. TIMSS Advanced students reaching the international benchmarks varied by student characteristics in both advanced mathematics and physics and varied by school characteristics in advanced mathematics (sections 5 and 6, figures 5-5, 5-6, 5-7, 6-5, 6-6, and 6-7).
  - In advanced mathematics, higher percentages of male students reached each of the three international benchmarks than U.S. TIMSS Advanced students overall. In contrast, lower percentages of female students and Black students reached each of the three benchmarks than students overall. Additionally, higher percentages of White students and suburban students reached the *High* and *Intermediate* benchmarks compared to students overall, whereas lower percentages of Hispanic and rural students did so. Also, a higher percentage of students of Two or more races reached the *Intermediate* benchmark and a lower percentage of rural students reached the *Advanced* benchmark than U.S. students overall. There were no measurable differences in the percentages of students reaching any of the benchmarks by school control.
  - In physics, higher percentages of male students reached each of the three international benchmarks than U.S. TIMSS Advanced students overall. In contrast, lower percentages of female, Black, and Hispanic students reached all three benchmarks than students overall. Additionally, higher percentages of White students reached the *High* and *Intermediate* benchmarks compared to U.S. students overall. Unlike advanced mathematics, there were no measurable differences in the percentages of U.S. students reaching any of the physics benchmarks by school locale. Nor were there differences by school control.
- In both advanced mathematics and physics, the average percent correct for U.S. students on TIMSS Advanced items was higher in two of the three content domains (and in specific topic areas) than on advanced mathematics and physics items overall (sections 5 and 6, tables 5-4 and 6-4, figures 5-9 and 6-9).
  - In advanced mathematics, the U.S. average percent correct was higher on the items in *algebra* and *calculus* (46 and 47 percent, respectively) and lower on the items in *geometry* (38 percent) compared to advanced mathematics overall (44 percent correct).
  - Within the three advanced mathematics content domains, the U.S. average percent correct was notably higher than advanced mathematics overall in the *algebra* topic area of *equations and inequalities* (53 percent), as well as in the *algebra* topic area of *functions* and all three topic areas in *calculus.* Topic areas in which U.S. average percent correct was lower were *expressions and operations* in *algebra* and both topic areas in *geometry.*
  - In physics, the U.S. average percent correct was higher on the items in *mechanics and thermodynamics* and w*ave phenomena and atomic/nuclear physics* (44 and 43 percent, respectively) and lower on the items in *electricity and magnetism* (36 percent), compared to physics items overall (42 percent).

- Within the three physics content domains, the U.S. average percent correct was notably higher than physics overall in the topic areas of *forces and motion* and *laws of conservation* in *mechanics and thermodynamics* (48 and 49 percent, respectively), as well as in *wave phenomena.* Topic areas in which U.S. average percent correct was lower were *heat and temperature* in *mechanics and thermodynamics* and both topic areas in *electricity and magnetism.*

- U.S. performance on TIMSS Advanced mathematics and physics items ranged widely and was not strictly related to the level of topic coverage. Level of topic coverage (high, moderate, or low) was based on coverage in the curricula of eligible courses and whether the topic was reported as taught by the time of the assessment; topic coverage was markedly greater for advanced mathematics than for physics (sections 5 and 6, figures 5-9 and 6-9). Even for topics reported as taught to most students, performance across the individual items within and across topics still varies for a variety of reasons, including how challenging the content covered by the topic is, the breadth of content covered by the topic, and the specific requirements of the items and their contexts.
  - Nearly three-quarters of the advanced mathematics topics (17 of 23) were in the high coverage category. Of the other 6 advanced mathematics topics, 3 were in the moderate coverage category and 3 were in the low coverage category. In contrast, the majority of the 23 physics topics were in the low-coverage category (16), compared to 4 in the high and 3 in the moderate coverage categories.
  - In advanced mathematics, item performance in the United States ranged from 1 to 76 percent correct across content domains. The topics in each coverage category generally spanned all content domains (with the exception being the moderate coverage category, which had no topics from *geometry*). However, there were no strong patterns observed between coverage levels and item performance. The topics at each coverage level had examples of relatively higher item performance and relatively lower item performance; in addition, there were some topics with wide-ranging item performance and others where item performance was more tightly clustered within topics. And while the items that required an extended constructed-response in high-coverage topics tended to be associated with relatively lower item performance, this was not uniformly the case. Of the three low-coverage mathematics topics, two were not covered in the standard-level IB mathematics course curriculum and one was expected to have been covered in a prerequisite course that AP calculus (and other calculus) teachers reported as not taught.
  - In physics, item performance in the United States ranged from 5 to 85 percent correct, and there were some patterns in topic coverage levels by content domain. For example, all the high-coverage, and two of the three moderate-coverage, topics were from the content domain of *mechanics and thermodynamics* and related to mechanics. In contrast, the low-coverage topics included all those from the content domains of *electricity and magnetism* and *wave phenomena and atomic/nuclear physics* (except *mechanical waves,* which was at the moderate-coverage level), as well as the three thermodynamics topics from the content domain of *mechanics and thermodynamics.* For these topics (most not covered at all in AP Physics 1), the range of item performance tended to be lower than for topics at the high- and moderate-coverage levels. Additionally, the low-coverage topics also included most of the lowest-performing physics items. Because AP Physics 1 was the highest physics course taken by 42 percent of U.S. students, this contributed to lower U.S. performance overall on items measuring these topics.

- U.S. students demonstrated some common approaches, misconceptions, and errors on the TIMSS Advanced mathematics and physics items, including those covering topics that had a high level of coverage across the U.S. TIMSS Advanced-eligible courses and where U.S. students performed relatively well on average (section 3 tables and supplementary tables in appendix A, section 5.2 and 6.2 tables and figures, and section 5.3 and 6.3 example item exhibits referenced below).
  - Advanced mathematics example items illustrated common approaches, misconceptions, and errors related to
    - **Solving problems in real-life contexts:** In general, solving problems in real-life contexts was more difficult for U.S. students than noncontextualized problems involving similar mathematical concepts. For example, in the context of maximizing a profit (exhibit 5-4), only 16 percent of U.S. students correctly differentiated the profit function, calculated the point at which the profit will be a maximum, and verified the solution to show that they indeed found the maximum point. More than half of U.S. students (55 percent) did not take a meaningful approach to the problem, and another 13 percent did not even attempt the item. Another item (exhibit 5-7) required students to apply their understanding of trigonometric functions to solve a contextualized problem involving changes in animal population. Thirty-six percent of U.S. students received full credit on this item by correctly calculating the maximum number of animals and also the time at which this will occur. In comparison, U.S. student performance was higher on a noncontextualized example item (exhibit 5-3) requiring students to determine the second derivative of a rational function (52 percent full credit). In both these latter example items, however, between about one-quarter and one-third of students did not make a meaningful attempt at the problems—although this was less common among students taking the highest level AP calculus course.
    - **Understanding concepts and procedures:** U.S. students did not always demonstrate a deep understanding of the mathematics concepts and procedures applied to solve problems. For example, in the context of comparing two rental plans (exhibit 5-5), students were required to solve simultaneous linear equations to find the point where they intersect and then to predict and explain what would happen under a different set of conditions. While the majority of U.S. students (57 percent) at least wrote the linear equations from the situation and applied the correct procedure to determine the point at which one plan would become cheaper, only about one-third (35 percent) understood the concepts well enough to explain that an equal increase in the initial cost for both plans will not change the difference between the y-intercepts or the slopes of the lines, and, therefore, the points at which the two lines intersect would not change.
    - **Understanding properties of vectors:** U.S. students did not perform well on most items in the *geometry* topic related to properties of vectors, nor in the *geometry* content domain, when compared to advanced mathematics overall. In the example item involving sums of vectors (exhibit 5-6), just 39 percent of U.S. students provided a correct response and earned full credit. Just as commonly, U.S. students selected an incorrect option (36 percent) that either confused the angle at which $\cos\Theta$ is zero, or erroneously used $\sin\Theta$ instead of $\cos\Theta$ in the formula for dot product of vectors. Another 9 percent of students each selected two other incorrect options, which demonstrated a similar minimal understanding of the properties of vectors. Although this topic is covered in the intended curriculum or prerequisite courses for

IB mathematics and AP calculus, teachers reported relatively lower percentages of students having been taught the topic—81 percent of U.S. students overall.

- Physics example items illustrated common approaches, misconceptions, and errors related to
  - **Applying Newton's laws of motion:** While U.S. students performed relatively well in the topic area of forces and motion, many had difficulty applying Newton's laws of motion in problem-solving situations. For example, in the context of a ball thrown upward (exhibit 6-2), 41 percent of U.S. students overall incorrectly indicated that there is no acceleration at the moment that the ball stops moving upward and reverses direction at its maximum height. In addition, 36 percent of students did not know that the amount of time the ball travels is the same on the way up as it is on the way down, with more than half of these students (19 percent overall) incorrectly indicating that the ball takes longer to go up than to come down because the ball is decelerating (slowing down) on the way up. These types of misconceptions were less common among students who had taken an AP physics C course (21 percent and 14 percent demonstrating the two different misconceptions) than U.S. students overall.
  - **Understanding electric and magnetic fields:** Average U.S. performance was lowest on the items in *electricity and magnetism* compared to physics overall, and many U.S. students demonstrated a lack of understanding of electric and magnetic fields. In one item (exhibit 6-5), for example, less than half of U.S. students were able to identify the direction of the electric force on a charged object in an electric field. On another item (exhibit 6-6), only about one-quarter of U.S. students could predict the change in the motion of an electron beam moving in an applied magnetic field. These topics are not well covered across all U.S. physics courses, but even in the higher level AP courses where these topics are covered, some students confused the direction of force on charged particles due to electric fields (in the direction of the electric field) with the direction of force due to magnetic fields (perpendicular to the direction of the magnetic field)—albeit at lower frequency than U.S. students overall.
  - **Solving quantitative problems:** The TIMSS Advanced physics assessment includes both qualitative (conceptual) problems and quantitative problems that require calculations. Many of the quantitative problems require students to show their work, including any equations used, for full credit. For example, one item involving a collision of two skiers (exhibit 6-3) required students to use the law of conservation of momentum to determine the final velocity of the skiers. About half of U.S. students (52 percent) demonstrated that they conceptually understood how to solve the problem by writing out the correct equations to receive full credit, but 17 percent either made a calculation error or showed incomplete work. Such errors were made by both AP and non-AP physics students with similar frequency.

7.2 Conclusions and future research

Together, these results expand upon the overview of U.S. performance provided in NCES' initial report on TIMSS Advanced (Provasnik et al. 2016). They focus specifically on U.S. students and highlight differences in coursetaking patterns by sex and race/ethnicity, noting the higher enrollment in the most advanced AP courses by male students and by White students. This pattern is further reflected in differences in performance favoring these groups and adds to an existing of body of research that demonstrates the relationship of AP coursework and/or AP-exam-taking with a host of positive outcomes including college entrance exam scores, enrollment, grades, course completion, persistence, and on-time completion rates (NCES 2019, Patterson et al. 2011, Mattern et al. 2013, Mustafa and Compton 2017). Further research using TIMSS Advanced might more fully explore the relationship of coursetaking and achievement gaps at the individual student level, examining performance differences between groups within courses.

Differences in coursetaking patterns across student subgroups by sex, race/ethnicity, and school locale are likely to be impacted by a number of factors, including students' access to the specific advanced mathematics and physics courses offered in their schools and student enrollment choices in these courses. Schools that offer AP and IB courses (and particularly the highest level AP Calculus BC and AP Physics C courses) may vary within and across states and districts. Male and female students in the TIMSS Advanced sample generally attend the same high schools, so gender differences are more likely to reflect student enrollment choices (Broughman et al. 2019, Mitchell et al. 2017). In the case of race/ethnicity and school locale differences, the percentage of students taking different courses may reflect both student enrollment choices and a possible lack of student access to the higher level AP courses in their schools. This report did not fully explore the impact of equity in student access and student choice in relating coursetaking patterns and curricular coverage to performance on TIMSS Advanced, and this is another area that would benefit from further research.

The results in this report also demonstrate content domains and topic areas of relative strength and weakness for U.S. students, which are sometimes but not always closely related to specific topic coverage in the curricula of the courses they have taken. While this report explores students' opportunity to learn mainly through its examination of the curriculum and teacher questionnaire data that show if a topic was taught, other factors likely bear on their performance such as how they are taught. Further research related to teaching and teacher qualifications and training—though not the focus of this report—might more fully explore these factors and their relationship to achievement in advanced subject matter in mathematics and physics. In addition, further research is needed on how well schools with advanced course offerings are staffed and capable of delivering quality instruction in terms of both human and other instructional resources.

The diagnostic data in this report from the example items helps demonstrate students' approaches, misconceptions, and errors and thus can inform what changes in curriculum or teaching might be beneficial. This report contributes to the literature on student misconceptions and can be extended in further research that looks at patterns of student misconceptions in multiple countries across grade levels and time by using data from TIMSS Advanced and TIMSS grade 4 and grade 8 as was done for a recent IEA report (Neidorf et al. 2019). The diagnostic information provided by the example items in this report is based on just a few of the hundreds of TIMSS and TIMSS Advanced items that are available to explore student misconceptions in mathematics and science across a range of topics at each grade level.

It is hoped that this report has contributed to our understanding of U.S. student performance in advanced STEM subjects and will spur additional research, including the analysis of item-level diagnostic data, to improve U.S. high school students' educational opportunities and college or career readiness in advanced mathematics and physics.

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Appendix A: Supplemental Data

Exhibit A-1. Description of content included in the intended curricula of TIMSS Advanced-eligible advanced mathematics courses in the United States: 2014–15

Advanced mathematics courses	Content covered
AP Calculus AB[1]	The curriculum for AP Calculus AB is divided into three main areas covering the following topics: • *Functions, Graphs, and Limits:* one-sided limits; limits at infinity; the limits of a sequence; infinite limits; using limits to determine continuity of a function; intermediate value theorem; mean value theorem • *Derivatives:* derivatives at a point including slope of a curve, tangent line to a curve, and rate of change; derivative as a function; characteristics of functions and their derivatives; increasing and decreasing of functions; mean value theorem; point of inflection; application of derivatives, including optimization and rates of change; differential equations • *Integrals:* antiderivative; integration by substitution; application of integrals (including finding the area of a region, the volume of a solid with known cross sections, the average value of a function, the distance traveled along a line, and accumulated change from rate of change); separable differential equations
AP Calculus BC[1]	AP Calculus BC covers all the topics of AP Calculus AB (see above), but at a faster pace and including the following additional topics: • *Functions, Graphs, and Limits:* parametric, polar, and vector functions • *Derivatives:* analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration; Euler method; L'Hospital's rule • *Integrals:* application of integrals, including polar curves and curves given in parametric form; solving logistic differential equations • *Polynomial Approximations and Series:* convergence and divergence of a series; series with constants, including geometric series, harmonic series, and alternating series with error bound; Maclaurin series for common functions; general Taylor series and representation; radius and interval of convergence; operations on power series
IB Mathematics	The curricula for the IB mathematics courses cover: *Standard level:* algebra; functions and equations; circular functions and trigonometry; vectors; statistics and probability; calculus *Higher level:* the standard-level topics, as well as additional topics in four areas—sets, relations, and groups; discrete mathematics; calculus; and statistics and probability—from which students can choose one area to study
Other calculus courses	Curricula for other calculus courses vary across states, districts, and schools but typically cover topics in differential and integral calculus and analytic geometry.

[1] Course curricula have been revised for the 2016–17 academic year, with topics added to both the AB and BC courses.
NOTE: AP stands for Advanced Placement. IB stands for International Baccalaureate. The AP and IB courses have specific curricula that are intended to be taught to all students regardless of the state, district, or school in which they take them. Additionally, AP and higher level IB courses enable students passing the associated exam to potentially earn college credit and/or qualify for more advanced college courses. For TIMSS Advanced 2015, eligible courses were identified using the definitions from the School Codes for the Exchange of Data (SCED) course classification system. Descriptions of courses and their content in school catalogues were reviewed to determine course eligibility. These descriptions are the basis of the summary provided for the other calculus courses. The TIMSS Advanced assessments were administered in the spring of the 2014–15 school year; thus, the courses represent what was offered in schools in the 2014–15 school year or the prior year.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA)'s Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015; The College Board, http://apcentral.collegeboard.com/apc/public/courses/descriptions/index.html; International Baccalaureate, http://www.ibo.org/programmes/diploma-programme/curriculum/.

Exhibit A-2. Description of content included in the intended curricula of TIMSS Advanced-eligible physics courses in the United States: 2014–15

Physics courses	Content covered
AP Physics B *(Prior to 2014–15)*	The curriculum for AP Physics B was divided into five main areas covering the following topics: • *Newtonian mechanics:* kinematics; Newton's laws of motion; work, energy, and power; systems of particles and linear momentum; circular motion and rotation; oscillations and gravitation • *Fluid and thermal physics:* fluid mechanics; temperature and heat; kinetic theory and thermodynamics • *Electricity and magnetism:* electrostatics; conductors and capacitors; electric circuits; magnetic fields; electromagnetism • *Waves and optics:* wave motion; physical optics; geometric optics • *Atomic and nuclear physics:* atomic physics and quantum effects; nuclear physics
AP Physics 1 & 2 *(2014–15)*	Collectively, the curricula for AP Physics 1 & 2 cover: *AP Physics 1:* Newtonian mechanics (including rotational and harmonic motion); circular motion and gravitation; work, energy, and conservation of energy; conservation of linear and angular momentum; mechanical waves and sound; electrostatics; electric circuits (DC circuits with resistors only) *AP Physics 2:* some topics in mechanics in more depth as well as more advanced topics including principles of fluids; thermodynamics; electric fields and potential; DC and RC circuits (including capacitors and dielectrics); magnetism and electromagnetic induction; electromagnetic radiation; physical and geometric optics; topics in modern physics (quantum, atomic, and nuclear physics)
AP Physics C	The curricula for AP Physics C courses cover: *AP Physics C-Mechanics:* the content under mechanics in AP Physics 1 and 2, but in greater depth with calculus applications *AP Physics C-Electricity and Magnetism:* the content under electricity and magnetism in AP Physics 1 & 2, but in greater depth with calculus applications
IB Physics[1]	The curricula for IB physics courses cover: *Standard level:* physics and physical measurement; mechanics; thermal physics; oscillations and waves; electric currents; fields and forces; atomic and nuclear physics; energy, power, and climate change; as well as two areas students choose for additional study from among seven options: sight and wave phenomena; quantum and nuclear physics; digital technology; relativity and particle physics; astrophysics; communication; and electromagnetic waves. *Higher level:* the standard-level topics, as well as additional required topics in motion in fields; thermal physics; wave phenomena; electromagnetic induction; quantum physics and nuclear physics; and digital technology; as well as two areas students choose for additional study from among six options: astrophysics; communications; electromagnetic waves; relativity; medical physics; and particle physics
Other physics courses	Curricula for other physics courses vary across states, districts, and schools but typically cover topics in Newtonian mechanics (forces, motion, and laws of conservation); heat, temperature and thermodynamics; electricity and magnetism; wave phenomena; and atomic and nuclear physics (to varying degrees).

[1] Course curricula have been revised for the 2016–17 academic year, thus some topics may have changed from those listed above.
NOTE: AP stands for Advanced Placement. IB stands for International Baccalaureate. The AP and IB courses have specific curricula that are intended to be taught to all students regardless of the state, district, or school in which they take them. Additionally, AP and higher level IB courses enable students passing the associated exam to potentially earn college credit and/or qualify for more advanced college courses. For TIMSS Advanced 2015, eligible courses were identified using the definitions from the School Codes for the Exchange of Data (SCED) course classification system. Descriptions of courses and their content in school catalogues were reviewed to determine course eligibility. These descriptions are the basis of the summary provided for the other physics courses. The TIMSS Advanced assessments were administered in the spring of the 2014–15 school year; thus, the courses represent what was offered in schools in the 2014–15 school year or in the prior year.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA)'s Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015; The College Board, http://apcentral.collegeboard.com/apc/public/courses/descriptions/index.html; International Baccalaureate, http://www.ibo.org/programmes/diploma-programme/curriculum/.

Table A-1. Number of TIMSS Advanced mathematics assessment items and the distribution of score points across the TIMSS Advanced mathematics topics in each content domain: 2015

Advanced mathematics content domains, topic areas, and topics	Number of items[1]	Number of score points[2]	Percentage of score points in content domain[3]
Algebra	**37**	**43**	**100**
Expressions	**13**	**13**	**30**
1. Operations with exponential, logarithmic, polynomial, rational, and radical expressions	2	2	5
2. Operations with complex numbers	2	2	5
3. Evaluating algebraic expressions (e.g., exponential, logarithmic, polynomial, rational, and radical)	5	5	12
4. The nth term of arithmetic and geometric sequences and the sums of finite and infinite series	4	4	9
Equations and inequalities	**15**	**21**	**49**
1. Linear and quadratic equations and inequalities as well as systems of linear equations and inequalities	4	5	12
2. Exponential, logarithmic, polynomial, rational, and radical equations	6	8	19
3. Using equations and inequalities to solve contextual problems	5	8	19
Functions	**9**	**9**	**21**
1. Equivalent representations of functions, including composite functions, as ordered pairs, tables, graphs, formulas, or words	4	4	9
2. Properties of functions, including domain and range	5	5	12
Calculus	**34**	**43**	**100**
Limits	**8**	**9**	**21**
1. Limits of functions, including rational functions	4	4	9
2. Conditions for continuity and differentiability of functions	4	5	12
Derivatives	**18**	**25**	**58**
1. Differentiation of functions (including polynomial, exponential, logarithmic, trigonometric, rational, and radical functions); differentiation of products, quotients, and composite functions	5	6	14
2. Using derivatives to solve problems (e.g., in optimization and rates of change)	3	4	9
3. Using first and second derivatives to determine slope and local extrema, and points of inflection	4	7	16
4. Using first and second derivatives to sketch and interpret graphs of functions	6	8	19
Integrals	**8**	**9**	**21**
1. Integrating functions (including polynomial, exponential, trigonometric, and rational functions)	2	2	5
2. Evaluating definite integrals, and applying integration to compute areas and volumes	6	7	16
Geometry	**30**	**34**	**100**
Noncoordinate and coordinate geometry	**16**	**18**	**53**
1. Properties of geometric figures in two and three dimensions	6	8	24
2. Using coordinate geometry to solve problems in two dimensions	5	5	15
3. Properties of vectors and their sums and differences	5	5	15
Trigonometry	**14**	**16**	**47**
1. Trigonometric properties of triangles (sine, cosine, and tangent)	5	6	18
2. Trigonometric functions and their graphs	4	4	12
3. Solving problems involving trigonometric functions	5	6	18

[1] This is the number of assessment items in each content domain, topic area, and topic.
[2] This is the maximum number of score points for each item, summed across the items in each content domain, topic area, and topic. All multiple-choice items are worth 1 score point, while constructed-response items may be worth either 1 or 2 score points.
[3] Data are provided in terms of the percentage of score points (instead of percentage of items) in order to be consistent with the way the target percentages are described in the framework.
NOTE: The data in this table are based on the items included in scaling and, thus, will differ slightly from the TIMSS Advanced international report, which did not exclude dropped items from its item counts. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015. NOTE: Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table A-2. Number of TIMSS Advanced physics assessment items and the distribution of score points across the TIMSS Advanced physics topics in each content domain: 2015

Physics content domains, topic areas, and topics	Number of items[1]	Number of score points[2]	Percentage of score points in content domain[3]
Mechanics and thermodynamics	**38**	**46**	**100**
Forces and motion	**19**	**20**	**43**
1. Applying Newton's laws and laws of motion	4	4	9
2. Forces, including frictional force, acting on a body	7	7	15
3. Forces acting on a body moving in a circular path; the body's centripetal acceleration, speed, and circling time	4	4	9
4. Law of gravitation in relation to movement of celestial objects	4	5	11
Laws of conservation	**9**	**12**	**26**
1. Kinetic and potential energy; conservation of mechanical energy	4	6	13
2. Law of conservation of momentum; elastic and inelastic collisions	3	4	9
3. First law of thermodynamics	2	2	4
Heat and temperature	**10**	**14**	**30**
1. Heat transfer and specific heat capacities	6	8	17
2. Law of ideal gases; expansion of solids and liquids in relation to temperature change	4	6	13
Electricity and magnetism	**28**	**31**	**100**
Electricity and electric circuits	**14**	**15**	**48**
1. Electrostatic attraction or repulsion between isolated charged particles–Coulomb's law	3	3	10
2. Charged particles in an electric field	6	7	23
3. Electrical circuits; using Ohm's law and Joule's law	5	5	16
Magnetism and electromagnetic induction	**14**	**16**	**52**
1. Charged particles in a magnetic field	6	8	26
2. Relationship between magnetism and electricity; magnetic fields around electric conductors; electromagnetic induction	4	4	13
3. Faraday's and Lenz's laws of induction	4	4	13
Wave phenomena and atomic/nuclear physics	**35**	**38**	**100**
Wave phenomena	**19**	**20**	**53**
1. Mechanical waves; the relationship between speed, frequency, and wavelength	6	6	16
2. Electromagnetic radiation; wavelength and frequency of various types of waves (radio, infrared, visible light, x-rays, gamma rays)	5	5	13
3. Thermal radiation, temperature, and wavelength	4	4	11
4. Reflection, refraction, interference, and diffraction	4	5	13
Atomic and nuclear physics	**16**	**18**	**47**
1. Structure of the atom and its nucleus; atomic number and atomic mass; electromagnetic emission and absorption and the behavior of electrons	3	3	8
2. Wave-particle duality and the photoelectric effect	4	4	11
3. Types of nuclear reactions and their role in nature (e.g., in stars) and society; radioactive isotopes	6	7	18
4. Mass-energy equivalence in nuclear reactions and particle transformations	3	4	11

[1] This is the number of assessment items in each content domain, topic area, and topic.

[2] This is the maximum number of score points for each item, summed across the items in each content domain, topic area, and topic. All multiple-choice items are worth 1 score point, while constructed-response items may be worth either 1 or 2 score points.

[3] Data are provided in terms of the percentage of score points (instead of percentage of items) in order to be consistent with the way the target percentages are described in the framework.

NOTE: The data in this table are based on the items included in scaling and, thus, will differ slightly from the TIMSS Advanced international report, which did not exclude dropped items from its item counts. Detail may not sum to totals because of rounding.

SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table A-3a. Extent of TIMSS Advanced *algebra* topic coverage in the intended curriculum by course type and the overall percentage of U.S. TIMSS Advanced students who were taught these topics, by topic area: 2015

	AP Calculus				IB Mathematics[1]				Other Calculus courses		Overall percentage of U.S. students taught *algebra* topics[2]
	BC		AB		Higher level		Standard level				
	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	
Total algebra	**9/9**	**100**	**9/9**	**99**	**9/9**	**100**	**8/9**	**100**	**—**	**98**	**99**
Expressions and operations	**4/4**	**99**	**4/4**	**98**	**4/4**	**100**	**3/4**	**100**	**—**	**96**	**98**
1. Operations with exponential, logarithmic, polynomial, rational, and radical expressions	⊙	100	⊙	100	●	100	●	100	—	99	100
2. Operations with complex numbers	⊙	100	⊙	100	●	100	○	100	—	99	100
3. Evaluating algebraic expressions (e.g., exponential, logarithmic, polynomial, rational, and radical)	⊙	100	⊙	100	●	100	●	100	—	99	100
4. The nth term of arithmetic and geometric sequences and the sums of finite and infinite series[3]	⊙	98	⊙	92	●	100	●	100	—	90	93
Equations and inequalities	**3/3**	**100**	**3/3**	**100**	**3/3**	**100**	**3/3**	**100**	**—**	**99**	**100**
1. Linear and quadratic equations and inequalities as well as systems of linear equations and inequalities	⊙	100	⊙	100	⊙	100	⊙	100	—	99	100
2. Exponential, logarithmic, polynomial, rational, and radical equations[4]	⊙	100	⊙	100	●	100	●	100	—	99	100
3. Using equations and inequalities to solve contextual problems[5]	⊙	—	⊙	—	●	—	●	—	—	—	—
Functions	**2/2**	**100**	**2/2**	**100**	**2/2**	**100**	**2/2**	**100**	**—**	**100**	**100**
1. Equivalent representations of functions, including composite functions, as ordered pairs, tables, graphs, formulas, or words	⊙	100	⊙	100	●	100	●	100	—	100	100
2. Properties of functions, including domain and range	⊙	100	⊙	100	●	100	●	100	—	100	100

● Topic area is covered in the course curriculum.
⊙ Topic reflects foundational knowledge expected to have been covered in a prerequisite mathematics course.
○ Topic is not included in the course curriculum.
— Not available.

[1] IB mathematics teachers often teach both the standard-level and higher-level courses in their schools, and their responses to the questionnaire may be based on the topics covered in the higher-level course. Therefore, some topics indicated as not included in the standard-level curriculum may be identified as having been taught to all IB mathematics students.

[2] "Overall percentage of students taught *algebra* topics" reflects all U.S. TIMSS Advanced students, including those taking AP, IB and other TIMSS Advanced-eligible non-AP, non-IB advanced mathematics courses.

[3] The intended curricula for both AP Calculus AB and BC include this topic as foundational knowledge expected to have been covered in a prerequisite mathematics course. In addition, AP Calculus BC covers more advanced topics related to finite and infinite series not included in AP Calculus AB.

[4] Two framework topics (#1 and #2 in the *equations and inequalities* topic area) were combined into a single topic in the teacher questionnaire; data for the combined topic are presented under both topics #1 and #2.

[5] Framework topic was not included in the teacher questionnaire.

NOTE: This table provides the disaggregated data on *algebra* topics for the main report table that summarizes advanced mathematics topic coverage at the topic area level (table 3-2). "Topics covered" reflects the intended curriculum based on the overlap of TIMSS Advanced *algebra* topics with Advanced Placement (AP) Calculus course descriptions available from the College Board and the core curriculum specified for the International Baccalaureate (IB) Mathematics standard-level and higher-level courses. The intended curriculum is not indicated for other TIMSS Advanced-eligible non-AP, non-IB calculus courses, since the curricula for these courses vary across states, districts, and schools. The ratios shown under each course type reflect the proportion of topics in a given topic area (and for the *algebra* content domain overall) that are covered in the curriculum for that course (or are considered foundational knowledge expected to have been covered in a prerequisite mathematics course). The Common Core State Standards in Mathematics were used to determine foundational knowledge covered in prerequisite mathematics courses. The "percentage of students taught" is based, for each topic, on the number of students whose teachers reported that the students in their advanced mathematics class were "mostly taught this year" or "mostly taught before this year" the respective topic. For students with more than one advanced mathematics teacher, students are counted as "taught" if any of their teachers indicated that the topic was taught in the current or a previous year by the time of the assessment. Percentages are shown for each topic as well as the average across the topics in each topic area and in the *algebra* content domain overall. Detail may not sum to totals because of rounding.

SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015; The College Board, http://apcentral.collegeboard.com/apc/public/courses/descriptions/index.html; International Baccalaureate, http://www.ibo.org/programmes/diploma-programme/curriculum/; The Common Core State Standards Initiative, http://www.corestandards.org/wp-content/uploads/Math_Standards1.pdf.

Table A-3b. Extent of TIMSS Advanced *calculus* topic coverage in the intended curriculum by course type and the overall percentage of U.S. TIMSS Advanced students who were taught these topics, by topic area: 2015

	AP Calculus				IB Mathematics[1]				Other Calculus courses		Overall percentage of U.S. students taught *calculus* topics[2]
	BC		AB		Higher level		Standard level				
	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	
Total calculus	**8/8**	**100**	**8/8**	**99**	**8/8**	**100**	**7/8**	**100**	**—**	**94**	**98**
Limits	**2/2**	**100**	**2/2**	**100**	**2/2**	**100**	**1/2**	**100**	**—**	**100**	**100**
1. Limits of functions, including rational functions	●	100	●	100	●	100	●	100	—	100	100
2. Conditions for continuity and differentiability of functions	●	100	●	100	●	100	○	100	—	100	100
Derivatives	**4/4**	**100**	**4/4**	**100**	**4/4**	**100**	**4/4**	**100**	**—**	**98**	**100**
1. Differentiation of functions (including polynomial, exponential, logarithmic, trigonometric, rational, and radical functions); differentiation of products, quotients, and composite functions	●	100	●	100	●	100	●	100	—	99	100
2. Using derivatives to solve problems (e.g., in optimization and rates of change)	●	100	●	100	●	100	●	100	—	99	100
3. Using first and second derivatives to determine slope and local extrema, and points of inflection	●	100	●	100	●	100	●	100	—	97	99
4. Using first and second derivatives to sketch and interpret graphs of functions[3]	●	—	●	—	●	—	●	—	—	—	—
Integrals	**2/2**	**100**	**2/2**	**98**	**2/2**	**100**	**2/2**	**100**	**—**	**83**	**95**
1. Integrating functions (including polynomial, exponential, trigonometric, and rational functions)	●	100	●	98	●	100	●	100	—	83	95
2. Evaluating definite integrals, and applying integration to compute areas and volumes[4]	●	100	●	98	●	100	●	100	—	83	95

● Topic area is covered in the course curriculum.

⊙ Topic reflects foundational knowledge expected to have been covered in a prerequisite mathematics course.

○ Topic is not included in the course curriculum.

— Not available.

[1] IB mathematics teachers often teach both the standard-level and higher-level courses in their schools, and their responses to the questionnaire may be based on the topics covered in the higher-level course. Therefore, some topics indicated as not included in the standard-level curriculum may be identified as having been taught to all IB mathematics students.

[2] "Overall percentage of U.S. students taught *calculus* topics" reflects all U.S. TIMSS Advanced students, including those taking AP, IB and other TIMSS Advanced-eligible non-AP, non-IB advanced mathematics courses.

[3] Framework topic was not included in the teacher questionnaire.

[4] Two framework topics (#1 and #2 in the *integrals* topic area) were combined into a single topic in the teacher questionnaire; data for the combined topic are presented under both topics #1 and #2.

NOTE: This table provides the disaggregated data on *calculus* topics for the main report table that summarizes advanced mathematics topic coverage at the topic area level (table 3-2). "Topics covered" reflects the intended curriculum based on the overlap of TIMSS Advanced *calculus* topics with Advanced Placement (AP) Calculus course descriptions available from the College Board and the core curriculum specified for the International Baccelaureate (IB) Mathematics standard-level and higher-level courses. The intended curriculum is not indicated for other TIMSS Advanced-eligible non-AP, non-IB calculus courses, since the curricula for these courses vary across states, districts, and schools. The ratios shown under each course type reflect the proportion of topics in a given topic area (and for the *calculus* content domain overall) that are covered in the curriculum for that course (or are considered foundational knowledge expected to have been covered in a prerequisite mathematics course). The Common Core State Standards in Mathematics were used to determine foundational knowledge covered in prerequisite mathematics courses. The "percentage of students taught" is based, for each topic, on the number of students whose teachers reported that the students in their advanced mathematics class were "mostly taught this year" or "mostly taught before this year" the respective topic. For students with more than one advanced mathematics teacher, students are counted as "taught" if any of their teachers indicated that the topic was taught in the current or a previous year by the time of the assessment. Percentages are shown for each topic as well as the average across the topics in each topic area and in the *calculus* content domain overall. Detail may not sum to totals because of rounding.

SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015; The College Board, http://apcentral.collegeboard.com/apc/public/courses/descriptions/index.html; International Baccalaureate, http://www.ibo.org/programmes/diploma-programme/curriculum/; The Common Core State Standards Initiative, http://www.corestandards.org/wp-content/uploads/Math_Standards1.pdf.

Table A-3c. Extent of TIMSS Advanced *geometry* topic coverage in the intended curriculum by course type and the overall percentage of U.S. TIMSS Advanced students who were taught these topics, by topic area: 2015

	AP Calculus				IB Mathematics[1]				Other Calculus courses		Overall percentage of U.S. students taught *geometry* topics[2]
	BC		AB		Higher level		Standard level				
	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	
Total geometry	**6/6**	**97**	**6/6**	**96**	**6/6**	**100**	**6 / 6**	**100**	**—**	**92**	**96**
Noncoordinate and coordinate geometry	**3/3**	**94**	**3/3**	**94**	**3/3**	**100**	**3/3**	**100**	**—**	**88**	**93**
1. Properties of geometric figures in two and three dimensions	⊙	98	⊙	99	⊙	100	⊙	100	—	100	99
2. Using coordinate geometry to solve problems in two dimensions[3]	⊙	98	⊙	99	⊙	100	⊙	100	—	100	99
3. Properties of vectors and their sums and differences[4]	⊙	87	⊙	83	●	100	●	100	—	66	81
Trigonometry	**3/3**	**100**	**3/3**	**100**	**3/3**	**100**	**3/3**	**100**	**—**	**98**	**100**
1. Trigonometric properties of triangles (sine, cosine, and tangent)	⊙	100	⊙	100	⊙	100	⊙	100	—	100	100
2. Trigonometric functions and their graphs	⊙	100	⊙	100	●	100	●	100	—	97	99
3. Solving problems involving trigonometric functions[5]	⊙	—	⊙	—	●	—	●	—	—	—	—

● Topic area is covered in the course curriculum.
⊙ Topic reflects foundational knowledge expected to have been covered in a prerequisite mathematics course.
○ Topic is not included in the course curriculum.
— Not available.

[1] IB mathematics teachers often teach both the standard-level and higher-level courses in their schools, and their responses to the questionnaire may be based on the topics covered in the higher-level course. Therefore, some topics indicated as not included in the standard-level curriculum may be identified as having been taught to all IB mathematics students.
[2] "Overall percentage of students taught *geometry* topics" reflects all U.S. TIMSS Advanced students, including those taking AP, IB and other TIMSS Advanced-eligible non-AP, non-IB advanced mathematics courses.
[3] Two framework topics (#1 and #2 in the *noncoordinate and coordinate geometry* topic area) were combined into a single topic in the teacher questionnaire; data for the combined topic are presented under both topics #1 and #2.
[4] The intended curricula for both AP Calculus AB and BC include this topic as foundational knowledge expected to have been covered in a prerequisite mathematics course. In addition, AP Calculus BC covers more advanced topics related to vectors not included in AP Calculus AB.
[5] Framework topic was not included in the teacher questionnaire.
NOTE: This table provides the disaggregated data on *geometry* topics for the main report table that summarizes advanced mathematics topic coverage at the topic area level (table 3-2). "Topics covered" reflects the intended curriculum based on the overlap of TIMSS Advanced *geometry* topics with Advanced Placement (AP) Calculus course descriptions available from the College Board and the core curriculum specified for the International Baccelaureate (IB) Mathematics standard-level and higher-level courses. The intended curriculum is not indicated for other TIMSS Advanced-eligible non-AP, non-IB calculus courses, since the curricula for these courses vary across states, districts, and schools. The ratios shown under each course type reflect the proportion of topics in a given topic area (and for the *geometry* content domain overall) that are covered in the curriculum for that course (or are considered foundational knowledge expected to have been covered in a prerequisite mathematics course). The Common Core State Standards in Mathematics were used to determine foundational knowledge covered in prerequisite mathematics courses. The "percentage of students taught" is based, for each topic, on the number of students whose teachers reported that the students in their advanced mathematics class were "mostly taught this year" or "mostly taught before this year" the respective topic. For students with more than one advanced mathematics teacher, students are counted as "taught" if any of their teachers indicated that the topic was taught in the current or a previous year by the time of the assessment. Percentages are shown for each topic as well as the average across the topics in each topic area and in the *geometry* content domain overall. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015; The College Board, http://apcentral.collegeboard.com/apc/public/courses/descriptions/index.html; International Baccalaureate, http://www.ibo.org/programmes/diploma-programme/curriculum/; The Common Core State Standards Initiative, http://www.corestandards.org/wp-content/uploads/Math_Standards1.pdf.

Table A-4a. Extent of TIMSS Advanced *mechanics and thermodynamics* topic coverage in the intended curriculum by course type and the overall percentage of U.S. TIMSS Advanced students who were taught these topics, by topic area: 2015

	AP Physics C[1]				AP Physics 1 and 2				AP Physics B		IB Physics[2]				Other Physics courses		Overall percentage of U.S. students taught *mechanics and thermodynamics* topics[3]
	Electricity & Magnetism		Mechanics		Physics 2		Physics 1				Higher level		Standard level				
	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	
Total mechanics and thermodynamics	**6/9**	**85**	**6/9**	**83**	**9/9**	**95**	**6/9**	**83**	**9/9**	**95**	**9/9**	**100**	**8/9**	**100**	**—**	**89**	**87**
Forces and motion	**4/4**	**100**	**4/4**	**97**	**4/4**	**100**	**4/4**	**99**	**4/4**	**100**	**4/4**	**100**	**4/4**	**100**	**—**	**93**	**98**
1. Applying Newton's laws and laws of motion	⊙	100	●	100	⊙	100	●	100	●	100	●	100	●	100	—	100	100
2. Forces, including frictional force, acting on a body	⊙	100	●	100	⊙	100	●	100	●	100	●	100	●	100	—	100	100
3. Forces acting on a body moving in a circular path; the body's centripetal acceleration, speed, and circling time	⊙	100	●	100	⊙	100	●	98	●	100	●	100	●	100	—	86	97
4. The law of gravitation in relation to movement of celestial objects	⊙	100	●	88	⊙	100	●	97	●	100	●	100	◖	100	—	86	95
Laws of conservation	**2/3**	**86**	**2/3**	**82**	**3/3**	**94**	**2/3**	**82**	**3/3**	**96**	**3/3**	**100**	**2/3**	**100**	**—**	**95**	**87**
1. Kinetic and potential energy; conservation of mechanical energy	⊙	100	●	97	⊙	100	●	99	●	100	●	100	●	100	—	100	99
2. Law of conservation of momentum; elastic and inelastic collisions	⊙	100	●	100	●	100	●	99	●	100	●	100	●	100	—	100	100
3. First law of thermodynamics	○	58	○	48	●	82	○	48	●	88	●	100	○	100	—	86	63
Heat and temperature	**0/2**	**77**	**0/2**	**56**	**2/2**	**88**	**0/2**	**51**	**2/2**	**84**	**2/2**	**100**	**2/2**	**100**	**—**	**71**	**63**
1. Heat transfer and specific heat capacities	○	55	○	57	●	89	○	49	●	88	●	100	●	100	—	76	64
2. Law of ideal gases; expansion of solids and liquids in relation to temperature change	○	53	○	55	●	87	○	52	●	79	●	100	◖	100	—	66	62

● Topic is covered in the course curriculum.

◖ Basic introduction to the topic (or a portion of the topic) is included in the course curriculum.

⊙ Topic reflects foundational concepts expected for AP Physics 2 that are covered in AP Physics 1 and those expected for AP Physics C-Electricity & Magnetism that are covered in C-Mechanics.

○ Topic is not included in the course curriculum.

— Not available.

[1] The AP Physics C course curriculum covers a specific set of topics in mechanics and in electricity and magnetism. The extent to which other TIMSS Advanced topics have been covered in AP Physics C or prior physics courses varies across states, districts, and schools.

[2] IB physics teachers often teach both the standard-level and higher-level courses in their schools, and their responses to the questionnaire may be based on the topics covered in the higher-level course. Therefore, some topics indicated as not included in the standard-level curriculum may be identified as having been taught to all IB physics students.

[3] "Overall percentage of students taught *mechanics and thermodynamics* topics" reflects all U.S. TIMSS Advanced students, including those taking AP, IB and other TIMSS Advanced-eligible non-AP, non-IB physics courses.

NOTE: This table provides the disaggregated data on *mechanics and thermodynamics* topics for the main report table that summarizes advanced mathematics topic coverage at the topic area level (table 3-4). "Topics covered" reflects the intended curriculum based on the overlap of TIMSS Advanced *mechanics and thermodynamics* topics with Advanced Placement (AP) Physics course descriptions available from the College Board and the core curriculum specified for the International Baccelaureate (IB) Physics standard-level and higher-level courses. The intended curriculum is not indicated for other TIMSS Advanced-eligible non-AP, non-IB second-year physics courses, since the curricula for these courses vary across states, districts, and schools. The ratios shown under each course type reflect the proportion of topics in a given topic area (and for the *mechanics and thermodynamics* content domain overall) that are covered in the curriculum for that course (or are considered foundational knowledge expected to have been covered in a prior physics course). The "percentage of students taught" is based, for each topic, on the number of students whose teachers reported that the students in their physics class were "mostly taught this year" or "mostly taught before this year" the respective topic. For students with more than one physics teacher, students are counted as "taught" if any of their teachers indicated that the topic was taught in the current or a previous year by the time of the assessment. Percentages are shown for each topic as well as the average across the topics in each topic area and in the *mechanics and thermodynamics* content domain overall. Detail may not sum to totals because of rounding.

SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015; The College Board, http://apcentral.collegeboard.com/apc/public/courses/descriptions/index.html; International Baccalaureate, http://www.ibo.org/programmes/diploma-programme/curriculum/.

Table A-4b. Extent of TIMSS Advanced *electricity and magnetism* topic coverage in the intended curriculum by course type and the overall percentage of U.S. TIMSS Advanced students who were taught these topics, by topic area: 2015

	AP Physics C[1]				AP Physics 1 and 2				AP Physics B		IB Physics[2]				Other Physics courses		Overall percentage of U.S. students taught *electricity and magnetism*[3]
	Electricity & Magnetism		Mechanics		Physics 2		Physics 1				Higher level		Standard level				
	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	
Total electricity and magnetism	**6/6**	**92**	**0/6**	**64**	**6/6**	**86**	**2/6**	**49**	**6/6**	**87**	**6/6**	**100**	**4/6**	**100**	**—**	**65**	**66**
Electricity and electric circuits	**3/3**	**99**	**0/3**	**72**	**3/3**	**87**	**2/3**	**68**	**3/3**	**97**	**3/3**	**100**	**3/3**	**100**	**—**	**76**	**79**
1. Electrostatic attraction or repulsion between isolated charged particles–Coulomb's law	●	100	○	78	●	92	◖	75	●	99	●	100	●	100	—	72	82
2. Charged particles in an electric field	●	97	○	70	●	85	○	59	●	95	●	100	●	100	—	72	74
3. Electrical circuits; using Ohm's law and Joule's law	●	100	○	68	●	85	◖	71	●	98	●	100	●	100	—	85	81
Magnetism and electromagnetic induction	**3/3**	**84**	**0/3**	**56**	**3/3**	**84**	**0/3**	**29**	**3/3**	**76**	**3/3**	**100**	**1/3**	**100**	**—**	**54**	**54**
1. Charged particles in a magnetic field	●	98	○	62	●	85	○	31	●	77	●	100	●	100	—	58	58
2. Relationship between magnetism and electricity; magnetic fields around electric conductors; electromagnetic induction	●	82	○	56	●	85	○	31	●	76	●	100	○	100	—	58	55
3. Faraday's and Lenz's laws of induction	●	74	○	51	●	84	○	25	●	76	●	100	○	100	—	44	49

● Topic is covered in the course curriculum.
◖ Basic introduction to the topic (or a portion of the topic) is included in the course curriculum.
⊙ Topic reflects foundational concepts expected for AP Physics 2 that are covered in AP Physics 1 and those expected for AP Physics C-Electricity & Magnetism that are covered in C-Mechanics.
○ Topic is not included in the course curriculum.

— Not available.
[1] The AP Physics C course curriculum covers a specific set of topics in mechanics and in electricity and magnetism. The extent to which other TIMSS Advanced topics have been covered in AP Physics C or prior physics courses varies across states, districts, and schools.
[2] IB physics teachers often teach both the standard-level and higher-level courses in their schools, and their responses to the questionnaire may be based on the topics covered in the higher-level course. Therefore, some topics indicated as not included in the standard-level curriculum may be identified as having been taught to all IB physics students.
[3] "Overall percentage of U.S. students taught *electricity and magnetism* topics" reflects all U.S. TIMSS Advanced students, including those taking AP, IB and other TIMSS Advanced-eligible non-AP, non-IB physics courses.
NOTE: This table provides the disaggregated data on *electricity and magnetism* topics for the main report table that summarizes advanced mathematics topic coverage at the topic area level (table 3-4). "Topics covered" reflects the intended curriculum based on the overlap of TIMSS Advanced *electricity and magnetism* topics with Advanced Placement (AP) Physics course descriptions available from the College Board and the core curriculum specified for the International Baccelaureate (IB) Physics standard-level and higher-level courses. The intended curriculum is not indicated for other TIMSS Advanced-eligible non-AP, non-IB second-year physics courses, since the curricula for these courses vary across states, districts, and schools. The ratios shown under each course type reflect the proportion of topics in a given topic area (and for the *electricity and magnetism* content domain overall) that are covered in the curriculum for that course (or are considered foundational knowledge expected to have been covered in a prior physics course). The "percentage of students taught" is based, for each topic, on the number of students whose teachers reported that the students in their physics class were "mostly taught this year" or "mostly taught before this year" the respective topic. For students with more than one physics teacher, students are counted as "taught" if any of their teachers indicated that the topic was taught in the current or a previous year by the time of the assessment. Percentages are shown for each topic as well as the average across the topics in each topic area and in the *electricity and magnetism* content domain overall. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015; The College Board, http://apcentral.collegeboard.com/apc/public/courses/descriptions/index.html; International Baccalaureate, http://www.ibo.org/programmes/diploma-programme/curriculum/.

Table A-4c. Extent of TIMSS Advanced *wave phenomena and atomic/nuclear physics* topic coverage in the intended curriculum by course type and the overall percentage of U.S. TIMSS Advanced students who were taught these topics, by topic area: 2015

	AP Physics C[1]				AP Physics 1 and 2						IB Physics[2]						Overall percentage of U.S. students taught *wave phenomena and atomic/nuclear physics* topics[3]
	Electricity & Magnetism		Mechanics		Physics 2		Physics 1		AP Physics B		Higher level		Standard level		Other Physics courses		
	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	
Total wave phenomena and atomic/nuclear physics	**0/8**	**67**	**0/8**	**55**	**7/8**	**86**	**2/8**	**46**	**7/8**	**83**	**8/8**	**100**	**7/8**	**96**	**—**	**72**	**62**
Wave phenomena	**0 / 4**	**70**	**0 / 4**	**64**	**3 / 4**	**89**	**2 / 4**	**56**	**3 / 4**	**87**	**4 / 4**	**100**	**4 / 4**	**96**	**—**	**79**	**69**
1. Mechanical waves; the relationship between speed, frequency, and wavelength	○	89	○	81	⊙	92	●	80	●	99	●	100	●	100	—	99	87
2. Electromagnetic radiation; wavelength and frequency of various types of waves (radio, infrared, visible light, x-rays, gamma rays)	○	68	○	74	●	92	○	61	●	97	●	100	●	94	—	73	73
3. Thermal radiation, temperature, and wavelength	○	48	○	40	○	89	○	39	○	66	●	100	●	94	—	70	53
4. Reflection, refraction, interference, and diffraction	○	76	○	61	●	82	◐	43	●	88	●	100	●	95	—	75	63

● Topic is covered in the course curriculum.

◐ Basic introduction to the topic (or a portion of the topic) is included in the course curriculum.

⊙ Topic reflects foundational concepts expected for AP Physics 2 that are covered in AP Physics 1 and those expected for AP Physics C-Electricity & Magnetism that are covered in C-Mechanics.

○ Topic is not included in the course curriculum.

See notes at end of table.

Table A-4c. Extent of TIMSS Advanced *wave phenomena and atomic/nuclear physics* topic coverage in the intended curriculum by course type and the overall percentage of U.S. TIMSS Advanced students who were taught these topics, by topic area: 2015—Continued

	AP Physics C[1]				AP Physics 1 and 2				AP Physics B		IB Physics[2]				Other Physics courses		Overall percentage of U.S. students taught *wave phenomena and atomic/nuclear physics* topics[3]
	Electricity & Magnetism		Mechanics		Physics 2		Physics 1				Higher level		Standard level				
	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	Topics covered	Percentage of students taught	
Atomic and nuclear physics	**0 / 4**	**63**	**0 / 4**	**46**	**4 / 4**	**84**	**0 / 4**	**36**	**4 / 4**	**79**	**4 / 4**	**100**	**3 / 4**	**97**	**—**	**65**	**54**
1. The structure of the atom and its nucleus; atomic number and atomic mass; electromagnetic emission and absorption and the behavior of electrons	○	76	○	63	●	92	○	67	●	81	●	100	●	100	—	76	73
2. Wave-particle duality and the photoelectric effect	○	62	○	43	●	79	○	28	●	79	●	100	○	94	—	61	50
3. Types of nuclear reactions and their role in nature (e.g., in stars) and society; radioactive isotopes[4]	○	62	○	43	●	79	○	28	●	79	●	100	●	94	—	61	50
4. Mass-energy equivalence in nuclear reactions and particle transformations	○	52	○	33	●	85	○	20	●	76	●	100	●	100	—	60	45

● Topic is covered in the course curriculum.
◖ Basic introduction to the topic (or a portion of the topic) is included in the course curriculum.
⦿ Topic reflects foundational concepts expected for AP Physics 2 that are covered in AP Physics 1 and those expected for AP Physics C-Electricity & Magnetism that are covered in C-Mechanics.
○ Topic is not included in the course curriculum.
— Not available.

[1] The AP Physics C course curriculum covers a specific set of topics in mechanics and in electricity and magnetism. The extent to which other TIMSS Advanced topics have been covered in AP Physics C or prior physics courses varies across states, districts, and schools.

[2] IB physics teachers often teach both the standard-level and higher-level courses in their schools, and their responses to the questionnaire may be based on the topics covered in the higher-level course. Therefore, some topics indicated as not included in the standard-level curriculum may be identified as having been taught to all IB physics students.

[3] "Overall percentage of U.S. students taught *wave phenomena and atomic/nuclear physics* topics" reflects all U.S. TIMSS Advanced students, including those taking AP, IB and other TIMSS Advanced-eligible non-AP, non-IB physics courses.

[4] Two framework topics (#2 and #3 in the *atomic and nuclear physics* topic area) were combined into a single topic in the teacher questionnaire; data for the combined topic are presented under both topics #2 and #3.

NOTE: This table provides the disaggregated data on *wave phenomena and atomic/nuclear physics* topics for the main report table that summarizes advanced mathematics topic coverage at the topic area level (table 3-4). "Topics covered" reflects the intended curriculum based on the overlap of TIMSS Advanced *wave phenomena and atomic/nuclear physics* topics with Advanced Placement (AP) Physics course descriptions available from the College Board and the core curriculum specified for the International Baccelaureate (IB) Physics standard-level and higher-level courses. The intended curriculum is not indicated for other TIMSS Advanced-eligible non-AP, non-IB second-year physics courses, since the curricula for these courses vary across states, districts, and schools. The ratios shown under each course type reflect the proportion of topics in a given topic area (and for the *wave phenomena and atomic/ nuclear physics* content domain overall) that are covered in the curriculum for that course (or are considered foundational knowledge expected to have been covered in a prior physics course). The "percentage of students taught" is based, for each topic, on the number of students whose teachers reported that the students in their physics class were "mostly taught this year" or "mostly taught before this year" the respective topic. For students with more than one physics teacher, students are counted as "taught" if any of their teachers indicated that the topic was taught in the current or a previous year by the time of the assessment. Percentages are shown for each topic as well as the average across the topics in each topic area and in the *wave phenomena and atomic/nuclear physics* content domain overall. Detail may not sum to totals because of rounding.

SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015; The College Board, http://apcentral.collegeboard.com/apc/public/courses/descriptions/index.html; International Baccalaureate, http://www.ibo.org/programmes/diploma-programme/curriculum/.

Appendix B: Data Tables for Report Figures

Table B-1. Percentage of U.S. TIMSS Advanced mathematics students, by sex and course type: 2015

Course type[1]	U.S. total	Males	Females
Total AP calculus courses	**76**	**77**	**74**
AP Calculus BC	19	21	17
AP Calculus AB	56	56	57
Total non-AP mathematics courses	**24**	**23**	**26**
IB Mathematics[2]	‡	‡	‡
Other calculus courses[3]	23	22	25

‡ Reporting standards not met (sample size < 62).
[1] Course type reflects the highest level mathematics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other calculus courses" category only if they have not taken an AP or IB course.
[2] Includes higher-level and standard-level IB mathematics courses.
[3] Includes other calculus courses (including "honors" and "regents" courses).
NOTE: Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-2. Percentage of U.S. TIMSS Advanced mathematics students, by race/ethnicity and course type: 2015

Course type[1]	U.S. total	White	Black	Hispanic	Asian	Two or more races
Total AP calculus courses	**76**	**71**	**79**	**86**	**82**	**80**
AP Calculus BC	19	19	6	12	27	33
AP Calculus AB	56	52	72	74	55	47
Total non-AP mathematics courses	**24**	**29**	**21**	**14**	**18**	**20**
IB Mathematics[2]	‡	‡	‡	‡	‡	‡
Other calculus courses[3]	23	28	20	13	13	19

‡ Reporting standards not met (sample size < 62).
[1] Course type reflects the highest level mathematics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other calculus courses" category only if they have not taken an AP or IB course.
[2] Includes higher-level and standard-level IB mathematics courses.
[3] Includes other calculus courses (including "honors" and "regents" courses).
NOTE: Reporting standards were not met for Native Hawaiian/Pacific Islander and Native American/Alaska Native students (sample size < 62). Data for these students are not shown separately in the table and associated figure, though data are included in the U.S. total. Black includes African American and Hispanic includes Latino. Racial categories exclude Hispanic origin. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-3. Percentage of U.S. TIMSS Advanced mathematics students, by school locale and course type: 2015

Course type[1]	U.S. total	Urban	Suburban	Town	Rural
Total AP calculus courses	**76**	**71**	**84**	**67**	**70**
AP Calculus BC	19	18	28	16	5
AP Calculus AB	56	53	56	52	65
Total non-AP mathematics courses	**24**	**29**	**16**	**33**	**30**
IB Mathematics[2]	‡	‡	‡	‡	‡
Other calculus courses[3]	23	22	16	33	30

‡ Reporting standards not met (sample size < 62).

[1] Course type reflects the highest level mathematics course taken. If students took another course in addition to an Advanced Placement (AP) or IB course, they are included in the AP or IB category. Students are included in the “other calculus courses” category only if they have not taken an AP or IB course.

[2] Includes higher-level and standard-level IB mathematics courses.

[3] Includes other calculus courses (including “honors” and “regents” courses).

NOTE: Urban is defined as territories inside an urbanized area and inside a principal city. Suburban is defined as territories inside an urbanized area but outside a principal city. Town is defined as territories inside an urban cluster but outside an urbanized area. Rural is defined as territories that are not in an urbanized area or urban cluster. For additional information, see https://nces.ed.gov/programs/edge/docs/NCES_LOCALE_USERSMANUAL_2016012.pdf. Detail may not sum to totals because of rounding.

SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015

Table B-4. Percentage of U.S. TIMSS Advanced physics students, by sex and course type: 2015

Course type[1]	U.S. total	Males	Females
Total AP physics courses[2]	**83**	**83**	**81**
AP Physics C	25	30	16
AP Physics B	12	9	16
AP Physics 2	4	5	4
AP Physics 1	42	39	46
Total non-AP physics courses	**17**	**17**	**19**
IB Physics[3]	6	6	6
Other physics courses[4]	12	11	13

[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the “other physics courses” category only if they have not taken an AP or IB course.

[2] AP Physics C includes courses in mechanics (C-M) and electricity and magnetism (C-E/M). AP Physics B was a 1-year physics course that was discontinued after the 2013–14 school year and was replaced with the 2-year course sequence AP Physics 1 and 2 beginning in 2014–15. Data in the AP Physics B category are generally for students whose took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering a 1-year course identified as AP Physics B in 2014–15.

[3] Includes higher-level and standard-level IB physics courses.

[4] Includes other types of second-year physics courses (including “honors” and “regents” courses).

NOTE: Detail may not sum to totals because of rounding.

SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-5. Percentage of U.S. TIMSS Advanced physics students, by race/ethnicity and course type: 2015

Course type[1]	U.S. total	White	Black	Hispanic	Asian	Two or more races
Total AP physics courses[2]	**83**	**83**	**78**	**91**	**79**	**76**
AP Physics C	25	28	14	20	26	15
AP Physics B	12	9	19	11	16	18
AP Physics 2	4	6	2	2	3	4
AP Physics 1	42	40	42	58	33	39
Total non-AP physics courses	**17**	**17**	**22**	**9**	**21**	**24**
IB Physics[3]	6	2	11	5	17	3
Other physics courses[4]	12	15	11	4	4	21

[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the “other physics courses” category only if they have not taken an AP or IB course.
[2] AP Physics C includes courses in mechanics (C-M) and electricity and magnetism (C-E/M).
[3] Includes higher-level and standard-level IB physics courses. AP Physics B was a 1-year physics course that was discontinued after the 2013–14 school year and was replaced with the 2-year course sequence AP Physics 1 and 2 beginning in 2014–15. Data in the AP Physics B category are generally for students whose took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering a 1-year course identified as AP Physics B in 2014–15.
[4] Includes other types of second-year physics courses (including “honors” and “regents” courses).
NOTE: Reporting standards were not met for Native Hawaiian/Pacific Islander and Native American/Alaska Native students (sample size < 62). Data for these students are not shown separately in the table and associated figure, though data are included in the U.S. total. Black includes African American and Hispanic includes Latino. Racial categories exclude Hispanic origin. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-6. Percentage of U.S. TIMSS Advanced physics students, by school locale and course type: 2015

Course type[1]	U.S. total	Urban	Suburban	Town	Rural
Total AP physics courses[2]	**83**	**55**	**88**	**83**	**100**
AP Physics C	25	14	38	17	26
AP Physics B	12	13	14	4	47
AP Physics 2	4	2	6	6	#
AP Physics 1	42	26	29	56	28
Total non-AP physics courses	**17**	**45**	**12**	**17**	**0**
IB Physics[3]	6	22	3	#	#
Other physics courses[4]	12	23	9	17	#

Rounds to zero.
[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the “other physics courses” category only if they have not taken an AP or IB course.
[2] AP Physics C includes courses in mechanics (C-M) and electricity and magnetism (C-E/M).
[3] Includes higher-level and standard-level IB physics courses. AP Physics B was a 1-year physics course that was discontinued after the 2013–14 school year and was replaced with the 2-year course sequence AP Physics 1 and 2 beginning in 2014–15. Data in the AP Physics B category are generally for students who took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering a 1-year course identified as AP Physics B in 2014–15.
[4] Includes other types of second-year physics courses (including courses such as “honors” or “regents” courses).
NOTE: Urban is defined as territories inside an urbanized area and inside a principal city. Suburban is defined as territories inside an urbanized area but outside a principal city. Town is defined as territories inside an urban cluster but outside an urbanized area. Rural is defined as territories that are not in an urbanized area or urban cluster. For additional information, see https://nces.ed.gov/programs/edge/docs/NCES_LOCALE_USERSMANUAL_2016012.pdf. There were no rural students who had taken AP Physics 2 or non-AP physics courses (including IB Physics and other physics courses) as their highest course. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-7. Average advanced mathematics scores of U.S. TIMSS Advanced students, by content domain and sex: 2015

Sex	Overall advanced mathematics scale	Algebra	Calculus	Geometry
U.S. average	**485**	**478**	**504**	**455**
Male	500	490	517	474
Female	470 ▼	466 ▼	492 ▼	435 ▼

▼ Female average score is significantly lower than male average score (*p* < .05).
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-8. Average advanced mathematics scores of U.S. TIMSS Advanced students, by content domain and race/ethnicity: 2015

Race/ethnicity	Overall advanced mathematics scale	Algebra	Calculus	Geometry
U.S. average	**485**	**478**	**504**	**455**
White	495	486	514	468
Black	400 ▼	403 ▼	407 ▼	358 ▼
Hispanic	440 ▼	432 ▼	463 ▼	402 ▼
Asian	506	503	521	479
Two or more races	525 ▲	524 ▲	546 ▲	493

▲ Subgroup average score is significantly higher than average score of White students (*p* < .05).
▼ Subgroup average score is significantly lower than average score of White students (*p* < .05).
NOTE: Reporting standards were not met for Native Hawaiian/Pacific Islander and Native American/Alaskan Native students (sample size < 62). Data for these students are not shown separately in the figure, but are included in the U.S. average. Black includes African American and Hispanic includes Latino. Racial categories exclude Hispanic origin.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-9. Average advanced mathematics scores of U.S. TIMSS Advanced students, by content domain and school locale: 2015

School locale	Overall advanced mathematics scale	Algebra	Calculus	Geometry
U.S. average	**485**	**478**	**504**	**455**
Urban	478	474	496	448
Suburban	503	494	523	473
Town	485	478	506	458
Rural	428 ▼	423 ▼	440 ▼	398 ▼

▼ Subgroup average score is significantly lower than average score of suburban students (*p* < .05).

NOTE: Urban is defined as territories inside an urbanized area and inside a principal city. Suburban is defined as territories inside an urbanized area but outside a principal city. Town is defined as territories inside an urban cluster but outside an urbanized area. Rural is defined as territories that are not in an urbanized area or urban cluster. For additional information, see https://nces.ed.gov/programs/edge/docs/NCES_LOCALE_USERSMANUAL_2016012.pdf.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-10. Percentage distributions of U.S. TIMSS Advanced students reaching the TIMSS Advanced international benchmarks in advanced mathematics compared to the U.S. total, by course type: 2015

Course type[1]	All U.S. TIMSS Advanced students	Reaching the *Advanced* benchmark (625)	Reaching the *High* benchmark (550)	Reaching the *Intermediate* benchmark (475)	Below the *Intermediate* benchmark
AP calculus courses					
AP Calculus BC	19	55 ▲	41 ▲	29 ▲	6 ▼
AP Calculus AB	56	38 ▼	48 ▼	54	60
Non-AP mathematics courses					
IB Mathematics[2]	1	1	1	1	2
Other calculus courses[3]	23	6 ▼	10 ▼	16 ▼	32 ▲

▲ Percentage is significantly higher than U.S. sample percentage (*p* < .05).

▼ Percentage is significantly lower than U.S. sample percentage (*p* < .05)

[1] Course type reflects the highest level mathematics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other calculus courses" category only if they have not taken an AP or IB course.

[2] Includes higher-level and standard-level IB mathematics courses.

[3] Includes other calculus courses (including "honors" and "regents" courses).

SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-11. U.S. performance on TIMSS Advanced advanced mathematics items, by level of topic coverage: 2015

		Percent correct[1]					
		Item 1	Item 2	Item 3	Item 4	Item 5	Item 6
High level of topic coverage							
Algebra topics	Operations with exponential, logarithmic, polynomial, rational, and radical expressions	34	42	†	†	†	†
	Evaluating algebraic expressions (e.g., exponential, logarithmic, polynomial, rational, and radical)	25	30	33	46	71	†
	Linear and quadratic equations and inequalities as well as systems of linear equations and inequalities	26	43	48	58	†	†
	Exponential, logarithmic, polynomial, rational, and radical equations	22	29	38	41	55	61
	Using equations and inequalities to solve contextual problems	36	49	51	57	61	†
	Equivalent representations of functions, including composite functions, as ordered pairs, tables, graphs, formulas, or words	54	64	72	76	†	†
	Properties of functions, including domain and range	26	39	45	52	52	†
Calculus topics	Limits of functions, including rational functions	37	56	59	59	†	†
	Differentiation of functions (including polynomial, exponential, logarithmic, trigonometric, rational, and radical functions); differentiation of products, quotients, and composite functions	48	55	58	59	71	†
	Using derivatives to solve problems (e.g., in optimization and rates of change)	16	17	31	†	†	†
	Using first and second derivatives to determine slope and local extrema, and points of inflection	21	34	56	62	†	†
	Using first and second derivatives to sketch and interpret graphs of functions	35	45	55	57	59	65
Geometry topics	Properties of geometric figures in two and three dimensions	21	23	32	40	45	74
	Using coordinate geometry to solve problems in two dimensions	23	43	43	50	60	†
	Trigonometric properties of triangles (sine, cosine, and tangent)	13	30	36	44	58	†
	Trigonometric functions and their graphs	37	52	58	61	†	†
	Solving problems involving trigonometric functions	1	13	28	33	44	†
Moderate level of topic coverage							
Algebra topics	The nth term of arithmetic and geometric sequences and the sums of finite and infinite series	35	41	48	61	†	†
Calculus topics	Integrating functions (including polynomial, exponential, trigonometric, and rational functions)	12	66	†	†	†	†
	Evaluating definite integrals, and applying integration to compute areas and volumes	30	37	45	52	63	65
Low level of topic coverage							
Algebra topics	Operations with complex numbers	34	41	†	†	†	†
Calculus topics	Conditions for continuity and differentiability of functions	21	44	52	62	†	†
Geomery topics	Properties of vectors and their sums and differences	21	31	39	40	59	†

† Not applicable.
[1] Percent correct is the percentage of students receiving credit on each item. For multiple-choice and short constructed-response items (each worth 1 score point), thsi reflects the percentage of students who provided a correct answer. For extended constructed-response items, this reflects the weighted percentage of students receiving full credit (2 points) or partial credit (1 point).
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-12. Percentage distribution of U.S. TIMSS Advanced students across response types for TIMSS Advanced mathematics example item 1, by course type: 2015

Course type[1]	Percentage by response type				
	Option B (correct)	Option A	Option C	Option D (diagnostic)	Blank[2]
Total AP calculus courses	**70**	**1**	**6**	**20**	**2**
AP Calculus BC	81	1	2	14	3
AP Calculus AB	66	2	8	22	2
Total non-AP mathematics courses	**47**	**6**	**9**	**33**	**5**
IB Mathematics[3]	†	†	†	†	†
Other calculus courses[4]	46	6	9	34	5

† Not applicable.
[1] Course type reflects the highest level mathematics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other calculus courses" category only if they have not taken an AP or IB course.
[2] Blank includes both students who omitted the item and those who did not reach the item.
[3] Includes both higher-level and standard-level IB mathematics courses.
[4] Includes other calculus courses (including "honors" or "regents" courses).
NOTE: Percentages are based on the full set of responses, including students who did not reach the item. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-13. Percentage distribution of U.S. TIMSS Advanced students across response types for TIMSS Advanced mathematics example item 2, by course type: 2015

Course type[1]	Percentage by response type				
	Correct	Partial	Incorrect		
	Code 20	Code 10 (diagnostic)	Code 77 (diagnostic)	Code 79	Blank[2]
Total AP calculus courses	**54**	**10**	**7**	**26**	**3**
AP Calculus BC	69	10	3	13	5
AP Calculus AB	50	10	8	30	2
Total non-AP mathematics courses	**45**	**14**	**6**	**33**	**2**
IB Mathematics[3]	†	†	†	†	†
Other calculus courses[4]	47	15	6	31	2

† Not applicable.
[1] Course type reflects the highest level mathematics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other calculus courses" category only if they have not taken an AP or IB course.
[2] Blank includes both students who omitted the item and those who did not reach the item.
[3] Includes both higher-level and standard-level IB mathematics courses.
[4] Includes other calculus courses (including "honors" or "regents" courses).
NOTE: Percentages are based on the full set of responses, including students who did not reach the item. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-14. Percentage distribution of U.S. TIMSS Advanced students across response types for TIMSS Advanced mathematics example 3, by course type: 2015

Course type[1]	Percentage by response type						
	Correct		Incorrect				
	Code 10	Code 11	Code 70 (diagnostic)	Code 71	Code 72	Code 79	Blank[2]
Total AP calculus courses	**14**	**5**	**7**	**0**	**8**	**53**	**14**
AP Calculus BC	22	5	9	0	5	46	13
AP Calculus AB	11	5	7	0	9	55	14
Total non-AP mathematics courses	**5**	**3**	**5**	**0**	**13**	**63**	**12**
IB Mathematics[3]	†	†	†	†	†	†	†
Other calculus courses[4]	6	3	5	0	13	62	11

† Not applicable.
[1] Course type reflects the highest level mathematics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other calculus courses" category only if they have not taken an AP or IB course.
[2] Blank includes both students who omitted the item and those who did not reach the item.
[3] Includes both higher-level and standard-level IB mathematics courses.
[4] Includes other calculus courses (including "honors" or "regents" courses).
NOTE: Percentages are based on the full set of responses, including students who did not reach the item. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-15. Percentage distribution of U.S. TIMSS Advanced students across response types for TIMSS Advanced mathematics example item 4, by course type: 2015

Course type[1]	Percentage by response type				
	Correct	Partial		Incorrect	
	Code 20	Code 10 (diagnostic)	Code 11	Code 79	Blank[2]
Total AP calculus courses	**34**	**24**	**5**	**30**	**7**
AP Calculus BC	45	26	7	21	1
AP Calculus AB	31	23	4	32	9
Total non-AP mathematics courses	**37**	**15**	**6**	**34**	**8**
IB Mathematics[3]	†	†	†	†	†
Other calculus courses[4]	36	15	6	35	8

† Not applicable.
[1] Course type reflects the highest level mathematics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other calculus courses" category only if they have not taken an AP or IB course.
[2] Blank includes both students who omitted the item and those who did not reach the item.
[3] Includes both higher-level and standard-level IB mathematics courses.
[4] Includes other calculus courses (including "honors" or "regents" courses).
NOTE: Percentages are based on the full set of responses, including students who did not reach the item. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-16. Percentage distribution of U.S. TIMSS Advanced students across response types for TIMSS Advanced mathematics example item 5, by course type: 2015

Course type[1]	Percentage by response type				
	Option C (correct)	Option A	Option B (diagnostic)	Option D	Blank[2]
Total AP calculus courses	**41**	**8**	**36**	**9**	**6**
AP Calculus BC	50	5	32	11	2
AP Calculus AB	38	10	37	8	7
Total non-AP mathematics courses	**31**	**12**	**37**	**10**	**10**
IB Mathematics[3]	†	†	†	†	†
Other calculus courses[4]	30	13	37	10	10

† Not applicable.

[1] Course type reflects the highest level mathematics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the “other calculus courses” category only if they have not taken an AP or IB course.

[2] Blank includes both students who omitted the item and those who did not reach the item.

[3] Includes both higher-level and standard-level IB mathematics courses.

[4] Includes other calculus courses (including “honors” or “regents” courses).

NOTE: Percentages are based on the full set of responses, including students who did not reach the item. Detail may not sum to totals because of rounding.

SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-17. Percentage distribution of U.S. TIMSS Advanced students across response types for TIMSS Advanced example item 6, by course type: 2015

Course type[1]	Percentage by response type				
	Correct	Partial		Incorrect	
	Code 20	Code 10 (diagnostic)	Code 11 (diagnostic)	Code 79	Blank[2]
Total AP calculus courses	**40**	**10**	**6**	**33**	**12**
AP Calculus BC	59	14	4	18	4
AP Calculus AB	34	8	6	37	15
Total non-AP mathematics courses	**25**	**9**	**7**	**32**	**28**
IB Mathematics[3]	†	†	†	†	†
Other calculus courses[4]	24	9	7	31	29

† Not applicable.

[1] Course type reflects the highest level mathematics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the “other calculus courses” category only if they have not taken an AP or IB course.

[2] Blank includes both students who omitted the item and those who did not reach the item.

[3] Includes both higher-level and standard-level IB mathematics courses.

[4] Includes other calculus courses (including “honors” or “regents” courses).

NOTE: Percentages are based on the full set of responses, including students who did not reach the item. Detail may not sum to totals because of rounding.

SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-18. Average physics scores of U.S. TIMSS Advanced students, by content domain and sex: 2015

Sex	Overall physics scale	Mechanics and thermodynamics	Electricity and magnetism	Wave phenomena and atomic/ nuclear physics
U.S. average	**437**	**462**	**380**	**431**
Male	455	480	401	446
Female	409 ▼	434 ▼	346 ▼	406 ▼

▼ Female average score is significantly lower than male average score (*p* < .05).
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-19. Average physics scores of U.S. TIMSS Advanced students, by content domain and race/ ethnicity: 2015

Race/ethnicity	Overall physics scale	Mechanics and thermodynamics	Electricity and magnetism	Wave phenomena and atomic/n uclear physics
U.S. average	**437**	**462**	**380**	**431**
White	463	488	415	454
Black	334 ▼	357 ▼	250 ▼	343 ▼
Hispanic	390 ▼	423 ▼	317 ▼	389 ▼
Asian	433	459	381	414
Two or more races	470	480	377	481

▲ Subgroup average score is significantly higher than average score of White students (*p* < .05).
▼ Subgroup average score is significantly lower than average score of White students (*p* < .05).
NOTE: Reporting standards were not met for Native Hawaiian/Pacific Islander and Native American/Alaskan Native students (sample size < 62). Data for these students are not shown separately in the figure, but are included in the U.S. average. Black includes African American and Hispanic includes Latino. Racial categories exclude Hispanic origin.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-20. Average physics scores of U.S. TIMSS Advanced students, by content domain and school locale: 2015

School locale	Overall physics scale	Mechanics and thermodynamics	Electricity and magnetism	Wave phenomena and atomic/ nuclear physics
U.S. average	**437**	**462**	**380**	**431**
Urban	418	440	360	419
Suburban	460	484	411	451
Town	451	473	389	444
Rural	435	466	368	413

NOTE: Urban is defined as territories inside an urbanized area and inside a principal city. Suburban is defined as territories inside an urbanized area but outside a principal city. Town is defined as territories inside an urban cluster but outside an urbanized area. Rural is defined as territories that are not in an urbanized area or urban cluster. For additional information, see https://nces.ed.gov/programs/edge/docs/NCES_LOCALE_USERSMANUAL_2016012.pdf. There are no measurable differences between the average physics scores of U.S. TIMSS Advanced suburban students and students from other locales.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-21. Percentage distributions of U.S. TIMSS Advanced students reaching international benchmarks in physics compared to the U.S. total, by course type: 2015

Course type[1]	All U.S. TIMSS Advanced students	Reaching the *Advanced* benchmark (625)	Reaching the *High* benchmark (550)	Reaching the *Intermediate* benchmark (475)	Below the *Intermediate* benchmark
AP physics courses					
AP Physics C-E/M[2]	10	34 ▲	26 ▲	19 ▲	4 ▼
AP Physics C-M	15	22	23	20 ▲	12 ▼
AP Physics B[3]	12	10	10	12	12
AP Physics 2	4	6	6	6	3 ▼
AP Physics 1	42	12 ▼	21 ▼	30 ▼	50 ▲
Non-AP physics courses					
IB Physics[4]	6	5	3	3	8
Other physics courses[5]	12	10	11	10	12

▲ Percentage is significantly higher than U.S. sample percentage ($p < .05$).

▼ Percentage is significantly lower than U.S. sample percentage ($p < .05$).

Rounds to zero.

† Not applicable.

[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the “other physics courses” category only if they have not taken an AP or IB course.

[2] AP Physics C course in electricity and magnetism (C-E/M). A large majority of the students whose highest physics course was AP Physics C-E/M had also taken an AP Physics C course in mechanics (C-M) (89 percent), whether sequentially or in a combined course.

[3] AP Physics B was a 1-year physics course that was discontinued after the 2013–14 school year and was replaced with the 2-year course sequence AP Physics 1 and 2 beginning in 2014–15. Data in the AP Physics B category are generally for students who took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering a 1-year course identified as AP Physics B in 2014–15.

[4] Includes higher-level and standard-level IB physics courses.

[5] Includes other second-year physics courses (including “honors” and “regents” courses).

NOTE: Detail may not sum to totals because of rounding.

SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-22. U.S. performance on TIMSS Advanced physics items, by level of topic coverage: 2015

		Percent correct[1]						
		Item 1	Item 2	Item 3	Item 4	Item 5	Item 6	Item 7
High level of topic coverage								
Mechanics and thermodynamics topics	Applying Newton's laws and laws of motion	45	51	64	85	†	†	†
	Forces, including frictional force, acting on a body	23	25	42	49	56	59	73
	Kinetic and potential energy; conservation of mechanical energy	13	41	60	84	†	†	†
	Law of conservation of momentum; elastic and inelastic collisions	44	46	50	†	†	†	†
Moderate level of topic coverage								
Mechanics and thermodynamics topics	Forces acting on a body moving in a circular path; the body's centripetal acceleration, speed and circling time	31	43	49	56	†	†	†
	The law of gravitation in relation to movement of celestial objects	13	36	50	60	†	†	†
Wave phenomena and atomic/nuclear physics topics	Mechanical waves: the relationship between speed, frequency, and wavelength	41	51	53	56	64	69	†
Low level of topic coverage								
Mechanics and thermodynamics topics	First law of thermodynamics	31	71	†	†	†	†	†
	Heat transfer and specific heat capacities	13	20	26	29	53	69	†
	Law of ideal gases; expansion of solids and liquids in relation to temperature change	23	30	43	49	†	†	†
Electricity and magnetism topics	Electrostatic attraction or repulsion between isolated charged particles—Coulomb's law	16	35	41	†	†	†	†
	Charged particles in an electric field	14	43	43	44	53	68	†
	Electrical circuits; using Ohm's law and Joule's law	29	31	36	43	45	†	†
	Charged particles in a magnetic field	5	19	23	25	34	45	†
	Relationship between magnetism and electricity; magnetic fields around electric conductors; electromagnetic induction	15	36	38	74	†	†	†
	Faraday's and Lenz's laws of induction	13	28	40	71	†	†	†
Wave phenomena and atomic/nuclear physics topics	Electromagnetic radiation; wavelength and frequency of various types of waves (radio, infrared, visible light, x-rays, gamma rays)	34	35	36	55	65	†	†
	Thermal radiation, temperature, and wavelength	11	25	46	57	†	†	†
	Reflection, refraction, interference, and diffraction	22	27	34	55	†	†	†
	The structure of the atom and its nucleus: atomic number and atomic mass; electromagnetic emission and absorption and the behavior of electrons	35	40	41	†	†	†	†
	Wave-particle duality and the photoelectric effect	47	51	59	69	†	†	†
	Types of nuclear reactions and their role in nature (e.g., in stars) and society; radioactive Isotopes	9	26	41	42	53	59	†
	Mass-energy equivalence in nuclear reactions and particle transformations	9	29	42	†	†	†	†

† Not applicable.

[1] Percent correct is the percentage of students receiving credit on each item. For multiple-choice and short constructed-response items (each worth 1 score point), thsi reflects the percentage of students who provided a correct answer. For extended constructed-response items, this reflects the weighted percentage of students receiving full credit (2 points) or partial credit (1 point).

SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-23a. Percentage distribution of U.S. TIMSS Advanced students across response types for TIMSS Advanced physics example item 1, part A, by course type: 2015

Course type[1]	Percentage by response type				
	Option D (correct)	Option A (diagnostic)	Option B	Option C	Blank[2]
Total AP physics courses[3]	**55**	**37**	**3**	**4**	**2**
AP Physics C	76	21	1	2	1
AP Physics C-E/M[4]	75	23	0	3	0
AP Physics C-M	77	19	1	2	1
AP Physics B	45	49	#	5	0
AP Physics 2	53	46	0	0	2
AP Physics 1	46	42	5	5	3
Total non-AP physics courses	**28**	**65**	**5**	**2**	**0**
IB Physics[5]	23 !	63 !	9 !	5 !	0 !
Other physics courses[6]	31 !	66 !	3 !	0 !	0 !

Rounds to zero.
! Interpret with caution (sample size < 62, but > 30).
[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other physics courses" category only if they have not taken an AP or IB course.
[2] Blank includes both students who omitted the item and those who did not reach the item.
[3] AP Physics C includes courses in mechanics (C-M) and electricity and magnetism (C-E/M). AP Physics B was a 1-year physics course that was discontinued after the 2013–14 school year and was replaced with the 2-year course sequence AP Physics 1 and AP Physics 2 beginning in 2014–15. Data in the AP Physics B category are generally for students who took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering courses identified as AP Physics B in 2014–15.
[4] A large majority of the students whose highest course was AP Physics C-E/M had also taken AP Physics C-M (89 percent), whether sequentially or in a combined course.
[5] Includes both higher-level and standard-level IB physics courses.
[6] Includes other second-year physics courses (including "honors" or "regents" courses).
NOTE: Percentages are based on the full set of responses, including students who did not reach the item. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-23b. Percentage distribution of U.S. TIMSS Advanced students across response types for TIMSS Advanced physics example item 1, part B, by course type: 2015

Course type[1]	Percentage by response type			
	Correct	Incorrect		
	Code 10	Code 78	Code 79	Blank[2]
Total AP physics courses[3]	**67**	**17**	**14**	**1**
AP Physics C	73	14	12	1
AP Physics C-E/M[4]	73	11	16	0
AP Physics C-M	73	16	10	1
AP Physics B	75	18	6	2
AP Physics 2	61	24	15	0
AP Physics 1	61	19	19	2
Total non-AP physics courses	**49**	**30**	**18**	**4**
IB Physics[5]	39 !	30 !	21 !	9 !
Other physics courses[6]	54 !	29 !	17 !	1 !

! Interpret with caution (sample size < 62, but > 30).
[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other physics courses" category only if they have not taken an AP or IB course.
[2] Blank includes both students who omitted the item and those who did not reach the item.
[3] AP Physics C includes courses in mechanics (C-M) and electricity and magnetism (C-E/M). AP Physics B was a 1-year physics course that was discontinued after the 2013–14 school year and was replaced with the 2-year course sequence AP Physics 1 and AP Physics 2 beginning in 2014–15. Data in the AP Physics B category are generally for students who took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering courses identified as AP Physics B in 2014–15.
[4] A large majority of the students whose highest course was AP Physics C-E/M had also taken AP Physics C-M (89 percent), whether sequentially or in a combined course.
[5] Includes both higher-level and standard-level IB physics courses.
[6] Includes other second-year physics courses (including "honors" or "regents" courses).
NOTE: Percentages are based on the full set of responses, including students who did not reach the item. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-24. Percentage distribution of U.S. TIMSS Advanced students across response types for TIMSS Advanced physics example item 2, by course type: 2015

	Percentage by response type					
	Correct	Partial		Incorrect		
Course type[1]	Code 20	Code 10 (diagnostic)	Code 11	Code 77	Code 79	Blank[2]
Total AP physics courses[3]	**39**	**16**	**1**	**3**	**36**	**6**
AP Physics C	53	19	2	4	20	2
AP Physics C-E/M[4]	49	22	#	6	20	4
AP Physics C-M	57	17	4	2	20	1
AP Physics B	29	13	0	4	41	13
AP Physics 2	37	15	0	1	41	6
AP Physics 1	33	15	1	2	44	6
Total non-AP physics courses	**18**	**16**	**#**	**3**	**57**	**6**
IB Physics[5]	6 !	8 !	0 !	0 !	80 !	6 !
Other physics courses[6]	24 !	21 !	# !	4 !	44 !	6 !

Rounds to zero.

! Interpret with caution (sample size < 62, but > 30).

[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the “other physics courses” category only if they have not taken an AP or IB course.

[2] Blank includes both students who omitted the item and those who did not reach the item.

[3] AP Physics C includes courses in mechanics (C-M) and electricity and magnetism (C-E/M). AP Physics B was a 1-year physics course that was discontinued after the 2013–14 school year and was replaced with the 2-year course sequence AP Physics 1 and AP Physics 2 beginning in 2014–15. Data in the AP Physics B category are generally for students who took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering courses identified as AP Physics B in 2014–15.

[4] A large majority of the students whose highest course was AP Physics C-E/M had also taken AP Physics C-M (89 percent), whether sequentially or in a combined course.

[5] Includes both higher-level and standard-level IB physics courses.

[6] Includes other second-year physics courses (including “honors” or “regents” courses).

NOTE: Percentages are based on the full set of responses, including students who did not reach the item. Detail may not sum to totals because of rounding.

SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-25. Percentage distribution of U.S. TIMSS Advanced students across response types for TIMSS Advanced physics example item 3, by course type: 2015

Course type[1]	Percentage by response type: Correct – Code 20	Partial – Code 10	Partial – Code 11 (diagnostic)	Incorrect – Code 70	Incorrect – Code 71	Incorrect – Code 79	Blank[2]
Total AP physics courses[3]	**13**	**7**	**17**	**#**	**11**	**38**	**15**
AP Physics C	18	8	22	1	9	32	10
AP Physics C-E/M[4]	26	7	25	0	11	21	10
AP Physics C-M	14	8	21	1	7	40	10
AP Physics B	23	18	20	0	14	21	3
AP Physics 2	25	9	32	0	5	25	3
AP Physics 1	5	3	11	#	12	46	23
Total non-AP physics courses	**5**	**3**	**16**	**0**	**9**	**46**	**21**
IB Physics[5]	3 !	2 !	9 !	0 !	9 !	53 !	24 !
Other physics courses[6]	6 !	4 !	19 !	0 !	9 !	42 !	20 !

Rounds to zero.

! Interpret with caution (sample size < 62, but > 30).

[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the “other physics courses” category only if they have not taken an AP or IB course.

[2] Blank includes both students who omitted the item and those who did not reach the item.

[3] AP Physics C includes courses in mechanics (C-M) and electricity and magnetism (C-E/M). AP Physics B was a 1-year physics course that was discontinued after the 2013–14 school year and was replaced with the 2-year course sequence AP Physics 1 and AP Physics 2 beginning in 2014–15. Data in the AP Physics B category are generally for students who took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering courses identified as AP Physics B in 2014–15.

[4] A large majority of the students whose highest course was AP Physics C-E/M had also taken AP Physics C-M (89 percent), whether sequentially or in a combined course.

[5] Includes both higher-level and standard-level IB physics courses.

[6] Includes other second-year physics courses (including “honors” or “regents” courses).

NOTE: Percentages are based on the full set of responses, including students who did not reach the item. Detail may not sum to totals because of rounding.

SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-26. Percentage distribution of U.S. TIMSS Advanced students across response types for TIMSS Advanced physics example item 4, by course type: 2015

Course type[1]	Percentage by response type				
	Option C (correct)	Option A (diagnostic)	Option B	Option D	Blank[2]
Total AP physics courses[3]	**43**	**40**	**8**	**8**	**1**
AP Physics C	56	31	6	6	1
AP Physics C-E/M[4]	61	25	8	6	0
AP Physics C-M	52	36	5	5	1
AP Physics B	35	47	9	3	6
AP Physics 2	65	23	5	7	0
AP Physics 1	35	45	10	11	0
Total non-AP physics courses	**40**	**33**	**16**	**12**	**#**
IB Physics[5]	37 !	35 !	16 !	12 !	1 !
Other physics courses[6]	42 !	31 !	15 !	12 !	0 !

Rounds to zero.
! Interpret with caution (sample size < 62, but > 30).
[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other physics courses" category only if they have not taken an AP or IB course.
[2] Blank includes both students who omitted the item and those who did not reach the item.
[3] AP Physics C includes courses in mechanics (C-M) and electricity and magnetism (C-E/M). AP Physics B was a 1-year physics course that was discontinued after the 2013–14 school year and was replaced with the 2-year course sequence AP Physics 1 and AP Physics 2 beginning in 2014–15. Data in the AP Physics B category are generally for students who took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering courses identified as AP Physics B in 2014–15.
[4] A large majority of the students whose highest course was AP Physics C-E/M had also taken AP Physics C-M (89 percent), whether sequentially or in a combined course.
[5] Includes both higher-level and standard-level IB physics courses.
[6] Includes other second-year physics courses (including "honors" or "regents" courses).
NOTE: Percentages are based on the full set of responses, including students who did not reach the item. Detail may not sum to totals because of rounding.
SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-27. Percentage distribution of U.S. TIMSS Advanced students across response types for TIMSS Advanced physics example item 5, by course type: 2015

Course type[1]	Percentage by response type: Option B (correct)	Option A	Option C (diagnostic)	Option D	Blank[2]
Total AP physics courses[3]	**24**	**22**	**34**	**20**	**1**
AP Physics C	32	22	23	21	1
AP Physics C-E/M[4]	36	35	15	13	0
AP Physics C-M	30	13	29	27	1
AP Physics B	32	39	24	5	0
AP Physics 2	40	38	16	5	1
AP Physics 1	14	15	46	25	1
Total non-AP physics courses	**18**	**16**	**45**	**21**	**0**
IB Physics[5]	24 !	5 !	47 !	24 !	0 !
Other physics courses[6]	14 !	22 !	44 !	19 !	0 !

! Interpret with caution (sample size < 62, but > 30).

[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other physics courses" category only if they have not taken an AP or IB course.

[2] Blank includes both students who omitted the item and those who did not reach the item.

[3] AP Physics C includes courses in mechanics (C-M) and electricity and magnetism (C-E/M). AP Physics B was a 1-year physics course that was discontinued after the 2013–14 school year and was replaced with the 2-year course sequence AP Physics 1 and AP Physics 2 beginning in 2014–15. Data in the AP Physics B category are generally for students who took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering courses identified as AP Physics B in 2014–15.

[4] A large majority of the students whose highest course was AP Physics C-E/M had also taken AP Physics C-M (89 percent), whether sequentially or in a combined course.

[5] Includes both higher-level and standard-level IB physics courses.

[6] Includes other second-year physics courses (including "honors" or "regents" courses).

NOTE: Percentages are based on the full set of responses, including students who did not reach the item. Detail may not sum to totals because of rounding.

SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Table B-28. Percentage distribution of U.S. TIMSS Advanced students across response types for TIMSS Advanced physics example item 6, by course type: 2015

Course type[1]	Percentage by response type				
	Option A (correct)	Option B (diagnostic)	Option C	Option D	Blank[2]
Total AP physics courses[3]	**42**	**23**	**26**	**8**	**1**
AP Physics C	53	23	18	4	2
AP Physics C-E/M[4]	61	19	15	2	2
AP Physics C-M	46	26	21	6	2
AP Physics B	41	9	37	13	0
AP Physics 2	48	29	15	6	2
AP Physics 1	35	26	30	8	1
Total non-AP physics courses	**44**	**23**	**23**	**7**	**2**
IB Physics[5]	42	27	19	6	6
Other physics courses[6]	46	21	26	8	0

! Interpret with caution (sample size < 62, but > 30).

[1] Course type reflects the highest level physics course taken. If students took another course in addition to an Advanced Placement (AP) or International Baccalaureate (IB) course, they are included in the AP or IB category. Students are included in the "other physics courses" category only if they have not taken an AP or IB course.

[2] Blank includes both students who omitted the item and those who did not reach the item.

[3] AP Physics C includes courses in mechanics (C-M) and electricity and magnetism (C-E/M). AP Physics B was a 1-year physics course that was discontinued after the 2013–14 school year and was replaced with the 2-year course sequence AP Physics 1 and AP Physics 2 beginning in 2014–15. Data in the AP Physics B category are generally for students who took this as their highest physics course in 2013–14 or prior. However, a small number of schools still reported offering courses identified as AP Physics B in 2014–15.

[4] A large majority of the students whose highest course was AP Physics C-E/M had also taken AP Physics C-M (89 percent), whether sequentially or in a combined course.

[5] Includes both higher-level and standard-level IB physics courses.

[6] Includes other second-year physics courses (including "honors" or "regents" courses).

NOTE: Percentages are based on the full set of responses, including students who did not reach the item. Detail may not sum to totals because of rounding.

SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS) Advanced, 2015.

Appendix C: Technical Notes

This appendix briefly describes features of the TIMSS Advanced 2015 assessments, with a particular focus on the U.S. implementation and the specific data sources and methods used for this report. For further details, see the additional Technical Notes available on the NCES TIMSS website at http://nces.ed.gov/timss/timss15technotes.asp, as well as the IEA's Methods and Procedures in TIMSS Advanced 2015 (at http://timss.bc.edu/publications/timss/2015-a-methods.html) and NCES' *TIMSS and TIMSS Advanced 2015 Technical Report* (at https://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2018020).

Sampling and response rates

TIMSS Advanced is a sample-based assessment, meaning that while only a sample of students take the assessments, they are selected in such a way as to allow the results to be generalizable to a larger target population. The TIMSS Advanced target populations are based on standardized definitions, and the sampling is conducted following standardized and refereed international procedures.

TIMSS Advanced participating countries and other education systems drew probability samples of students in their final year of secondary school— International Standard Classification of Education (ISCED) Level 3—who were taking or had taken courses in advanced mathematics or who were taking or had taken courses in physics. (For additional information on ISCED levels, see http://uis.unesco.org/en/topic/international-standard-classification-education-isced.)

First, samples of schools that offered one or more of the eligible advanced mathematics or physics courses were selected. Then a random sample of students who were currently taking or had previously taken the eligible courses was selected from within those schools. In the United States, two samples of twelfth-graders were drawn to represent the nation—one for advanced mathematics and one for physics. The eligible courses that define the target populations had to cover most, if not all, of the advanced mathematics and physics topics that were outlined in the assessment frameworks. In the United States, this was defined as a calculus course for eligibility for the advanced mathematics population and an advanced physics course for the physics population. These included Advanced Placement (AP) calculus and physics courses, International Baccalaureate (IB) mathematics and physics courses, and other state-, district- or school-specific calculus and second-year physics courses. The U.S. national samples included both public and private schools, randomly selected and weighted to be representative of the nation's advanced mathematics and physics students at the end of high school. (See the section on sampling weights and standard errors in this appendix for definitions.)

In total, the TIMSS Advanced U.S. national sample in 2015 consisted of 241 schools for advanced mathematics and 165 schools for physics (of the original sample of 348 schools for both subjects). The weighted school response rate for the United States for advanced mathematics was 72 percent before the use of substitute schools and 76 percent with the inclusion of substitute schools. The weighted school response rate for the United States for physics was 65 percent before the use of substitute schools and 68 percent with the inclusion of substitute schools. The U.S. national sample consisted of 2,954 students in advanced mathematics and 2,932 students in physics. The weighted student response rate was 87 percent for advanced mathematics and 85 percent for physics. Student response rates are based on a combined total of students from both sampled and substitute schools. As indicated by footnotes in the cross-education system figures in the Highlights report, the United States did not satisfy guidelines for sample participation rates in TIMSS Advanced.

As required by NCES standards, a nonresponse bias analysis was conducted because the U.S. school-level response rate for both subjects in TIMSS Advanced fell below 85 percent of the sampled schools. The purpose of the analysis was to examine whether the participation status of schools was related to various characteristics and thus introduced the potential for bias in the results. For TIMSS Advanced, these analyses suggest that there was little potential for nonresponse bias in the advanced mathematics sample based on the characteristics studied. It also suggests that, while there was some evidence that the use of substitute schools has not reduced the potential for bias, it did not add to it substantially. Moreover, after the application of school nonresponse adjustments, there was little evidence of remaining potential bias in the final sample. In physics, however, the results suggest that there was some potential for nonresponse bias in the sample based on the characteristics studied. It also suggests that, while there was some evidence that the use of substitute schools reduced the potential for bias, it did not reduce it substantially. Moreover, after the application of school nonresponse adjustments, there was some evidence of remaining potential bias in the final sample with the largest bias in locale.

See the sections on International Requirements for Sampling, Data Collection and Response Rates and on Sampling in the United States in the Technical Notes at http://nces.ed.gov/timss/timss15technotes.asp for additional information. For additional detail on the nonresponse bias analysis, see the full NCES technical report (at https://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2018020).

Assessment and questionnaires

The 2015 assessment instruments for TIMSS Advanced were developed by international Item Review Committees and included items submitted by participating education systems and other subject matter experts. In TIMSS Advanced, about 33 percent of the items were from previous assessments and about 67 percent were developed for the 2015 assessment. Items were reviewed by representatives of each country for possible bias. To further examine potential biases and design issues in the TIMSS Advanced assessments, nearly all participating countries field-tested the assessment items in 2014. After the field test, items that did not meet the established measurement criteria or were otherwise found to include intrinsic biases were dropped for the main assessment.

Each participating country translated the international version of the assessment (in English) into the language(s) spoken in their country or adapted the assessments to reflect any variations necessary in different English-speaking countries. The translations were then verified by the IEA Secretariat. The translation and translation verification processes occur twice—first before the field test and then before the commencement of data collection.

The 2015 assessment instruments for TIMSS Advanced were organized in booklets and were constructed and distributed such that each student responded to only a portion of the items and that each item was administered to a random sample of students. The TIMSS Advanced assessments consisted of 6 booklets for advanced mathematics and 6 booklets for physics, each requiring approximately 90 minutes. In both subjects, items were assembled into 9 unique blocks (each containing approximately 10 items across the content domains). Each block appeared in two booklets to permit linking of student responses from the various booklets. Each student completed only one booklet (either advanced mathematics or physics) consisting of 3 blocks of items.

After the cognitive assessment, students then completed a 30-minute questionnaire designed to provide information about their backgrounds, attitudes, and experiences in school. Principals in schools where TIMSS Advanced was administered completed questionnaires designed to provide information on their school's structure, resources, curriculum and instruction, climate, and policies. The advanced mathematics and physics teachers of students participating in the assessments also completed questionnaires on these topics and on their own educational background and experiences.

This report uses data from the student questionnaire to conduct various subgroup analyses (i.e., those by sex and race/ethnicity) as well as data from the teacher questionnaires on the TIMSS Advanced mathematics and physics topics covered in the classes taken by students in the TIMSS Advanced population. The race/ethnicity categories in the U.S. student questionnaires include White, Black/African American, Asian, American Indian or Alaska Native, and Native Hawaiian or other Pacific Islander. Students were asked which of the categories best describes them and instructed to select all that apply; those who selected more than one of these groups were categorized as "Two or more races." In the teacher questionnaires, students' advanced mathematics or physics teachers were asked to indicate whether each framework topic was: "mostly taught this year," "mostly taught before this year," or "not yet taught or just introduced."

In addition to the student, teacher and school questionnaires, TIMSS Advanced also administers country-level curriculum questionnaires focused primarily on the organization and content of the curriculum in advanced mathematics and physics. The curriculum questionnaires were completed in each participating country with input from subject matter and curriculum experts. For each topic covered in the TIMSS Advanced framework, the curriculum questionnaire asks countries to indicate whether, according to their curriculum, students in the advanced mathematics or physics courses being assessed by TIMSS Advanced should have been taught the topic by the end of the school year (in their current course or before). In the United States (which does not have a national curriculum), the questionnaire was completed by reviewing the curriculum frameworks for the various TIMSS Advanced-eligible courses in the U.S. to determine the extent to which each of TIMSS Advanced topics were covered. This included the AP course guidelines from the College Board, the core curriculum guides from the IB program, as well as the curriculum standards of the five most populous states (New York, California, Texas, Massachusetts, and Florida). The United States' curriculum questionnaires were completed by mathematics and science experts at the American Institutes for Research under contract to NCES. The results were then reviewed by external curriculum experts at the College Board and the Council of Chief State School Officers. The completed curriculum questionnaires for the United States and other countries participating in TIMSS Advanced can be found at http://timssandpirls.bc.edu/timss2015/international-results/advanced/timss-advanced-2015/mathematics/programs-and-curriculum/united-states-description-of-advanced-mathematics-programs-and-curriculum/ for mathematics and at http://timssandpirls.bc.edu/timss2015/international-results/advanced/timss-advanced-2015/physics/curriculum/united-states-description-of-physics-programs-and-curriculum/ for physics.

This report uses topic coverage data from research conducted while compiling responses for the TIMSS Advanced 2015 U.S. national curriculum questionnaires and from the teacher questionnaires for the results on *intended* and *implemented* curricula in section 3. The *intended* curriculum research determined whether the TIMSS Advanced topics are covered in the different AP and IB advanced mathematics and physics courses. Each topic was classified as fully covered, partially covered, not covered, or reflecting foundational concepts expected to have been covered in a prerequisite course. For the AP calculus courses, the Common Core State Standards in Mathematics were used to determine foundational knowledge covered in prerequisite mathematics courses for the TIMSS Advanced topics in algebra and geometry. The *intended* curriculum is not indicated for the non-AP, non-IB courses eligible for TIMSS Advanced, because the curricula for these courses vary across states, districts, and schools. The *implemented* curriculum is based on the percentage of U.S. TIMSS Advanced students whose advanced mathematics or physics teachers reported that the TIMSS Advanced topics were "mostly taught this year" or "mostly taught before this year" (i.e., by the time of the assessment). The third response option in the teacher questionnaire ("not yet taught or just introduced") indicated either topics not covered in the curriculum or topics included in the curriculum that had not yet been taught (or were just introduced) by the time of the assessment. These responses were treated as "not taught" for the analyses of *implemented* curriculum. For students with more than one advanced mathematics or physics teacher, students were counted as "taught" if any of their teachers indicated that the topic was taught in the current or a prior year.

See the sections on Test Development and the Student, Teacher, and School Questionnaires in the Technical Notes at http://nces.ed.gov/timss/timss15technotes.asp for more information about the field test, assessment design, and questionnaires. See Chapter 7 on Translation and Translation Verification in *TIMSS Advanced in Methods and Procedures for TIMSS Advanced 2015*. The contextual questionnaires for TIMSS Advanced can be found at https://timssandpirls.bc.edu/timss2015/advanced-questionnaires/index.html.

Reporting results

In TIMSS Advanced 2015, results are generally reported in two ways: scale scores and international benchmarks of achievement. The TIMSS Advanced scales in advanced mathematics and physics range from 0 to 1,000, with a fixed scale centerpoint of 500 and a standard deviation of 100.[1] For both subjects, TIMSS Advanced provides overall scale scores as well as subscale scores for each content and cognitive domain. In addition to these scale scores as the basic form of measurement, TIMSS Advanced describes student performance in terms of the percentage of students reaching three international benchmarks (*Advanced, High,* and *Intermediate*) in advanced mathematics and physics. These international benchmarks provide a way to interpret the scale scores and to understand how students' proficiency varies at different points on the scales, because each successive point, or benchmark, is associated with particular kinds of knowledge and skills that students must successfully demonstrate. See the Weighting, Scaling, and Plausible Values and International Benchmarks sections of the Technical Notes at http://nces.ed.gov/timss/timss15technotes.asp for more information.

[1] As described in section 4, the scale centerpoint represents the international mean of the overall achievement distribution in the first assessment year (1995).

In addition to scale scores and international benchmarks of achievement, this report also uses item-level statistics to provide results on student performance on individual assessment items. These statistics include the "percent correct," or the weighted percentage of students receiving full credit or partial credit on an assessment item. For multiple-choice and short constructed-response items, correct responses are awarded 1 score point; for extended constructed-response items, correct responses are awarded 2 score points, and partial responses are awarded 1 score point. Thus, students providing partial responses to an extended constructed-response item receive a weight of 0.5 when computing the overall percent correct for the item. For the example items included in the report, the item-level statistics include the percentages of students providing a correct response, partial responses (for extended constructed-response items), incorrect responses, and blank responses (for omitted or not-reached items). In addition to the percentages of students at the correct, partial, and incorrect levels, this report uses data on the distributions across the different response options for multiple-choice items and specific types of partial or incorrect responses for constructed-response items. TIMSS Advanced uses a two-digit scoring system (e.g., 20, 10, 11, 70). The first digit for a correct or partial response code indicates the number of score points given to the response (1 or 2), while the first digit for incorrect response codes is 7. The second digit (0–5) designates a specific type of correct, partial, or incorrect response. The code 79 is used for "other incorrect responses" not covered by a specific incorrect code (e.g., 70, 71). The definitions of the two-digit diagnostic codes are specific to each item (i.e., there is no general definition for a code that applies to all items). TIMSS Advanced 2015 used diagnostic codes in the constructed-response item scoring guides to track particular misconceptions or errors described in this report. In most cases, these diagnostic codes were applied internationally, although in some cases they were applied only in the United States.

For detailed information about the item-level statistics produced by TIMSS, see Chapter 11 (Reviewing the TIMSS Advanced 2015 Achievement Item Statistics) of the IEA's *Methods and Procedures in TIMSS Advanced 2015* (at http://timss.bc.edu/publications/timss/2015-a-methods.html). For more information, see the Scoring and Scoring Reliability section in the Technical Notes at https://nces.ed.gov/timss/timss15technotes.asp. Scoring guides are shown for example items in this report and in the examples shown in the TIMSS Advanced international results at http://timssandpirls.bc.edu/timss2015/international-results/.

Sampling weights and standard errors

Sampling weights are necessary to compute statistically sound estimates. Adjusted survey weights adjust for the probabilities of selection for individual schools and classrooms and for school or student nonresponse. As with any study, there are limitations that should be taken into consideration. Estimates produced using data from TIMSS Advanced 2015 are subject to two types of error: nonsampling errors and sampling errors. The sources of nonsampling errors are typically problems such as unit and item nonresponse, the differences in respondents' interpretations of the meaning of survey questions, and mistakes in data preparation. Sampling errors arise when a sample of the population, rather than the whole population, is used to estimate some statistic. Different samples from the same population would likely produce somewhat different estimates of the statistic in question. This uncertainty is referred to as sampling variance and is usually expressed as the standard error of a statistic estimated from sample data. Standard errors for all statistics reported in this report are available in the associated web tables at https://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2020051. See the sections on Weighting, Scaling, and Plausible Values and Data Limitations in the Technical Notes at http://nces.ed.gov/timss/timss15technotes.asp.

Statistical comparisons

Comparisons made in this report have been tested for statistical significance. For example, in the commonly made comparisons between the performance of U.S students and international students (i.e., TIMSS Advanced scale centerpoint, international average or international median percentage of students reaching the international benchmarks) and between different subgroups of U.S. students (e.g., by sex, race/ethnicity groups, and course type) as well as differences in the percentages of students in different categories, tests of statistical significance were used to establish whether or not the observed differences were statistically significant. The tests for significance used were standard *t* tests. These fell into two categories according to the nature of the comparison being made: comparisons of independent samples and comparisons of nonindependent samples. For comparisons with the international median percentages reaching each international benchmark, each international median value was treated as a nonvarying point estimate (i.e., an estimate with a standard error of 0). Standard *t* tests were also used for the comparisons involving item-level statistics such as the average percent correct across items for advanced mathematics and physics overall and by content domain and topic area. The standard errors for the "average percent correct" values across sets of items were based on the student sampling variance using the jackknife repeated replication (JRR) technique. For each item, the percent correct was computed 151 times—once using the overall student weights and once for each of 150 replicate weights. The item percent correct values (using overall weights and each of the replicate weights) were then averaged across the set of items in each group (by content domain, topic area, and for advanced mathematics/physics overall). The average percent correct using the overall student weights were the values cited in the report tables. The differences between each replicate weight's average percent correct value against the overall student weight's average were used to calculate the standard errors for the average. The significance tests for comparing the average percent correct for each content domain and topic area with advanced mathematics/physics overall reflect these standard errors. A difference is "significant" if the probability (p) associated with the t test is less than .05. If a test is significant, it implies that the difference between the observed measures in the sample represents a real difference in the population. No adjustments were made for multiple comparisons. See the section on Statistical Procedures in the Technical Notes at http://nces.ed.gov/timss/timss15technotes.asp for more information.

Additional information

Results from the 2015 TIMSS and TIMSS Advanced assessments can be explored in more detail at http://nces.ed.gov/timss. The TIMSS 2015 website houses numerous resources—including summaries of key findings, web tables, example items, and technical notes—for exploring the TIMSS and TIMSS Advanced assessments. Additionally, the TIMSS International Data Explorer (IDE) gives users the ability to analyze TIMSS data and create customized tables and figures for the United States and other participating education systems. The TIMSS IDE is available at http://nces.ed.gov/timss/idetimss/.

The TIMSS international reports for mathematics, science, advanced mathematics, and physics are available online at http://timssandpirls.bc.edu/timss2015/international-results/.

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