A LONGITUDINAL ANALYSIS OF THE RELATIONSHIP BETWEEN MATHEMATICS-RELATED AFFECT AND ACHIEVEMENT IN FINLAND

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In this paper, a nationally representative longitudinal data of mathematics learning outcomes in Finland is analyzed in order to determine the direction of causality between mathematics-related affect and achievement. First, the results indicated that students' mathematical achievement, emotion, and self-efficacy were significantly stable over time. Different models were estimated to test the reciprocal relationship between students affect and achievement. The results indicated that mathematics achievement and self-efficacy have a reciprocal relation, where the dominant effect is from achievement to self-efficacy. The results indicate also a weaker unidirectional effect from achievement to emotion.

INTRODUCTION

Emotions, attitudes and motivation play an important role in contemporary research on mathematics education. Attitudes and motivation are important, because they determine how much people choose to study mathematics after it becomes optional and in many countries the society has a shortage of mathematically educated persons in scientific and technical fields. Moreover, the needs of society increasingly emphasize creativity, problem solving, and other higher-level cognitive processes, which are intrinsically intertwined with emotions. Although it is well known that mathematics-related affect and achievement are related, we do not yet understand well enough how these develop in interaction with each other.

In this study, we shall focus on two affective traits (enjoyment and self-efficacy) and their relationship with achievement in mathematics. There is much evidence for the positive correlation between these three (e.g. Hannula & Laakso, 2011; Roesken, Hannula & Pehkonen, 2011), but there is need to study further their interaction and development throughout the school years. The present paper will analyze the longitudinal development of mathematics-related achievement, enjoyment and self-efficacy in the Finnish comprehensive school.

Several studies have explored the relationship between mathematical affect and achievement. Ma and Kishor (Ma & Kishor, 1997a, b; Ma, 1999) have summarized much of that research in their meta-analyses. They found a negative correlation between mathematics anxiety and achievement that was consistent over gender groups, age groups and ethnic groups (Ma, 1999). Their results suggested that there would be a causal direction from liking mathematics to achievement in mathematics (Ma & Kishor, 1997a) and the positive correlation between self-concept and achievement in

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mathematics was found to decrease as students grow older (Ma & Kishor, 1997b). International comparative studies have also produced large databases for modeling the causal relationships of different variables. Williams & Williams (2009) studied the relationship between self-efficacy and performance in mathematics for 33 nations, finding a good fit to the data in 30 nations and support for a reciprocal determinism in 24 of these. Their results for Finland showed that the effect from mathematical self-efficacy to achievement is statistically significant, but small in comparison to most other countries, and the effect in the opposite direction is one of the largest in OECD. A longitudinal study in Finnish comprehensive schools (from grade 5 to grade 6 and from grade 7 to grade 8) suggested that the main causal direction would be the opposite: from self-efficacy to achievement. However, for the older subsample a significant effect was also found among female students from achievement to self-confidence, supporting the hypotheses of a reciprocal linkage (Hannula, Maijala & Pehkonen, 2004).

The longitudinal studies analyzing the relationship between affective and cognitive variables in mathematics are still few in number. When student socio-economic status, openness and conscientiousness of Italian students were controlled, the cross-lagged effects form self-efficacy (at the age of 13) to achievement (at the age of 16) was on the same level as the effect from achievement (at the age of 13) to self-efficacy (at the age of 16) (Capara, Vecchione, Alessandri, Gerbino, & Barbaranelli, 2011). A review of eight Japanese longitudinal studies (Minato & Kamada, 1996) found no predominance of either attitude or achievement in most of the studies. However, in the few instances that predominance was found, the causal direction was from attitude to achievement. An Australian longitudinal study measured also the students' motivational orientations and found effects between self-concept and achievement to be of similar magnitude for both directions, while the causal direction for achievement and motivation was from achievement to motivation (Seaton, Parker, Marsh, Craven & Yeung, 2013). This suggests that self-efficacy rather than motivational orientation is a primary determinant for the longitudinal development of mathematical competences. In addition, a dominant causal relationship from achievement to perceived usefulness of mathematics has been found in the Longitudinal Study of American Youth (Ma & Xu, 2004).

So far, we have found no longitudinal study including measures for both self-beliefs and emotions analyzing their reciprocal relationship with achievement. However, Green, Liem, Martin, Colmar, Marsh & McInerney (2012) included all three elements in a longitudinal design to test the self-system model of motivational development. In addition to academic self-concept, positive attitude towards school (emotion) and academic achievement test, they measured three types of motivation, and three behavioral measures. The Australian high school students responded to the survey twice, within one year intervals. The analysis of the data consisted of testing alternative models for both measurements separately and only then testing the model fit for a longitudinal design. Their analysis suggested that positive attitude – possibly together with behavioral variables – mediated the effect of self-concept and motivation on academic achievement. The model where test performance would be directly influenced by all other variables was rejected due to poor model fit in the first stage of the analysis without testing it in the longitudinal design.

Summarizing the aforementioned studies, there seems to be strong evidence for a reciprocal relationship between academic self-efficacy and achievement. There is mixed evidence for the dominant direction of this relationship and for its development. With respect to the relationship between mathematics-related emotions and achievement the evidence is even less clear, but it suggests a reciprocal linkage, with the dominant direction possibly from emotions to achievement.

In the present study, we will analyze longitudinal data from Finland to study the relationships between achievement in mathematics and two affective measures: enjoyment of mathematics and self-efficacy in mathematics in a longitudinal design. Our aim is to determine the dominant direction of effect between the chosen affective variables and achievement in mathematics.

METHODS

The data of this study has been collected by the Finnish National Board of Education (FNBE) to study the long-time development of Finnish comprehensive students' mathematics-related affect and achievement from the beginning of grade 3 to the end of grade 9. A nationally representative sample of intact grade 3 classes was selected for the first measurement in 2005. The same pupils were tested again in 2008 at the beginning of their sixth grade in their intact classes, hence increasing the sample size. At this stage, we reached 80 % of the original sample. A similar selection of intact classes of previously participated students was measured again in 2012 at the end of ninth grade, when we reached 60 % of the original sample. Total number students who took part in all three-time points was 3,502 (48% female). Metsämuuronen (2013) has reported the details of the sample, procedures, and instruments in the official assessment report. For the present analysis, we included also students who participated only the first two (n = 1,050), or the last two measurements (n = 654).

Measures

The mathematics tests were composed by expert panels to measure the attainment of Finnish National Core Curricula (FNBE 2004) and the three tests shared several linking items. To make test scores comparable across grade levels, item response theory (IRT) was applied using the link items across grade levels to compute estimate test scores from each grade level to a common metric scale (see Béguin, 2000). The reliabilities were calculated for the subsample that responded to all three measures: mathematics enjoyment scale (four items, e.g. "I like to study Mathematics"; α: t1 = 0.879, t2 = 0.879, t3 = 0.885) and mathematics self-efficacy scale (four items, e.g. "Mathematics is an easy subject"; α: t1 = 0.879, t2 = 0.879, t3 = 0.885).

Statistical procedures and model fit

Using Mplus 7.11 (Muthén & Muthén, 1998-2012), latent autoregressive and cross-lagged panel models were estimated. Latent autoregressive/cross-lagged models account for random measurement error by using multiple indictors at each time point. Using the cross-lagged model the reciprocal causal relationship between mathematics enjoyment, mathematics self-efficacy, and mathematics achievement can be estimated between different measurement time points. Model fit was evaluated with several fit indices: the chi-square difference test (Satorra & Bentler, 2001), the Comparative Fit Index (CFI > 0.90), and the Root Mean Square Error of Approximation (RMSEA < 0.08), and the Akaike (AIC: lower value indicates a better fit) (Brown, 2006). Missing data patterns were handled with Mplus feature of full information maximum likelihood (FIML). Analyzes was based on the Mplus robust maximum likelihood estimator (MLR), which is robust to non-normality and to control for the non-independence of observation (Muthén & Muthén, 1998-2012).

With respect to structural relations between students' self-efficacy, enjoyment and achievement over time, we initially estimated a baseline model with autoregressive but no cross-lagged paths. To account for the indicator-specific effects seeming common in longitudinal analyses because the same indicators are repeatedly measured (Geiser, 2013; Raffalovich & Bohrnstedt, 1987), we allowed for correlations between the measurement error (residual) variables that relate to the same indicator over time (e.g., Sörbom, 1975). We also allow residual to correlates for each time point to account for shared occasion-specific effects between the constructs at the same time point (Anderson & Williams, 1992; Geiser, 2013). The results indicate that the baseline autoregressive measurement model fits the data adequately well (model 1, table 1).

Additionally, because invariance of factor loadings over time is conceptually important we tested if the factor structure of self-efficacy and enjoyment were invariant across the three time points. To test the factorial invariance, we tested a model whereby all the factor loadings on self-efficacy and enjoyment were freely estimated (configural model) across the three measurement time points ($\chi 2$ = 3248.323, df = 296, CFI= 0.957, RMSEA = 0.038) with models whereby the factor loadings were constrained equal ($\chi 2$ = 4499.981, df = 312, CFI= 0.940, RMSEA = 0.044). There was support for the factorial invariance over the three measurement time points. For all subsequent analysis the factor loadings were constrained equal.

After establishing the stability model, we specified a structural model by including a cross-lagged path of measurement time point 1 and time point 2 in order to examine possible reciprocal relations between mathematics achievement, mathematics enjoyment and self-efficacy as depicted in Figure 1.

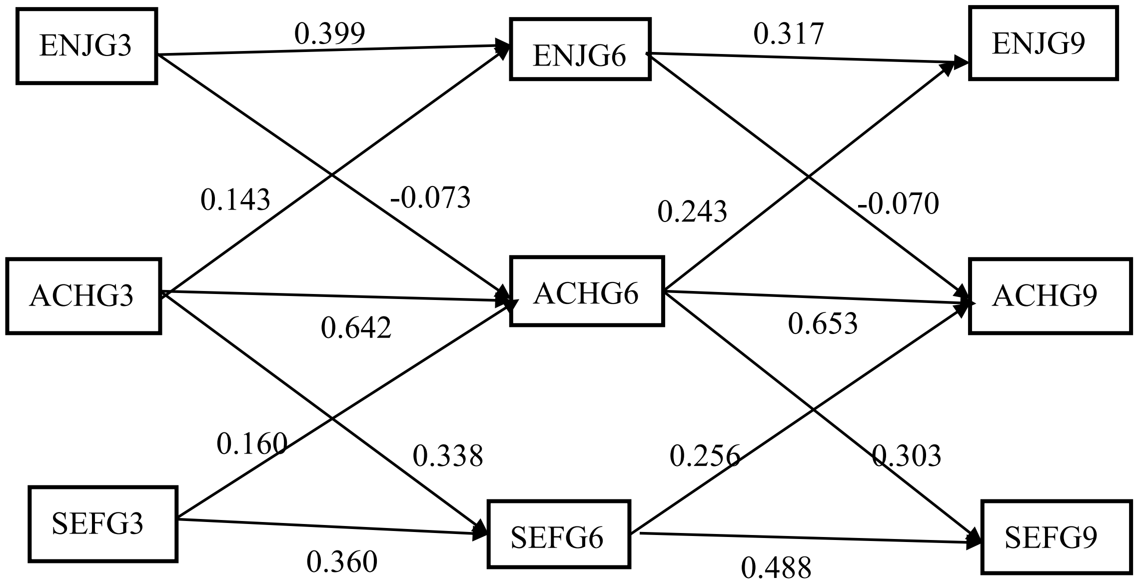

Figure 1: Latent autoregressive/cross lagged model measuring student's mathematics self-efficacy, enjoyment, and mathematics achievement on three measurement occasions (grade 3, grade 6 and grade 9). To avoid cluttering, only path estimates are shown. AchG3-AchG9 = Achievement at grade 3 to grade 9, EnjG3-EnjG9 = mathematics enjoyment at grade 3 to grade 9, SefG3-SefG9 = mathematics self-efficacy at grade 3 to grade 9.

RESULTS

Autoregressive effect of mathematics self-efficacy, enjoyment and achievement

The unidimensional path linking measurement at time point 1(grade 3) and subsequent grades is used to access the autoregressive/stability effect. As all autoregressive effects were statically significant, a significant portion of individual differences has remained stable over time. The stability effect was much stronger and consistent for mathematics self-efficacy (from $\beta = 0.360$ to $\beta = 0.488$) and mathematics achievement ($\beta = 0.642$ to $\beta = 0.653$). Moreover, the findings indicated that mathematics enjoyment at grade 3 influenced mathematics enjoyment on grade 6 but mathematics enjoyment in grade 6 had smaller impact on mathematics enjoyment at grade 9.

Cross-lagged effect between mathematics self-efficacy, enjoyment and achievement

Nonetheless, individual students' differences were not perfectly stable over time. This was further tested by comparing models with and without cross-lagged effects (table 1). First, the model without the cross-lagged structural path (M1) was compared with models with cross-lagged from students' achievement to students' affects (M2), and from affects to achievement (M3). As seen in table 1, the model fit and chi-square difference test indicated that models with cross-lagged effects account for the data better than the model without them. The model with all the cross-lagged effect (M4) was practically and significantly better than any other of the tested models. The comparison (AIC) between models M2 and M3 indicates a better model fit for M2. Also from the bidirectional cross-lagged model (M4), we can see that the cross-lagged effect from mathematics achievement to affects (β = 0.143-0.338) were larger than the corresponding cross-lagged effects from affect to achievement (β = -0.073- 0.256). These findings suggest that the longitudinal effect from achievement to affect is stronger than the effect to the opposing direction. Overall, the cross-lagged effects were consistently smaller in size compared to the autoregressive coefficients, indicating that cross-lagged effect was less important than the stability of all three measured variables.

Model	MLRχ^2	*df*	CFI	RMSEA	AIC	Comparison
No cross-lagged (M1)	4499.981	312	.940	.044	327912.090	
Cross-lagged from						M1 vs. M2
ACH_{T1}-$Affect_{T2}$	3503.168	308	.954	.039	326854.238	$\Delta\chi^2 = 990.001$,
ACH_{T2}-$Affect_{T3}$ (M2)						$\Delta df = 4$
$Affect_{T1}$-Ach_{T2}	4064.330	308	.946	.042	327460.032	M1 vs. M3
$Affect_{T2}$-Ach_{T3} (M3)						$\Delta\chi^2 =478.595$,
						$\Delta df = 4$
All cross-lagged	3187.793	304	.958	.037	326528.064	M1 vs. M4
paths (M4)						$\Delta\chi^2 = 1348.144$,
						$\Delta df = 8$
						M2 vs. M4
						$\Delta\chi^2 = 334.070$,
						$\Delta df = 4$
						M3 vs. M4
						$\Delta\chi^2 = 842.432$,
						$\Delta df = 4$

Table 1: Goodness-of-fit indices and chi-square difference tests of models tested. ACH= Achievement, Affect = mathematics enjoyment and mathematics self-efficacy, T1 = Time 1=Grade 3, T2 = Time 2= Grade 6, T3 = Time 3 =Grade 9. S = Scaling Correction Factor, CFI = Comparative fit index, RMSEA = Root Mean Square Error Of Approximation, robust maximum likelihood estimator (MLR), Akaike (AIC)

The correlations between the residual variables between the mathematics achievement, self-efficacy and enjoyment were statistically significant and small to medium (rs = 0.165- 0.410, ps < 0.001), but the correlations between mathematics self-efficacy and enjoyment were higher (rs = 0.647- 0.739, ps < 0.001). This indicated that a high amount of shared situation-specific effects influence the self-efficacy and mathematics enjoyment constructs at the same measurement time

DISCUSSION

The results of this longitudinal study support the view that mathematical self-efficacy and achievement are reciprocally linked and that the dominating direction of this relationship is from achievement to self-efficacy. Such a relation between self-efficacy and achievement could be characterized as evidence-based development of self-efficacy beliefs. Previous studies (Williams & Williams, 2010) suggest that this direction of the relationship may be characteristic for Finland. However, it should be noted that the effect of self-efficacy on achievement was larger for the older students. This supports the earlier hypothesis (Hannula, Maijala, & Pehkonen, 2004) that there might be a developmental trend from achievement-dominated relationship to a reciprocal relationship, which would eventually become a relationship dominated by self-efficacy beliefs.

Results of our study do not support the model suggested by Green et al. (2012), where the causal relation of these three variables would be from self-efficacy to achievement through emotions. In our data the cross-lagged effect was primarily from achievement to self-efficacy and we also found a unidirectional effect from achievement to emotions.

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