TEACHER SEMIOTIC MEDIATION AND STUDENT MEANING-MAKING: A PEIRCEAN PERSPECTIVE

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To interpret in detail the meaning-making in the classroom and the corresponding teacher semiotic mediation, we have resorted to Peirce's triadic Sign theory, interpreted by Sáenz-Ludlow and Zellweger. We present an example of the use of a few elements of that theory in the analysis of a classroom episode in which meaning is constructed with the teacher's semiotic mediation.

Meaning-making in the classroom (e. g., Antonini & Maracci, 2013) and the corresponding teacher semiotic mediation (e. g., Mariotti, 2012; Samper, Camargo, Molina & Perry, 2013) have gained importance as theoretic constructs for describing and explaining mathematics teaching and learning. In our case, they are fundamental due to the teaching approach (see details in Perry, Samper, Camargo & Molina, 2013) with which the pre-service mathematics teacher plane geometry course is developed in the Universidad Pedagógica Nacional (Colombia). This course has been the setting of our research, concerning teaching and learning proof, since 2004 (e. g., Camargo, Samper, Perry, Molina & Echeverry, 2009; Molina, Samper, Perry & Camargo, 2011). With the intention of interpreting in detail both phenomena, we have started to recur to elements of Peirce's triadic Sign theory based on Sáenz-Ludlow and Zellweger's (2012) elaborations. In this paper we analyze, in the light of such theoretical elaborations, a classroom episode in which geometric meaning is constructed with the teacher's semiotic mediation. We want to contribute to the determination of what it means to adopt a semiotic perspective of teaching and learning, inspired on Peirce's triadic sign idea. The analysis presented is part of an ongoing research on the use of conjectures as class content organizers, a project which is financed by the Colombian national science foundation, Colciencias.

SPECIFYING THEORETIC ELEMENTS

Peirce's distinctive contribution is to conceive SIGN activity (semiosis) as one in which three components are related: sign-object (*so*) that which is alluded to in a communication or thought, sign-vehicle (*sv*) the representation with which the object is alluded to (e. g. a word, gesture, graph), and sign-interpretant (*si*) that which is produced by the sign-vehicle in the mind of whoever perceives and interprets it.

Succinctly, we describe the semiosis that takes place in a verbal exchange constituted by two turns: in an *intra-interpretation* act (self-self), a person Y selects a particular aspect of a *sign-object* that is part of his *sign-interpretant*, encodes it and expresses it in

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a *sign-vehicle* addressed to a person X; in an *inter-interpretation* act (self-others) that takes place within his knowledge and experience, X decodes the *sign-vehicle* emitted by Y and constructs a sign-*interpretant* which determines a sign-object.

Delving deeper into the sign-object, there are three subcategories: Mathematical Real Object (*MRO*), immediate object (*io*), and dynamic object (*do*). The Mathematical Real Object is a historic-cultural object constructed by the community of mathematicians, which serves as reference for the community of mathematical discourse. The *sender's immediate object* is constituted by the specific aspect of the Mathematical Real Object that he wants to represent with a sign-vehicle. The *receiver's dynamic object* is constituted by the aspect interpreted from the sender's sign-vehicle. The immediate object is expressed in the sign-vehicle that carries it while the dynamic object is generated in the receiver's sign-interpretant. For this reason, for the analysis, it must be inferred from one or more sign-vehicles. This makes clearly distinguishing the dynamic object from the immediate object harder when the person changes his role from receiver to sender and there has not been a substantial change in the Real Object's aspect that the person is referring to.

In a dialogic interaction (a collective semiosis) in the classroom, which purpose is to make sense of an *MRO*, a sequence of SIGNS from different semiotic systems is naturally used. The *do*'s that emerge in the students' minds from the interpretation of these SIGNS will be, in a lesser or greater degree, in *accordance* with the teacher's intended *io*. The teacher's intentional *semiotic mediation* is constituted by all his deliberate actions that facilitate and guide the convergence of the students' evolving *do*'s to the intended *io* of the SIGNS. For this to happen, the teacher infers, interprets, and integrates, into one dynamic object, the most significant aspects of the *do*'s articulated by the students' that, in one way or another, he deems necessary in the evolution of their *do*'s as they try to make sense of the intended *MRO*. We call *didactic dynamic object* (*ddo*) this emerging and evolving dynamic object that is inferred and constructed by the teacher as a result of an intentional classroom interaction. The teacher uses his constructed *ddo*'s to make those didactical decisions necessary to facilitate the evolution of students' *do*'s so that they will approximate the intended *io*.

EPISODE CONTEXT

The episode took place in the plane geometry course developed during the second semester of 2013. The course is a second semester course of the pre-service teacher program and one of its intentions is that the students learn to prove and widen their view of proof. The teacher, coauthor of this paper, has ample experience in the respective curricular development.

The students, in groups of three, after solving the problem, "Given three non-collinear points A, B and C, does there exist a point D such that $\overline{AB}$ and $\overline{CD}$ bisect each other?", using Cabri, formulated a conjecture related to the construction carried out to solve the problem. One of the groups constructed the three non-collinear points A, B

and C, $\overline{AB}$, the midpoint M of $\overline{AB}$, line CM, the circle with center M and radius CM, and determined D as the intersection of the circle and line CM.

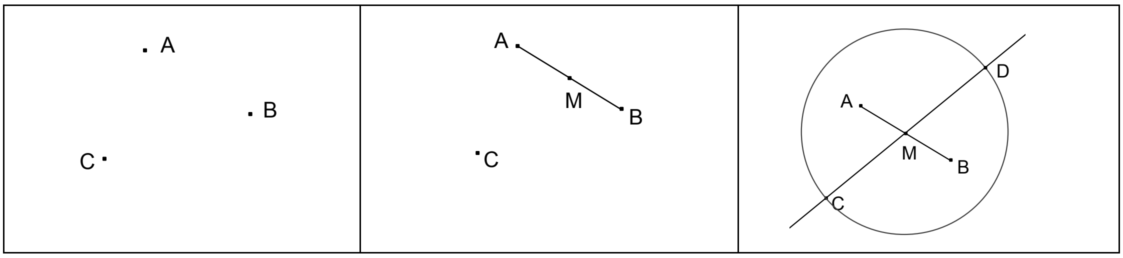

Figure 1

The conjecture the group presented, taking into account the teacher's request to specify in the conditional statement's antecedent the conditions for D, was: "Given three non-collinear points A, B and C, if D belongs to line CM, such that $\{D\} \neq \{C\}$, M midpoint of $\overline{AB}$ and $MD = MC$, then $\overline{AB}$ and $\overline{CD}$ bisect each other". Interacting in an instructional conversation (Perry, Samper, Camargo & Molina, 2013), teacher and students proved the conjecture. They then analyzed how the proof would change if the condition D belongs to line CM is substituted for D belongs to ray CM, concluding that different warrants would be involved which lead to different possible betweeness relations of points D, C and M. Immediately, the teacher questioned the existence of a point D with all the conditions imposed in the antecedent of the conjecture; due to this, they began the task of specifying what in the theory they then had to permitted them to construct each geometric object involved (i. e., validate the construction). Specifically, they could guarantee the construction of the segment, its midpoint and the ray or line, but not the construction of the circle used to determine D because the available theoretic system did not include geometric facts about circles.

The scene of the episode that we analyze in this paper is solving this difficulty, motivated from the theory. It starts with the teacher asking the class how to substitute the construction of the circle, that is, how to obtain point D with the required conditions without using the Cabri option "circle". The following are possible appropriate answers[1]: (i) Having points C and M, construct line CM to assign to them coordinates y and x, respectively, with $y > x$, use the defined metric to find the distance from C to M ($y - x$) conviniently construct the number z ($z = x + (2y - 2x) = 2y - x$); assign to z a point which turns out to be D. (ii) The procedure is the same as the previous one except that zero is assigned as C's coordinate, and therefore, the real number conveniently constructed is $2y$. (iii) Having points M and C, determine (without using coordinates) the distance (MC) between them; construct the ray opposite to ray MC; use the latter ray and the distance determined to locate point D on that ray.

[1] They are based on the Real-Number-Line Postulate which establishes that: (i) to each point of the line there corresponds a unique real number and (ii) to each real number there corresponds exactly one point of the line.

EPISODE ANALYSIS

With his *sv*, "How can we make point *D* appear without using the circle?", the teacher sets a task with the purpose of helping the students begin to make meaning of the Point-localization Theorem: "Given ray *CT* and a positive real number *z*, then there exists only one point *X* such that *X* belongs to ray *CT* and $CX = z$, reason why we consider this theorem and its proof, which is not part of the available theoretic system, as the *MRO* of the semiotic activity when the task is carried out. Now, the teacher's *MRO* is the process described in (iii). Specifically, the teacher wants the students to experience constructing the conditions that permit localizing a point *X*, since in each future situation in which the theorem is used they will have to begin by establishing the real number *z* and the ray on which the point will be located. The teacher's *sv* carries as *io* the possibility of defining a procedure to find *D* with a certain betweeness and at a certain distance from a specific point, without using circle. Taking into account the question's context, the *io* can be specified as: the possibility of defining a procedure to find *D* such that *M* is between *C* and *D*, and M is equidistant from *C* and *D*, without using a circle.

María[2] answers: "There we already **have a ray** [*CM*, (or a ray on line *CM*)], we could take measurements, first take a measurement from… *C* to *M* and then we take **the measurement from M to…** that is, take another measurement and **up to where that measurement gives us** (with the fingers of the extended hands, and these in perpendicular planes, she lets the right hand fall over the left one twice, **gesturing cutting**), there put point *D*". María's *sv* includes enunciation and gestures with her hands as she verbalizes. It carries as *io* a procedure which consists of measuring the distance between *M* and *C* and copying it starting at *M* on ray *CM* (which corresponds to transferring the measurement on the opposite ray of ray *MC*) to determine *D* such that *M* is between *C* and *D*. Above we have used boldface letters for the specific signs which back María's *io* description. It seems María's *si* includes an image of a physical compass capturing a distance and transporting it, and her *do* is consistent with the teacher's *io* in so far the student defines a procedure with the required conditions.

The teacher interprets María's answer as appropriate to continue constructing the details of the *io* he expects the students to approach; so his *ddo* includes the coordinates as resource to obtain the distance between two given points. The teachers' *sv*, "If we take measurements, what do we automatically need?" carries as *io* the same aspect of the procedure –find measurement– which María made reference to, but it also indicates that she must specify which geometric object she will use to obtain distance, request that although it does not carry, for any listener, explicitly an aspect of the procedure they are talking about, it can be understood by whoever knows that the use of coordinates together with a metric are required to find the distance between two points or the use of the Two points-number Postulate that declares the existence of the distance between any pair of different points. Ángela interpreted the first option. Her

[2] This and the other student names are pseudonyms.

sv, "Coordinates", carries as *io* the use of the points' coordinates to determine distance; in this moment of the conversation, her *do* seems to coincide with the teacher's intended *io*, who accepts her idea to use the coordinates to find the wanted distance. From the expression "take measurements" with which the teacher echoes María's proposal, we see that he accepts the path she suggests, using coordinates, although using the afore mentioned postulate is a more direct way to introduce the Point-localization Theorem. This is a teachers' didactic decision.

The next intervention is Dina's with the *sv*: "we can say […] that twice the distance from A to M is equal to the distance from A to B, right? Because it is a midpoint. So, using Ángela's idea, with coordinates, we can say that twice CM is equal to the distance from C to a D that I am going to place somewhere. Then, already there we are placing…". This *sv* carries two immediate objects: first, a relation between distance measurements implied by the midpoint of any segment ($2AM = AB$); second, the possibility of applying the relation mentioned to C and M, having obtained the required distances using the points' coordinates. We see that Dina's *si* includes a different condition to the one used so far ($CM = MD$), to characterize the midpoint of a segment ($2CM = CD$), condition she wants to use with coordinates to find D. Her *do* is a procedure to determine D as the endpoint of a segment with C as the other endpoint and M as midpoint, using the relation established by the Midpoint Theorem (i. e., If M is the midpoint of $\overline{AB}$ then $2AM = AB$ in terms of coordinates. Dina's *do* is relatively close to the teacher's intended *io* because it satisfies the condition of not using a circle to locate D, and also, when proposing the use of the afore mentioned theorem she is constructing the z mentioned in the Point-localization Theorem. That is, she substitutes the use of the circle with the use of the midpoint of $\overline{CD}$ not determined yet (which leads to having M between C and D) and using C as the initial point from which the constructed distance is put. As will be seen, this *do* is lacking proximity to the teacher's *io* in what concerns how to use the coordinates and for what.

The teacher points out that Dina and Ángela's ideas are pertinent. Besides, he clarifies that: "Then, we use coordinates to guarantee the distance that I want it to be", *sv* that carries as the teacher's *io* the role the coordinates will have in the procedure for finding D without using circles. The teacher's *si* includes the idea that the procedure that he aspires to establish in the class is developing adequately, and with his *ddo* he emphasizes that the procedure without circles requires coordinates not only for obtaining a distance but also for using it in determining the coordinate that will turn out to be that of the point that they want to locate.

With his next *sv*, "[…] what you (Dina) want is to use concrete numbers as coordinates. What concrete numbers do you want?", the teacher initiates the construction of an example of the obtainment of the coordinate that they want to determine using the coordinates of C and M. Here we do not analyze the interaction through which the example was constructed because its content is mainly of an arithmetic nature. It is enough to know that the coordinates of C and M were 2 and 4, respectively, and that they concluded, not without some difficulty for some students, that the coordinate of point *D* had to be 6. Molly explained the reason for this result with the following *sv*: "Because the distance from *C* to *M* is… two… units. That from *M* to *D* must also be two units, therefore *D*'s coordinate must be six". Molly's *do* seems to be a procedure consistent with: calculate *CM* and take into account the equidistance condition (*CM* = *MD*) to obtain *D*'s coordinate by adding *CM* to *M*'s coordinate.

Having finished developing the example, on the blackboard, written by the teacher, can be read: c(*C*) = 2, c(*M*) = 4, c(*D*) = 6 (i. e., the coordinate of *C* is two, etc.). With regard to this, the teacher emphasizes the difference between what is represented with the first two notations (having points *C* and *M*, to each a coordinate is assigned) and what is expressed in the third (having coordinate 6, to it a point is assigned which is precisely *D*). This description corresponds to his next *sv*: "These points (signaling the points in the notation c(*C*) = 2, c(*M*) = 4) already exist; we can give them those coordinates, right? After that we would have to say, this number exists, six, (points at the notation c(*D*) = 6) and to this number six, what do we do to it?" Various students respond the question correctly as they say: to six we have to "associate a point, *D*". From the teacher's intervention, we infer that his *si* includes an image of the difficulty students have to distinguish the conditions under which each of the items of the Real-Number-Line Postulate can be applied and the corresponding effect. His *ddo* emphasizes the distinction of each of the item's application; specifically he highlights that to determine point *D* it is necessary to first give the real number that will be his coordinate. The *do* of each of the various students who completed the teacher's comment is relatively consistent with the teacher's intended *io*. Later, already having assigned *D* to the coordinate 6, teacher and students agree that the equidistance condition alluded to by María at the beginning of this episode is satisfied.

Next, the teacher indicates that the procedure carried out in the example must be generalized. Various students propose designing as *x* and *y* the respective coordinates of points *C* and *M*. Antonio wants to designate *D*'s coordinate but the teacher changes the course of the conversation towards the number *z*, "First the number… Which one shall it be? A number will appear… then there exists the number *z*, and what condition should that *z* have?". The teacher's interaction with various students trying to refine an answer can be summarized as follows: *z* is a positive number because it is an absolute value, $|x-y|$. When the teacher asks whether what they have said about *z* is sufficient, Molly responds: "No, we must say that that number belongs to [is associated to] point *D*". With respect to such an answer, the teacher asks if they agree with that statement and although various students disagree, the comments they make indicate that they find it difficult obtain a general expression for *z*. From this we see that the teacher's *io* is closely related to, on the one hand, Molly's explanation on why, in the example, *D*'s coordinate should be 6, and on the other hand, the comment he made to emphasize that they have points *C* and *M* and coordinates are assigned to them, but with respect to the point they are searching for, first a positive number must be determined and then, the point assigned to it is precisely the one searched. The *do* that appears as a collective construction by the students that participate in the verbal exchange seems to be far from the teacher's intended *io* in three issues: (i) designating *D*'s coordinate without taking into account that they are looking for *D*; (ii) believing that the number *z* represents the distance between *M* and the point searched for; (iii) believing that the distance between *M* and the point searched for coincides with the coordinate that this point must have.

Seeing the difficulty the students have to obtain a general expression for the number *z*, makes the teacher simplify the situation by proposing that *C*'s coordinate be zero and *M*'s coordinate *y*, with $y > 0$. We also gives the value of *z* ($z = 2y$) and shows that effectively the distance from *C* to *M* is the same as that from *M* to the point to which *z* should be assigned to.

With the former explanation, they are ready to continue validating the construction, specifically the existence of *D*. In this process, orchestrated by the teacher, students are given the opportunity to respond correctly very punctual questions related with the procedure to determine *D*. These correct answers permit us to see signs of the beginning of a convergence of the *do*, constructed communally by those that participated in the exchange, towards the teacher's intended *io*.

234	Teacher:	[…] So, we can say the coordinate of point *C* is going to be equal to whom?
235	Juan and others:	To zero.
242	Teacher:	[…] Ready, coordinates for points *C* and *M* appeared. What do we have to do afterwards?
243	Jack:	Create the number *z*.
244	Teacher:	[…] What would we do with that number *z* afterwards?
245	Student:	Assign a point to it.
250	Teacher:	A unique point. What will it be called?
251	Various:	*D*
252	Teacher:	*D*, okay, such *D*…
253	Ángela	belongs to the line
254	Student:	*CM*
255	Teacher:	belongs to the line *CM*, okay, and…
256	Juan:	*D*'s coordinate is twice y.
257	Student:	*z*
258	Teacher:	*D*'s coordinate is equal to z. That is the correct way to write it. […]

FINAL REMARKS

The analysis presented is illustrative of the teacher's semiotic mediation characterization, using the elaboration that Saénz-Ludlow and Zellweger have done of Peirce's triadic Sign theory. In this paper we present an extension of that elaboration to include, as a central aspect of the mediation, the *didactic dynamic objects* (*ddo*'s) that emerge and evolve in the course of semiotic mediation, and that seek the convergence of the student's dynamic objects to the teacher's intended immediate object. Conceiving the teacher's semiotic mediation this way permits us to identify in greater detail the teacher's role in students' meaning-making.

It is important to mention that in the analysis carried out here we focused on the discursive student sign-vehicles but not on the gestures with which they accompany their interventions. This is due to the fact that the teacher's *ddo*'s seem to emerge principally from what the students say and not from what they do. Particularly, in María's first intervention with which the communicative exchange begins (when she proposes "take measurements") directs the semiosis through a path that permits advancing in meaning-making of the Point Localization Theorem, relating it to the Real-Number-Line Postulate. However, the gesture with which María accompanies her proposal seems to be a parody (acted out) of the Point-Localization Theorem, which if it had been discussed in class, would have led to a different semiosis and probably to meaning-making of the theorem relating it to the Two Points-Number Postulate as well as to the Real-Number-Line Postulate. This observation leads us to point out the importance of the teacher's didactic decisions, in the course of *semiotic mediation*—the semiosis that takes place in the classroom.

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