DEVELOPING CONCEPTUAL UNDERSTANDING OF PLACE VALUE: ONE PRESERVICE TEACHER’S JOURNEY

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This paper reports on a portion of a research study that examined the development of 43 preservice elementary school teachers’ conceptual understanding of place value, and highlights the experiences of one middle-performing preservice teacher. After participating in a research-based constructivist unit of instruction in place value, the findings showed that the preservice teachers demonstrated a statistically significant change in place value understanding. Common emergent mathematical qualities and qualities of disposition were identified in the qualitative analyses. These data provided insight into this preservice teacher’s thinking strategies.

INTRODUCTION

It is widely documented in the research literature that many elementary teachers lack sufficient depth of understanding of the mathematics they are expected to teach (Ball, 1990; NRC, 2001). Oftentimes, elementary teachers can reproduce mathematical procedures, but they do not understand why the procedures make sense conceptually (Ma, 2010). Thus strengthening the mathematical content knowledge for teaching and improving constructivist-based pedagogical practices in teacher education programs should be explored (e.g., Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, & Ball, 2008). Because place value is the foundation of number sense and the prerequisite to multidigit operational fluency (AMS, 2001; NCTM, 2000), it is an important topic for elementary teachers. Therefore the purpose of this research study was to examine the development of preservice elementary teachers’ conceptual understanding of place value within a constructivist framework.

THEORETICAL FRAMEWORK

The learning theory of constructivism provided the framework and the lens through which the research was conducted, informed by the works of many researchers (e.g., Cobb, 1996; Noddings, 1990). The core components of constructivism in the mathematics classroom were explicit in the study’s instructional sequence as follows: (a) role of student as active learner and as the authority on mathematical justification, (b) role of teacher as facilitator of learning and expert in questioning techniques, and (c) role of the classroom environment with a focus on discussion and problem solving.

The intervention for the research study was a constructivist instructional sequence designed by the researcher to develop conceptual understanding of place value. The place value instructional content was a blend of the works of Fosnot and Dolk (2005), McClain (2003), Safi (2009), and Yackel and Bowers (1997), in which place value was

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described in terms of three interrelated observable subconstructs: (a) quantification in the base ten numeration system, (b) invariance of number when composing and decomposing, and (c) the meaning of regrouping in multidigit addition and subtraction. The place value instruction built upon the theoretical conceptions of number (Fuson, 1990; Kamii, 1986), taking into consideration the complexity of place value, including key ideas such as the position of a digit, grouping, trading, and unitization. The researcher-developed assessment instruments as well as the interview protocols were anchored in this research literature on place value.

METHODOLOGY

Because the majority of current empirical research on preservice teachers' place value understanding has been purely qualitative, a mixed methods approach was used to collect data from 43 preservice elementary school teachers enrolled in the mathematics methods course. Quantitative place value data were collected from all 43 participants through administration of one pretest and two posttests. Data were analyzed using a repeated-measures analysis of variance (ANOVA) for correlated samples.

In the larger study, six participants were chosen to be interviewees based on their scores on the place value pretest—two low-performing, two middle-performing, and two high-performing. Qualitative data for these six participants, collected through two sets of interviews and document reviews in the form of homework and journal entries, were analyzed through a process of coding. The first cycle used provisional coding, using codes identified *a priori* adapted from Cobb and Wheatley's (1988) concepts of ten. Also included in the first cycle was initial coding, in which open-ended notes were made to characterize the preservice teachers' thinking strategies and record any salient affective observations. The second cycle used focused coding and inductive analysis to identify themes in the data (Patton, 2002), and the third cycle of coding added a layer of analysis to align with current national initiatives (e.g., Common Core Standards for Mathematical Practice, CCSS, 2011). As a result of this qualitative data analysis, six common emergent mathematical qualities and three common emergent qualities of disposition were identified, as shown in Figure 1.

Developing Quality
Mathematical Qualities
Flexibility, reversibility of composition and decomposition
Connections made between mathematics topics
Efficiency
Development of self-created notation
Improved mental mathematics proficiency
Precise vocabulary, e.g., groups, unitization
Qualities of Disposition
Comfort, trust, confidence in doing mathematics
Self-reflection, metacognition aided own understanding
Awareness of need for both procedural and conceptual knowledge

Figure 1: Common Developing Qualities of Six Interviewees

RESULTS AND DISCUSSION

Examining first the larger context of the study, prior to participating in the instructional sequence on place value, the 43 preservice teachers enrolled in the mathematics methods courses demonstrated *developing* levels of overall place value understanding but *limited* levels of base ten understanding. After participating in the place value instructional unit, the repeated-measures analysis of variance (ANOVA) of the pre- and posttest data showed that the preservice teachers' level of place value understanding had changed significantly, $F(2, 41) = 100.68$, $p < .001$, partial eta$^2 = .71$, placing the preservice teachers' level of place value knowledge between the *developing* and *full* levels of understanding. These ANOVA results suggest that the intervention of the constructivist instructional sequence was effective since the scores increased over time.

Though immersed in a larger study, this paper will focus on the journey of one preservice teacher, Liz, to illustrate a few of the emergent qualities identified in the qualitative analysis. In the original sampling, Liz was chosen as a middle-performing participant because her pretest score represented the median as compared to her classmates in her elementary mathematics methods course. In the present paper, Liz's journey is highlighted because of her ability to be self-reflective of her own mathematical learning.

Liz: Prior to Implementation of Instruction

Prior to Liz's participation in the instructional sequence, an initial interview was conducted to gain insight into the participants' thinking strategies on the pretest. This interview started with a focus on base ten items, beginning with pretest item 5, Figure 2. Liz's first few statements in this interview were characterized by her reliance on procedural thinking, as illustrated in the following excerpt.

5. Please consider the regrouped ones in the problem below:

$$\begin{array}{r} {}^{1\,1} \\ 389 \\ \underline{+\,475} \\ 864 \end{array}$$

a. What does the 1 above the 8 represent?
b. What does the 1 above the 3 represent?

Figure 2: Item 5 on pretest (Thanheiser, 2010).

Interviewer: My first question is about number 5: Please consider the regrouped ones in the problem below. I'm hoping that I've asked you questions you've never thought about before, like about these [regrouped] ones.

Liz: Some of these tripped me up because no one has ever made me clarify them before. I just know that's how it is?

Interviewer: That's right. And you know how to get the answer.

Liz: Right, exactly.

Interviewer: And you never really thought about it deeply. And then as a teacher of mathematics, this is something we should be thinking about. So, what does the 1 above the 8 represent? What are your thoughts?

Liz: I think I put down that it represented 1. I guess I was thinking that because I know that it's 8 + 7 + 1, which comes to 16, so I just assumed that it represents 1. Now I'm analyzing everything…I guess it could represent 10 because you could be adding it to the—no, that wouldn't make sense. Never mind. Because it would be 10, then it would be 18 + 7 and that wouldn't work. So I still don't know!

Interviewer: Okay. What about the other 1 [over the 3]? Do you think it means the same thing?

Liz: Yeah, as of right now, yeah, because—it's 4 + 3 + 1.

Liz seemed to be following the verbal representation of the addition algorithm to describe her thinking for item 5. Even though Liz used metacognition strategies to rethink her answer, talking aloud reconfirmed her misconception of the values of the regrouped ones. In a subsequent discussion of a base ten subtraction problem during this interview, her response reflected a lack of understanding of the underlying base ten base ten concepts, as she was unable to see the unitization of a regrouped 1 simultaneously having a value of 100 and 10 groups of ten.

The last portion of Liz's first interview was focused on base eight addition and subtraction problems set in the "Candy Factory" context, adapted from Bowers, Cobb, and McClain, 1999. On her pretest, Liz had solved these by converting into individual candy pieces, calculating in base ten, then repacking the candy back into base eight. With very little guidance from the interviewer during this first interview, Liz was able to begin using a more symbolic, efficient method of recording her trades in base eight for one subtraction problem. After obtaining her answer, she exclaimed, "Oh my gosh! That's crazy! I would never have thought of it that way, though." It was at this point in the interview that she articulated commonalities between the written algorithms across different place values with different bases. Thus, even before formal classroom instruction, Liz was beginning to exhibit some of the identified emergent qualities: *connections made between mathematics topics; efficiency* (in base eight computation); and *self-reflection, metacognition aided own understanding*.

Liz: During Implementation of Instruction

Liz's journal responses to daily journal prompts provided rich descriptions of turning points in her understanding. On Liz's third journal prompt, she was asked to describe one thing about place value that she didn't know before the unit. She had written: "That sometimes in subtracting, borrowing from another number can represent a couple things. When we take from the hundreds column, it is actually a group of 100, but we treat it like a ten." Here, Liz alluded to unitization between the tens and hundreds columns, and how these concepts provide meaning to the standard written algorithm for subtraction.

Liz: Following Implementation of Instruction

Liz's second interview was conducted after the administration of the first posttest, which showed an improvement of her overall place value understanding, especially in base ten concepts. When asked during this second interview to explain her thinking on posttest 1 item 6b, Figure 3, Liz's journey of understanding took an interesting turn. Following is the dialogue that took place regarding item 6b.

6. Please read over Ryan's work then answer the question which follows.

Below is the work of Ryan, a second grader, who solved this addition problem and this subtraction problem in May.

Problem A Problem B

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  1        2 1
 438       345
+ 47     -  52
 485       293
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a. Does the 1 in each of these problems represent the same amount? Please explain your answer.

b. Explain why in addition (as in Problem A) the 1 is added to the 5, but in subtraction (as in Problem B) 10 is added to the 2.

Figure 3: Item 6 on posttest 1, adapted from Thanheiser (2010).

Interviewer: Now tell me about the 1 in subtraction and how it might be different [from the 1 in the addition problem].

Liz: For this one, it's 345 and we can do 5 − 2, but we can't do 4 − 5. So you borrow from the hundreds column. You're borrowing 100 and making this 200. And you're moving it over to this column, so what it really is, it's still 345 because it's 200 + 145. But when you move it over to this column, you treat it as a ten, but it's 10 groups of ten.

Interviewer: Aha, I think you've answered my next question. This 1 you said came from here so it means 100 this time, not 10. But even though it means 100, you said it means 10 groups of ten. My next question was why don't we go 1 + 4, but here you go 10 + 4?

Liz: Because it's ten groups of ten, not one group of ten.

Interviewer: Yes! Ten groups of ten plus four groups of ten that's 14 − 5 and you end up with 9. But you're still in the tens column, so it's nine groups of ten. You got it.

Liz: I think I finally got it!

Interviewer: I don't think you said it right though on the second time around [posttest 1]. I think the second time around, you speak to the procedure, how it makes it easier, carrying, you're over the limit so you've got to go the next one.

Liz: Did I not talk about all those groups?

Interviewer: But you didn't tell me what you just told me here….

Liz: I think it was because I think it's taken until right now.

Liz's self-reflection during the second interview led her towards a more conceptual understanding of place value. She now spoke enthusiastically in terms of like groups of numbers, and therefore was unitizing, even though she still used the traditional "borrowing" language. Hence, Liz's comments provided evidence of three developing qualities: *precise vocabulary, e.g., groups, unitization; flexibility, reversibility of composition and decomposition* (of base ten numbers); and *comfort, trust, confidence in doing mathematics.*

Also during the second interview, Liz was asked to identify which classroom activities were most helpful in guiding her towards deeper understanding of base ten concepts. She cited that counting in base eight and finding base eight sums mentally were also helpful classroom activities:

Liz: I thought the counting itself takes a while to get used to just because you don't go to 10, you go to 8 and it starts over.…And I guess adding could be—I felt like I was doing grouping more in my head almost. When you had to add numbers, because if it was over—

Interviewer: If it was over a rod?

Liz: I was comfortable between 1 and 7, but once it went over 8, it was like wait, what does that represent now?

Interviewer: Right, like if it was 7 + 2?

Liz: Yeah, and then didn't we say it was like *one-ee-one*?

Interviewer: One rod and an extra one: one-ee-one.

Liz: It was weird because one-ee-one in my mind is eleven, but it wasn't eleven.

Interviewer: Because it's one group of eight and an extra. That's good. So it made you think?

Liz: Yeah, definitely made me think.

When Liz stated that she was beginning to group in her head, this was evidence that she was developing the quality *improved mental mathematics proficiency* (in base eight).

Near the end of the second interview, Liz reflected on her experiences thus far. The following excerpt illustrates the quality *awareness of need for both procedural and conceptual knowledge*. A distinction is made here that this does not refer to the preservice teachers' *acquisition* of both procedural and conceptual knowledge, which is indeed important (Hiebert & Lefevre, 1996). Instead, this quality was designated if the participant expressed an *awareness* (newly discovered for most) of the need for conceptual knowledge underlying the procedures with which they were proficient. In this excerpt, Liz's statements provide rich insight into her perception of mathematics.

> It's just funny how much I *didn't* know about those values [the regrouped ones]. Apparently, I didn't know anything [laughs] because I think I just did it! We used to play with those cubes, and I did that right I think. But I guess I never knew how it translated to the algorithm.

This was a very powerful statement for two reasons. First, Liz had never realized, prior to this experience, how physical actions with manipulatives are directly connected to the written algorithms that they represent. But even more importantly, Liz's statement about being able to correctly perform a written algorithm implied that she had previously thought that knowing how to do a procedure meant understanding the underlying mathematical concepts, which is not necessarily true.

Liz's Place Value Understanding: A Summary

Prior to the instructional sequence, Liz exhibited a procedural knowledge of place value operations that lacked a conceptual foundation. As Liz participated in the constructivist instructional sequence, her place value understanding shifted from procedural to conceptual, exhibiting improved place value conceptual understanding in all three subconstructs and in the unifying themes of place value: unitization, grouping and trading rules, and the position of the digit determines its value. By the end of the study, Liz's posttest scores placed her near *full* understanding of place value concepts.

CONCLUSION AND IMPLICATIONS

Preservice elementary teachers need rich mathematical experiences in their methods courses that provide them opportunities to discuss, invent, conjecture, and problem solve to increase their own conceptual understanding of place value. This place value understanding consequently provides a structure for the concepts underlying the written algorithms for whole number addition and subtraction. The participants' thinking strategies articulated in these qualitative analyses not only provide insight into the quantitative data, but these strategies can also help mathematics teacher educators anticipate their preservice teachers' place value misconceptions.

The results of this study have the following implications for possible future research: a longitudinal study could be designed to explore connections between preservice teachers' experiences and their students' achievement in place value, an instructional model for constructivism could be developed to allow mathematics educators to readily implement constructivist strategies, or the common emergent mathematical qualities could be further explored to develop more robust descriptions in the context of cultivating mathematical habits of mind in preservice teachers.

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