CHILDREN'S CONCEPTUAL KNOWLEDGE OF TRIANGLES MANIFESTED IN THEIR DRAWINGS

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When asked to draw different kinds of triangles, children reveal many creative ways to express variety. In this paper, the drawings of 81 children in the age between 4 and 6 will be examined and illustrated what kind of understanding of the concept "triangle" precedes the drawings. Therefore, different categories of the children's drawings were generated and also compared to their explanations of a triangle, which sometimes might not be in agreement with their drawings.

INTRODUCTION

"Shape is a fundamental construct in cognitive development in and beyond geometry" (Clements & Sarama, 2009, p. 199). According to Vollrath (1984), a comprehensive conception of geometric shapes is shown through different aspects like being able to name the shapes, give a definition of the shapes, show and illustrate further examples of this category and name all properties. Although this description was given for secondary school children and beyond, it is a good summary of what constitutes a comprehensive understanding of the concept of shape. In this paper it will be focused on the aspect of showing and illustrating many examples of geometrical shapes. This aspect is investigated through the drawings of the children.

THEORETICAL AND EMPRICAL BACKGROUND

It must be considered that in order to draw an object correctly, it demands the *knowledge* as well as the *ability* to put this knowledge down on paper, the so called drawing skills. If these are yet undeveloped, a child is not able to draw a geometric shape even though it might know how such a shape looks like. All developmental models concerning drawing skills (e.g. Piaget & Inhelder, 1967; Schuster, 2000) start with a so called "stage of scrawling", which becomes more realistic and more detailed on each stage. Piaget gives a very detailed description how the drawing skills of children develop: from the age of three the scrawls become more differentiated and shapes showing properties like "inside" or "outside" can be illustrated. Here, copies of a circle, square and triangle all look the same. From age four onwards, basic shapes such as square, rectangle, triangle, circle and ellipses can be drawn, but only from age 6 on, complex shapes can be drawn. If a child is not able to copy or draw a certain shape, Piaget interprets that it is due to a lack of knowledge, obviously not considering a lack of drawing skills, one of the reasons why his results are criticised (e.g. Freudenthal, 1983; Battista, 2007). In mathematics mainly the knowledge, which lies behind the drawing is important. Still, drawing a shape correctly demands knowledge and drawing skills and therefore, "wrong" drawings should not serve as indicator for

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lacking knowledge. Kläger (1990) highlights the importance to never regard drawings of children isolated but to always complement these through interviews, in order to gain insight in the perceptions of the children. For children cannot be "generalized", they draw what they see but also more or less than they see, they draw what they know but also more or less than they know (cf. Kläger, 1990, p. 15f.). The van Hieles (van Hiele & van Hiele, 1986), who also created a hierarchical developmental description, which other researchers prefer to interpret as levels (Battista, 2007), constitute that children realize shapes as whole entities from the age of four onwards and are not able to distinguish shapes by their properties before primary school and are consequently not able to draw specific properties before that.

There are several studies (e.g. Battista, 2007; Burger & Shaughnessy, 1986; Clements Swaminathan, Hannibal, & Sarama, 1999; Razel & Eylon, 1990) investigating young children's understanding of showing and illustrating further examples of geometric shapes not through drawings but by letting the children for example distinguish between examples and non-examples. These studies showed that children had more difficulties in recognizing triangles which were identified correctly in all of the different studies by approximately 60% of the children, compared to squares (80% - 90%) or to circles, which were identified correctly by nearly all of the children in these studies. There are no circles deviating from the prototype and square prototypes only occur concerning position, but there are several triangles deviating from the prototype and thus making it harder to be identified correctly in all variations. Therefore, it can be concluded that if it is harder to identify several types of triangles, it is also harder to draw several types of triangles – where knowledge is lacking it cannot be put into a representation. Some studies (e.g. Burger & Shaughnessy, 1986, Clements et al., 1999) indicate that children's prototype of a triangle seemed to be an isosceles triangle. They found that the majority of children did not identify a long and narrow, scalene triangle as a triangle, although they often admitted that it has three lines and three corners, something which might be seen in drawings as well.

The "drawing triangles task", which will be presented in the following, was already conducted by Burger and Shaughnessy (1986) with a smaller sample and a larger range between the ages (from preschool to college) than in the empirical study reported in this paper. They found that younger children often vary their drawings by ending up with "new inventions", as for example a triangle with "zic-zac-sides", older children vary their drawings more according to the nature of triangles (equilateral, isosceles, rectangular or general triangles). The study at hand complements these studies by examining whether children also prefer *drawing* prototypical triangles and what kind of triangles are drawn as variations. Additionally it will be examined whether these drawings are in line with their explanations of triangles. Furthermore, the competencies of the children are illustrated in the light of two different educational settings.

DESIGN

The study comprises 81 children, 34 from England and 47 from Germany in the age of four to six, who were interviewed at the beginning and at the end of one school year. The children in England were attending a primary school (for children from 4 to 11), where the children enter school in the year when they have their fifth birthday, but many children go to a reception class before that. The German children were attending a kindergarten where children from the age of three to six (up to primary school) can go to. In Germany (Baden-Württemberg), learning through play and an approach using "everyday mathematics" is at present the main concept for kindergarten education, whereas in England, elementary education is rather systematic and curriculum based and the expected competencies are described as "stepping stones".

The study was conducted in the form of qualitative interviews, taking about 30 minutes each. The order of the tasks – there were nine tasks altogether – as well as the material was predetermined but in accordance with the nature of qualitative interviews this order could be altered or complemented. There were two points of data collection, without a special intervention, one at the beginning of the school year 2008/ 2009 and one at the end of the school year. The English children, in contrast to the German children, were instructed in geometry during the year. Each child was interviewed individually, so copying the drawings from each other was not possible.

In this paper, the results of the "explaining triangles task" and of the "drawing triangles task" will be illustrated. First, the children were asked to "explain a triangle to someone who has never seen a triangle before". Later, after some other tasks, they were asked whether they could draw a triangle and afterwards a triangle that looks different than the first one and again a triangle that looks different than the first two, and so on. With this, the children's idea of triangles as well as their idea of diverseness was tested. Then they were asked to explain their drawings. Afterwards, it could be seen whether their first, general explanation of triangles were in line with their drawings and the explanations of their drawings.

In order to analyse the drawings and explanations of the children, different categories were generated and discussed. Besides the interpretation of the qualitative data as small case studies, also quantified details will be given to show tendencies and to suggest hypotheses because quantitative details can be one aspect of qualitative reality (Oswald, 2010, p. 186).

The underlying research questions are:

1. What kind of triangles do children draw when asked to draw a triangle?
2. In how far do they vary their triangles when asked to draw another one (and again another one and so on) that looks different than the first one (two,…)?
3. In how far do the explanations of the children match their drawings?
4. Can any differences be observed comparing the results of the two educational settings or the two points of data collection?

RESULTS

In the following, it first will be illustrated what kind of triangles the children drew as well as their way of varying different triangles, before it will be compared to the explanations of the children, all in the light of the two different educational settings. When examining the drawings of the children, it was first looked at what kinds of triangles the children did draw. It was distinguished between: (1) no triangle, (2) a "made-up" triangle (i.e. a non-triangle), (3) an equilateral triangle, (4) an isosceles triangle, (5) a rectangular triangle or (6) a general triangle (e.g. acute or obtuse angled). At both points of investigation, the majority of the children drew an isosceles triangle as their first triangle (38% of the English at the beginning and 47% at the end of the school year and 33% of the German at the beginning and 51% at the end of the school year). Only a few children drew an equilateral or a rectangular triangle as first triangle, but a general triangle (no specific one) was drawn by 24% of the English and 16% of the German children at the first investigation and by 38% of the English and 26% of the German children at the end of the school year. No child started with a "made-up" triangle, for example shapes with three corners but "zic-zac-sides", but often used such "inventions" in order to alter their triangle.

For the variations of the children, the following categories were generated.

Identity – Child draws the same or similar triangle again and again;

Area – Child draws triangles in different sizes;

Angular dimension – Child draws triangles with different angles;

Position – Child draws triangles in different positions and directions;

Combination – Child draws triangles that differ at least in two of the following attributes: area, angular dimension or position;

Objects from everyday life – Child draws objects from everyday life having geometric shapes (for example road signs);

Shape – Child draws different shapes (triangles and "own inventions");

Missing critical attributes – Child draws a shape that is missing some critical attributes of a triangle, as for example a third side.

The children's drawings were either grouped into one of the categories *identity*, *area*, *angular dimension*, *position*, or, if they drew at least two of these varieties, they were grouped into *combination*. Moreover, they could be additionally grouped into one of the other categories: *objects*, *shape* or *missing attributes*. Therefore, the overall percentages might be more than 100%.

It became obvious that most children connected "different" triangles with triangles that differ in their *area* dimension (see Table 1). Here, the triangles are all pointing upwards and are most of the time isosceles or equilateral.

	Identity		Area		Angular		Position		Comb.		Objects		Shape		Missing	
	E	G	E	G	E	G	E	G	E	G	E	G	E	G	E	G
2008	18	7	50	49	15	9	6	5	6	7	0	9	29	21	3	3
2009	12	14	71	44	24	28	9	12	6	12	0	9	9	21	3	3

Table 1: Triangle Drawings in Percentages

At the beginning of the school year, there were a few more English children than German children drawing triangles with different angles and positions. At the end of the school year, it was the other way round: now, slightly more German than English children diversified triangles according to angles and positions. Triangles as part of the geometric solids in everyday life (e.g. street signs or tents) were only drawn by the German children. At the beginning of the school year, there were more English children drawing triangles varying in their shapes, but later there were more German children drawing triangles varying in their shape. In both countries, only a few children left out critical attributes such as one side.

The children used different ways (like answering with gestures, through comparisons, and other informal or formal ways) to explain or define a triangle as reply to the question: “Could you explain a triangle to someone who has never seen a triangle before?”. It could also be that children used several ways for their explanations and so the overall percentage could again be more than 100%.

	No expl.		Informal expl.		Formal expl.		Other expl.	
	E	G	E	G	E	G	E	G
2008	15	30	9	30	62	16	12	30
2009	0	23	21	49	62	14	18	28

Table 2: Explanation of Triangles in Percentage

The results lead to the impression as if the concept knowledge of the English children is already more developed than that of the German children, because the English children explained the triangles more often, compared to 30% of the German children at the first and 23% at the second investigation, who did not explain the triangles at all. Moreover, the English children explained about four times more than the German children the triangles in a formal way, giving a definition as for example:”three straight sides and three corners”. Still, the drawings of the English children did often not fit the preceding explanations, as was the case for 24% at the first and for 12% at the second investigation. Here, only the informal and formal explanations were regarded (and no other explanations), because it is quite complicated to compare a gesture or a verbal comparison (e.g. “hat of a witch”), for example, with the properties of a drawing.

The “triangles” of Emma at the beginning of the school year for example (see Figure 1 below) do not all look like real triangles. Emma was explaining a triangle as having “three straight sides and three corners”, a correct definition not in line with two of her actual drawings:

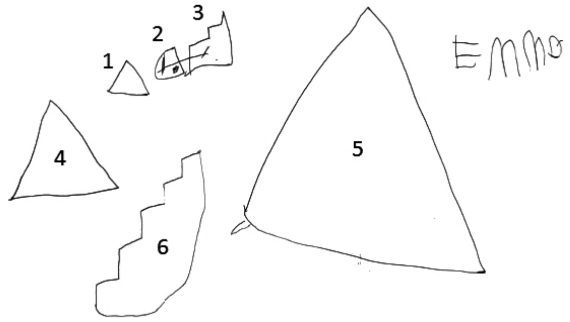

Figure 1: Emma (4,5 years), 2008, England

Interviewer:	Could you explain to me the difference between the triangles?
Emma:	This one is thinner (points to shape no. 6) and those two (points to no. 4 and no. 5) are fatter.
Interviewer:	Is there any other difference?
Emma:	No, they are all pointy. Some have two points and some have three points.

Emma might have seen the different spikes of the stairs as single triangles. So a triangle could also be part of any other object, her concept knowledge was still limited to that perception. However, at the end of the school year, Emma was drawing only triangles and no "made-up" shapes. Louis' triangles also deviate into non-triangles in the end. He explained a triangle in an informal but correct way, saying that it has "three points and it comes straight up and it comes straight down". He started with the triangle on the very left and ended with the shape on the right. When asked, whether all of these shapes are triangles, he answered:

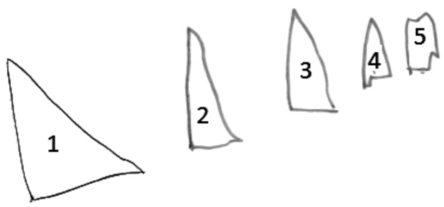

Figure 2: Louis (5,1 years), 2008, England

Louis:	Well, that one has got this bit (points to no. 4) and this one (no. 5) goes like that, it's all a bit strange... not a proper triangle.
Interviewer:	How many corners does this shape have? (points to no. 5)
Louis:	(counts). 1,2,3,4,5!
Interviewer:	And how many corners does a proper triangle have?
Louis:	1,2,3 ...three!.
Interviewer:	But this is still a triangle? (points to shape no. 5)
Louis:	(nods) Yes!

It seems that Louis discerns between proper triangles who have three points and "other shapes" who have more points but can still named as triangles. Other children, who drew "made-up" triangles also diverted the sides of the triangles and drew "wavy" or "rocky sides".

All the German children who were able to explain correctly (no matter whether this was informal or formal) what a triangle looks like, were also able to draw a triangle correctly. But there was no German child who knew a definition but was not able to connect it with a representation then. In contrast to this, to summarize the results above, the English children were often able to formulate a definition of a triangle but were not always able to connect this definition with a variety of representations.

DISCUSSION

The results revealed that the children in both educational settings mainly drew isosceles triangles, but it could not be detected whether their attempt was to draw an equilateral triangle and whether it just happened to limited drawing skills. It can be stated that prototype presentations were dominant not only for the first drawn triangle but also as varying triangles because most children varied their triangles through area size. It has to be discussed if the rare use of position as variation can be explained by the format of the paper and the horizontal orientation by drawing. Looking only on the drawings, no meaningful differences between the two educational settings can be asserted, except in drawing objects from everyday life or other shapes ("own inventions"). Although the English children were instructed in school and thus able to explain or define a triangle in most cases, their explanations did not always go in line with their drawings, presumably because they just knew the definition but could not connect it with a variety of representations.

CONCLUSION

Therefore, it can be concluded that instead of an isolated memorising of definitions and the limited use of only prototypical representations, which can rule children's thinking throughout their lives (Sarama & Clements, 2009, p. 216), already in preschool the focus should be more on the ability to connect a concept with many different representatives as examples. Teaching definitions should not be separated from showing different examples as well as from drawing shapes, otherwise it will be quite one-sided. Especially the drawing of triangles could be used not only as an assessment or research tool (as it was used here as well as in the studies before) but also as a teaching tool in everyday situations as well as in all kinds of teaching situations, because by drawing different triangles, different attributes can be easily demonstrated and consequently the drawing of triangles could help to build valid perceptions.

References

Battista, M. T. (2007). The development of geometric and spatial thinking. In F. K. Lester (Ed.), *Second handbook of research of mathematics teaching and learning* (pp. 843-908). Charlotte: Information Age.

Burger, W. F., & Shaughnessy, J.M. (1986). Characterizing the van Hiele levels of development in geometry. *Journal for Research in Mathematics Education*, *17*(1), 31-48.

Clements, D. H., & Sarama, J. (2009). *Early childhood mathematics education research: Learning trajectories for young children.* New York, London: Routledge.

Clements, D. H., Swaminathan, S., Hannibal, M. A. Z., & Sarama, J. (1999). Young children’s concepts of shape. *Journal for Research in Mathematics Education*, *30*(2), 192-212.

Freudenthal, H. (1983). *Didactical phenomenology of mathematical structure: Mathematics education library.* Dordrecht: Reidel.

Kläger, M. (1990). *Phänomen Kinderzeichnung: Manifestation bildnerischen Gestaltens*. Baltmannsweiler: Schneider.

Oswald, H. (2010). Was heißt qualitativ forschen? In B. Friebertshäuser, A. Langer, & A. Prengel (Eds.), *Handbuch Qualitative Forschungsmethoden in der Erziehungswissenschaft* (pp. 183-201). Weinheim, München: Juventa.

Piaget, J., & Inhelder, B. (1967). *A child's conception of space*. New York: Norton.

Razel, M., & Eylon, B. (1990). Development of visual cognition: Transfer effects of the Agam program. *Journal of Applied Developmental Psychology*, *11*, 459-485.

Schuster, M. (2000). *Psychologie der Kinderzeichnung* (3. überarbeitete Auflage). Göttingen: Hogrefe-Verlag.

van Hiele, P., & van Hiele, D. (1986). *Structure and insight: A theory of mathematics education*. New York: Academic Press.

Vollrath, H.-J. (1984). *Methodik des Begriffslehrens im Mathematikunterricht*. Stuttgart: Klett.