PRIOR TO SCHOOL MATHEMATICAL SKILLS AND KNOWLEDGE OF CHILDREN LOW-ACHIEVING AT THE END OF GRADE 1

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Recent psychological studies as well as research findings in mathematics education highlight the significance of early number skills for the child's achievement in mathematics at the end of primary school. In this context, first results from an ongoing four-year longitudinal study are reported. The study investigates the development of early numeracy understanding of 408 children from one year to school entry until the end of grade 2. The study seeks to identify children that struggle with respect to their mathematics learning after the first year of school mathematics and compare their performance with their number concept development one year prior to school as well as immediately prior to school entry.

INTRODUCTION

In their play, their everyday experiences at home, and in child care centres, children start developing mathematical knowledge and abilities a long time before entering formal education (e.g. Anderson, Anderson & Thauberger, 2008). However, the range of mathematical competencies children develop prior to school varies quite substantially. While most pre-schoolers manage to develop a wide range of informal knowledge and skills, there is a small number of children who tend to struggle with the acquisition of basic number-skills (e.g. Peter-Koop & Grüßing, 2014). Clinical psychological studies suggest that these children potentially at risk in learning school mathematics can already be identified one year prior to school entry by assessing their number concept development (e.g. Krajewski, 2005). These children benefit from interventions prior to school helping them to develop a foundation of knowledge and skills (e.g. Peter-Koop & Grüßing, 2014). This seems to be of crucial importance as findings from the SCHOLASTIK project (Weinert & Helmke, 1997) indicate that students who are low performing in mathematics from the beginning of primary school tend to stay in this position.

THEORIES ON NUMBER CONCEPT DEVELOPMENT

Research and curricula increasingly stress the importance of students' early engagement with sets, numbers and counting activities for their number concept development. Clements (1984) classified alternative models for number concept development that deliberately include early counting skills as skill integrations models. While Piaget emphasized that the understanding of number depends on operational competencies and that counting exercises do not have operational value and in this respect no conductive effect on conceptual number competence, research findings

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suggests the development of number skills and concepts result from the integration of number skills, such as counting, subitizing and comparing (Fusion, Secada, & Hall, 1983; Clements, 1984).

Krajewski & Schneider (2009) provide a theoretical model that is based on the assumption that the linkage of imprecise nonverbal quantity concepts with the ability to count forms the foundation for understanding several major principles of the number system. Their model depicts the acquisition of early numerical competencies via three developmental levels. On the first level (*basic numerical skills*) number words and number word sequence are isolated from quantities. Children compare quantities by using comparatives like "less", "more" or "the same amount". At the age of three to four years most children start to link number words to quantities and hence enter the second level (*quantity number concept*). The understanding of the linkage between quantities and number words is acquired in two phases: (a) an *imprecise quantity to number-word linkage* (e.g. 3 is "a bit" while 8 or 20 is "much" and 100 is "very much"), and (b) the *precise quantity to number-word linkage*, where quantity discrimination is based on counting. At this level children gain experiences with non-numerical relations between quantities as they increasingly understand part-whole and increase/decrease schemata (Resnick, 1989). At the third level (*linking quantity relations with number words*) children understand that the relationship between quantities also takes on a number-word reference. They realise that numerically indeterminate quantities can be divided into smaller amounts, and understand that this can also be represented with precise numbers. Furthermore they discover that two numerical quantities differ by a third numerical quantity. However, Krajewski and Schneider (2009) stress that children are not necessarily at the same developmental stage with respect to number words and number symbols and that the use of manipulatives also effects the children's performances on the different levels. Hence, with respect to their numerical development, it is very difficult to assign children exactly to one level.

In summary, Krajewski (2008) states that the quantity-number-competencies that children develop up to school entry build the foundations for their later understanding of school mathematics. While competencies on the third level reflect first computation skills and in this respect initial arithmetic understanding, the first to levels can be accounted as "preparatory mathematical skills" (ibid).

EARLY NUMBER-QUANTITY COMPETENCIES AND THEIR INFLUENCE ON LATER SCHOOL MATHEMATICS LEARNING

In their longitudinal study Krajewski & Schneider (2009) investigated the predictive validity of the quantity-number competencies of these developmental levels for mathematical school achievement. Their results indicate that quantity-number skills related to the second level measured in kindergarten predict about 25% of the variance in mathematical school achievement at the end of grade 4. Moreover, a subgroup analysis indicated that low-performing fourth-graders had already shown large deficits in their early quantity-number competencies. It can be concluded that these early

quantity-number competencies constitute an important prerequisite for the understanding of school mathematics.

An intervention study by Peter-Koop & Grüßing (2014) with a pre-/post-test design (one year prior and immediately before school entry) and follow-up tests at the end of grades 1 and 2 suggests that an eight months intervention had a long-term effect lasting until the end of grade 1. Children in the treatment group demonstrated increased skills in the areas addressed in the intervention, i.e. knowledge about numbers and sets as well as counting abilities, ordinal numbers, and part-whole-relationships. A total number of 854 children performed on a standardised test as well as an individual interview one year prior to entering grade 1 and the analysis of their results lead to the identification of 73 children potentially at risk learning school mathematics that took part in the intervention. Children with a migration background who speak at least one other language than German at home were overrepresented in the group of pre-schoolers potentially at risk learning school mathematics. This group, however, demonstrated the highest increases in their performance within the treatment group.

Since the study lacks a control group (due to missing parental consent with respect to their children not being given the opportunity to take part in the intervention group) it could not be investigated how many of the children identified to be potentially at risk learning school mathematics based on their number concept development one year prior to school would have shown at least an average performance at the end of grade 1 without participating in the intervention. Hence, the number concept development of 5- to 8-year old children in the transition from kindergarten to school is addressed in the ongoing longitudinal study (2011 – 2014) that is reported in this paper. In contrast to the previous intervention study, this study is recursive in nature, i.e. it seeks to identify the low-performing students at the end of grade 1. The longitudinal data from standardised tests and one-on-one early numeracy interviews one year prior to school and immediately before school entry is (and will further be) analysed to investigate whether these children already showed lower performance with respect to sets, numbers, quantities and counting than their better achieving peers in grade 1. It will further be analysed which areas these children – in contrast to their peers – did struggle with prior to school. The main questions addressed in the study are:

- Which children perform clearly below average at the end of grade 1?
- Which content areas do they struggle with the most?
- How did they perform one year prior and immediately before school entry?
- Which content areas did they struggle with the most prior to school?

METHODOLOGY

The data collection involves four measuring points (MP1 – MP4), i.e. one year prior to school, immediately before school entry, end of grade 1 and grade 2 (which will be conducted in June 2014). At each measuring point the children performed on both a standardised test on number concept development that is suitable for their respective age (OTZ, DEMAT 1+ / 2+) as well as on a not standardised task-based one-to-one

interview (EMBI-KiGa, EMBI) that focuses on the strategies that children apply on mathematical tasks or problems. Table 1 provides an overview of the study design.

Measuring points	Instruments	Participants
June 2011 MP 1	OTZ	children participating in the study (n = 538)
	EMBI-KiGa	children participating in the study (n = 538)
June 2012 MP 2	OTZ	children participating in the study (n = 495)
	EMBI-Kiga	children participating in the study (n = 495)
June 2013 MP 3	DEMAT 1+	all grade 1 classes with children participating in the study (n = 2250)
	EMBI	children participating in the study (n = 408)
June 2014 MP 4	DEMAT 2+	all grade 2 classes with children participating in the study
(to be conducted)	EMBI	children participating in the study

Table 1: Measuring points, instruments and number of participants in the study (for a detailed description of the instruments see Peter-Koop & Grüßing, 2014)

At MP3 and MP4 the whole learning group of children in the study is tested in order to compare the children's performance to their peers' and to diminish intra- and inter-group effects. Since the data collection is still in progress the analyses in this paper are only based on data from MP1 to MP3 while the whole learning group data has not been analysed yet. More detailed and complex analyses will be conducted after the completion of the data collection. For a total of 408 children (206 male, 202 female) complete data sets from the first three measuring points are available and provide the basis for the following initial analyses. Whereas 215 children (52.7%) in the sample only speak German at home, 193 children with migration background (47.3%) speak at least one language other than German at home.

In order to analyse the differences between the performances of the low-performing group of first-graders and their peers mean value comparisons have been computed using t-tests for independent samples.

RESULTS

Identification of low-performing children in the sample

In order to identify the children in the sample who are low performing at the end of grade 1 a cross mapping of the results in the standardised DEMAT 1+ and the EMBI-Interview was used to eliminate the children with low performance in only one of both tests. In this respect the DEMAT 1+ values provided a pre-selection of the lowest 20%, which was further validated with the children's performance on the EMBI. As a result 49 children (12% of the overall sample n=408) performed low in both, the standardised test and the interview (lowest 16% in EMBI scores) as well. This group of 49 children provides the basis for all further analyses.

In comparison with the complete sample children with non-German-speaking background are significantly ($p<0.001$) overrepresented in the group of

low-performers (35 of 49 children, i.e. 71.4%), while there is no major difference in the sex distribution (21 male, 28 female) to the overall sample.

Performance on the DEMAT 1+ subtests and EMBI interview parts

With respect to the standardised DEMAT 1+ the group of low-performing first-graders scored significantly ($p<0.001$) lower in all nine DEMAT 1+ subtests. This also holds true for their results on all four interview parts of the EMBI. On average the low-performers scored one to two points less in each of the four interview domains (see Figure 1). Apart from domain A (*counting*) the low- performing first-graders only get assigned the first point in each domain. The biggest difference between both groups is found in domain C (*strategies for addition and subtraction*), in which the mean difference accounts for more than two points.

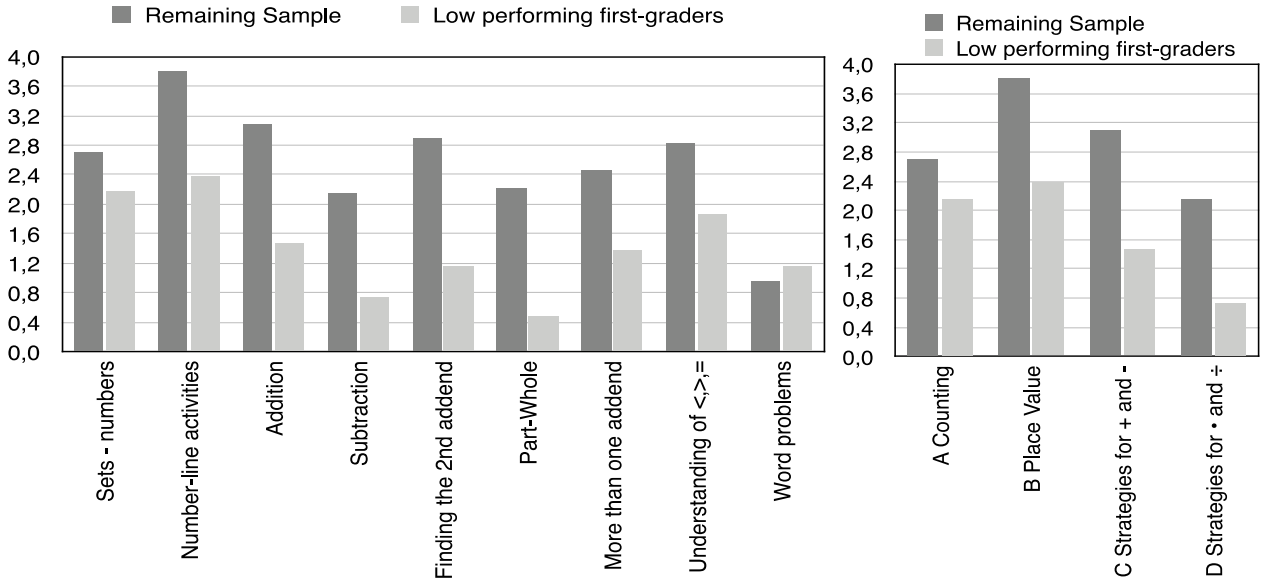

Figure 1: DEMAT 1+ subscales (left) and EMBI mean scores at MP 3

Performance prior to school (MP 1 and MP2)

The analysis of the data from MP1 and MP2 showed that the group of low- performing first-graders already performed lower prior to school entry. Their total scores on the OTZ (MP1: Low-performing first-graders: Mean: 12.96, SD: 5.156 – Remaining sample: Mean: 21.04, SD: 6.889; MP2: Low-performing first-graders: Mean: 21.92, SD: 5.235 – Remaining sample: Mean: 29.67, SD: 5.473) and their total scores on the EMBI-KiGa (MP 1: Low-performing first-graders: Mean: 3.159, SD: 1.775 – Remaining sample: Mean: 6.632, SD: 2.230; MP 2: Low-performing first-graders: Mean: 6.693, SD: 1.978 – Remaining sample: Mean: 8.972, SD: 1.337) show significant ($p < 0.001$) differences. While the overall scores at MP2 have improved for both groups as it was expected, the significant difference between the mean scores of both groups remains at an average difference of 2 points.

Analysis of the performance with respect to the different content-specific items in the EMBI-KiGa

The results on the EMBI-KiGa show that the group of low-performing first-graders performs significantly ($p<0.001$) worse in all content specific items apart from *one-to-one correspondence* ($p>0.1$) one year prior to school.

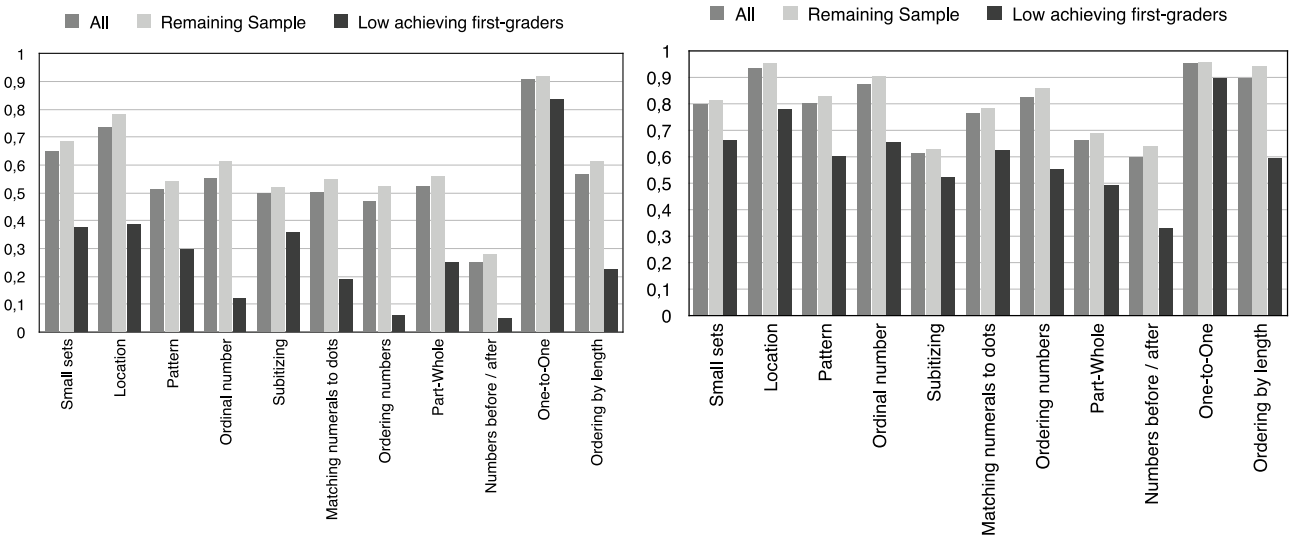

Figure 2: EMBI-KiGa subcategory mean scores MP1 (left) and MP2 (right)

They severely struggle with naming *numbers before and after* (mean=.051), *ordering numbers 0 to 9* (mean=.063) and *ordinal number* (mean=.122). While the group of low-performing first-graders showed overall improvements in all categories of the EMBI-KiGa from MP1 to MP2, they still score significantly ($p<0.001$) lower than their peers and there is still a major difference on their performance in the areas *numbers before/after* (mean=.326), *part-whole* (mean=.489), *ordering numbers* (mean=.551) *ordering by length* (mean=.591) and *ordinal number* (mean=.653).

DISCUSSION AND IMPLICATIONS

The first analyses of the data collected in MP1 to MP3 suggest that low-performing first-graders already demonstrate a significantly lower understanding of sets and numbers as well as less elaborate counting skills than their peers at both measuring points prior to school. Again, children with a migration background are overrepresented in this group (see Peter-Koop & Grüßing, 2014). At the end of grade 1 their performance is significantly lower in all subtests (DEMAT 1+) and in all content domains (EMBI). With respect to the DEMAT 1+ they particularly struggle with items on *subtraction, part-whole relationships, addition with more than one addend*, and *finding the second addend*. The subtests on *part-whole relationships, subtraction, addition with more than one addend*, *and word problems* proved to be the most difficult items for their better performing peers.

In contrast to the DEMAT 1+ that focuses on correct results, the EMBI seeks to identify strategies that children apply on mathematical tasks and problems. In this

perspective the identified group of low-performing first-graders demonstrates less elaborate strategies for addition and subtraction. This is in line with their understanding of number and their number skills prior to school. In order to solve problems such as 8+6 other than counting, an understanding of part-whole schema is required in order to add up to 10 and then on (8+2+4). While they still struggle with part-whole relationships in grade 1, they already demonstrated less insight into this concept than their peers prior to school. In addition, the low-performing first-graders demonstrate less insight in counting procedures and place value. This implies that their better achieving peers show significantly more elaborate knowledge and skills with respect to high numbers. In how far this can be compensated at the end of grade 2 so far remains unclear.

It is important to note that the group of low-performing first-graders experience special difficulties with respect to items that require elaborate language skills, i.e. *language of location, numbers before/after*, and ordinal numbers. This might explain the overrepresentation of children from a non-German-speaking background in this group. However, since the assessment of German language competencies has not been included in the study design, this concern needs further investigation.

Prior to school the lower-performing first-graders demonstrated significantly less knowledge and understanding of number symbols, which suggests that their command of the German language might only be one factor among others that would explain why they tend to struggle with the development of number skills and counting a lot more than their peers.

However, as the comparison of the results on the EMBI-KiGa suggests this group of children does improve from MP1 to MP2. Immediately before school entry they show about the same average scores as their peers did one year before school entry. This complies with findings from Aunola et al. (2004), who describe cumulation effects of number-related knowledge and skills deficits prior to school, i.e. pre-schoolers who only demonstrated weak competences in dealing with numbers and sets showed a slower development of their mathematical competencies in primary school with an increasing gap towards their peers who started school with higher number skills and knowledge.

In summary the initial results of the study in progress that are reported in this paper confirm previous findings that understanding and skills with respect to number and counting are important precursors for later achievement at school. The children that were identified as low-performers at the end of grade 1, prior to school demonstrated significantly lower knowledge and skills than their better achieving peers. However, these results provide only first insights into the development of number and counting skills. Further in-depth analyses of the individual development of the children will help to better understand and describe the factors that explain the difference in achievement in the transition from kindergarten to school.

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