ON THE STRUCTURE OF SECONDARY HIGH-SCHOOL TEACHERS´ BELIEF SYTEMS ON CALCULUS

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A teacher's instructional planning that is enacted in his classroom practice and that potentially impact on his students' knowledge and beliefs could be understood as an individual belief system dependent from his actual teaching and learning experience. Individual belief systems might be contradictory when we regard different teachers or one teacher concerning different mathematical disciplines. For this reason, this report focuses on thirty teachers´ beliefs about their teaching of a specific mathematical domain, i.e. calculus that is a central part of the (German) curriculum at upper secondary level. After a brief outline of the theoretical framework and methodology of this research project, results of the qualitative reconstruction of different aspects of teachers´ belief systems on calculus will be explained.

INTRODUCTION

Beliefs concerning both mathematics and teaching and learning of mathematics are a crucial part of the professional competence of mathematics teachers (Felbrich et al., 2012). The importance of gaining knowledge towards mathematics teachers' thinking or beliefs has been emphasised by many researchers in mathematics education in various settings and projects because teachers' beliefs about mathematics and the teaching and learning of mathematics have a high impact on their instructional practice (Philipp, 2007; Eichler, 2011, Felbrich et al., 2012), and, potentially impact on their students' learning (Stein et al. 2007). However, the vast body of research on teacher beliefs rarely considers that similar to the classification of mathematical subjects into fields such as algebra or probability theory – teachers´ beliefs on different mathematical domains such as geometry, stochastics or calculus may vary and may be associated with specific beliefs (Franke et al., 2007).

For this reason we focus on domain-specific beliefs of 30 secondary teachers referring to calculus, which is a central part of the German secondary curriculum, and the teaching and learning of calculus. Our specific interest in this paper concerns the structure of belief systems, i.e. the set of beliefs and different relations between beliefs that characterise calculus teachers' instructional planning (Eichler, 2011). Before we address the aforementioned reconstruction and relations, an outline is given about the theoretical framework of this research project and a brief description of those parts of the method being relevant for this paper. Finally we conclude the paper by reflecting on the main results and discuss possible directions of further research.

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THEORETICAL FRAMEWORK

The main constructs of our theoretical framework are teaching goals and teachers' beliefs. Firstly, according to Pajares (1992), we understand the term beliefs as an individual's personal conviction concerning a specific subject, which shapes an individual's ways of both receiving information about a subject and acting in a specific situation. We further follow Green (1971) referring the internal organisation of beliefs in a belief system involving the distinction of central beliefs, i.e. strongly held beliefs, and peripheral beliefs referring to an individual's belief system of lesser importance. The construct of belief systems also involves that beliefs are organised in clusters that are quasi-logically connected, which potentially includes also connections of beliefs that seem contradictory (ibid.). Finally, Green (ibid.) distinguishes primary beliefs and subordinated (derivative) beliefs in which enacting derivative beliefs serve as a means to an end for achieving primary beliefs.

According to the framework of Hannula (2012), both belief systems and goals are parts of mathematics-related affect that consists of cognitive, motivational and affective aspects. Hannula (ibid.) further describes beliefs or rather belief systems as a psychological aspect of mathematics-related affect as a trait and, hence representing a disposition. In contrast, he describes goals as a psychological aspect of mathematics-related affect as a state. Thus, goals refer to a "decision making during teaching" (Schoenfeld, 2011, p. 460). In contrast to the distinction of affect as a trait and affect as a state, we follow the so called Rubicon-model of Heckhausen and Gollwitzer (1987) in which goals are understood in a broader sense constituting a teacher's decision making (state of awareness referring to the choice of goals) before passing the Rubicon, i.e. when a teacher plans his classroom practice, and after passing the Rubicon, i.e. the teacher's decision making during his classroom practice (state of awareness when enacting the goals).

Following this framework, we understand teaching goals as specific form of beliefs and, in the same way, a system of different but related teaching goals as a teacher's belief system. These teaching goals are developed by a teacher when he plans his classroom practice and they are potentially enacted in his classroom practice. Finally, the enacted goals could be more or less changed based on the teachers' experience referring to their classroom practice and their students' learning (Stein et al., 2007).

To describe clusters of teaching goals or rather clusters of beliefs we refer, finally, to four so called mathematical world views proposed by Grigutsch et al. (1998) that are often used to conceptualise overarching teaching goals (e.g. Felbrich et al., 2012), i.e.

- a formalist (world) view in which mathematics is characterized by a logical and formal approach and in which accuracy and precision are important.
- a process-oriented view in which mathematics is defined as a heuristic and creative activity that allows solving problems using individual ways.
- an instrumentalist view in which mathematics is seen as a collection of rules and procedures to be memorized and applied according to the given situation.

- an application oriented view that accentuates the utility of mathematics for the real world.

In their research that was based on a questionnaire and that involved 400 German secondary teachers, Grigutsch et al. (1998) yield correlations between the four aspects of their mathematical world views as described in Figure 1.

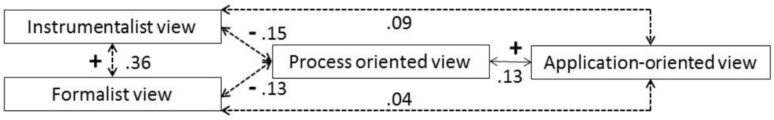

Figure 1: Correlations between the four world views.

On the basis of our theoretical framework the main focus of this paper is to describe the structure of calculus teachers' teaching goals beyond correlations, involving the identification of central and peripheral goals as well as primary and derivative goals.

METHOD

The sample for this study consists of 30 calculus teachers divided into three subsamples: 10 pre-service teachers, 10 teacher trainees and 10 experienced teachers. Since we do not focus on the development of teachers' beliefs (for this aspect see Erens & Eichler, 2013), in this paper, we make no distinction between the different grades of the teachers' experience. The teachers who participated in our study were recruited from different universities, teacher training colleges and schools across the south-western part of Germany. However, our sample is a theoretical sample (Glaser & Strauss, 1967), but not a representative sample.

We used semi-structured interviews for data collection. Topics of these interviews were several clusters of questions that concern the content of calculus teaching, the related goals, and reflections on the nature of calculus, on the possible influence of technology on the students' learning, or textbook(s) used by the teachers. Further, we use prompts to provoke teachers' beliefs. These prompts consist of fictive or real statements of teachers or students representing one of the four mathematical world views or tasks of textbooks that also represent the four world views.

For analysing the data, we used a qualitative coding method (Mayring, 2010) that is close to grounded theory (Glaser & Strauss, 1967). The codes gained by interpretation of each episode of the verbatim transcribed interviews indicate goals of calculus teaching. We used deductive codes derived from a theoretical perspective (cf. Grigutsch et al., 1998) and inductive codes for those goals we did not deduce from existing research concerning calculus education. The codings were conducted by at least two persons and we proved the interrater reliability to show an appropriate value.

RESULTS

The first step of analysing the structure of the teachers' system of goals referring to calculus was to identify central and peripheral teaching goals. We understand teaching goals to be central for a teacher if he reports these goals coherently through the whole interview and if he illustrates his goals with concrete examples of his classroom practice or concrete tasks. Since we described the process of identifying central and peripheral goals in detail elsewhere (Eichler & Erens, 2014), in this paper we only postulate different grades of centrality. Thus, we start with two central goals of Mr. P.

> Mr. P: Teaching calculus to me means to focus on the underlying concepts, discover connections between concepts und enable students to solve problems using individual ways. That´s really important to me and I would like to emphasize this point. But, as I said before, this aspect is always connected with applications on a task-level.

The application-orientation is a central overarching teaching goal of Mr. P that is in close proximity to the process-orientation. This relation between these two central goals is in line with the results of Grigutsch et al. (1998). However, referring to our whole sample, the nature of proximity of these two overarching teaching goals varies individually.

For Mr. P both views are inextricably intertwined and are, thus, *coordinated*. For other teachers application-orientated goals are *subordinated*, since for them the integration of applications as a principle of learning calculus is for reasons of student motivation:

> Mr. A.: I quite agree with the emphasis on applications in the given example. That is certainly a way to motivate them (students), but nevertheless one should not reduce genuine calculus or the teaching of calculus to that topic.

Again other teachers reckon that integrating real-world problems is an explicit part of their system of goals to which further goals are subordinated, e.g. process-oriented goals, or to which further goals are *super-ordinated*, e.g. goals representing the formalist view:

> Mr. B.: Examples for applications are quite suitable here, and with applications I always associate modelling of real data, […] increasingly introducing relevant applications into lessons may, for the students, succeed in a deeper insight into the concepts and ideas of calculus.

Although sometimes coordinated, sometimes subordinated and sometimes super-ordinated, within our data set the application-oriented and also process-oriented goals can be considered to have a certain “psychological strength” (Green, 1971, p. 47) and can thus be attributed in any case some degree of centrality in the respective teachers´ belief system. According to Green’s dimensions and the results of Grigutsch et al. (1998) one might hypothesize that particularly application-oriented goals that are central imply that teachers holding these goals rather see formalist aspects in calculus teaching as less essential or even contradict these. Though some teachers in our sample see formalist features of calculus concepts as a high barrier for student learners (mostly on a symbolical level), a general conclusion that application- or process-oriented problems are implicitly of higher importance than formality and logic cannot be drawn as the following quotations demonstrate:

Mr. A: Calculus is more than just dealing with application-oriented tasks. Then, for example, one would not regard the precision and exactness of calculus and use applications as a means to an end.

Mr. E: Problem-solving in calculus to me means: start with some kind of application in order to motivate students but then we first develop the formal and precise background we need as a sound footing before students can address more complex problems individually.

For these two teachers application-oriented goals and goals representing the formalist view are related. In the reverse direction, however, half a dozen teachers, who hold a consistent formalist view on calculus, either do not mention applications at all or mention these as a peripheral goal on the level of (given) textbook & exam tasks.

In order to reconstruct a teacher's belief system with any degree of credibility, we need various evidence emerging in different parts of the interview from which to draw these inferences. This consideration leads to the need to describe not only what a teacher believes about calculus but how the various goals are related to each other. So far we have described relations like coordination, subordination or super-ordination. However, in our data sample there is some evidence that individual teachers arrange their goals and beliefs into organized systems that make sense to them.

Mr. G1: Well, I daresay I could do calculus at school with a more theoretical and formal approach – similar to introducing concepts in algebra and topology. Maybe for some it would make things easier, but this will probably not be possible to implement in most courses

Mr. G2: I don´t emphasize the formal derivation of the integral with limits of upper and lower sums any more. From my own teaching orientation this (formal) prompt you showed me is absolutely congruous with my own approach to teach the integral. With logical rigour and formal exactness one often scares off the students. Therefore I demonstrate one example at the end but I do not let the students do these limits of sums in my lessons anymore.

Throughout the whole interview these two teachers (G1 trainee, G2 experienced) explicitly mention the central role of exactness and logical rigour as necessary ingredients of secondary level calculus courses. The two quotations however seem to confirm that different belief clusters may have a quasi-logical structure (cf. Green, p.44). The (self-)reported incongruity between instructional goals and the situation encountered in the classroom can be characterized as a conflict of goals. This incongruity may be "an observer´s perspective that does justice neither to the complexity of teaching, nor to the teachers´ attempts to relate sensibly to this complexity" as Leatham (2006, p. 95) and Skott (2009, p. 44) have tried to explain. Regarding our underlying framework, these remarks fit in with transformation process (i.e. passing the Rubicon) between intended and enacted teaching goals perceivable in the data.

As this report focuses on teachers´ beliefs towards calculus, which is, in Germany, the most central part of the mathematics syllabus at upper secondary level, teachers in our sample often mention normative aspects such as final exams which seem to have an impact on their actual teaching of calculus. Being asked to comment on the statement "I like calculus, because many exercises can be solved by similar procedures/patterns" from a student and a teacher perspective, Mr. G2 remarked:

> Mr. G2: Of course this naturally belongs to any calculus course at school level. Especially less gifted students need these rules and procedures in order to be successful in their final exams. This is the main objective for students and therefore practising these routines with exam tasks needs to be done in lessons, too. I don´t think these standardised tasks are exciting but these definitions and procedures are rather like a language that needs to be learned by students.

Taking this teacher as a paradigmatic example, it becomes apparent that the instrumentalist view is at most a peripheral goal in his belief system. The comparison of mathematical concepts and procedures to a language is somehow revealing. Derivation rules, basic skills and their application to routine tasks many teachers in our sample see as prerequisite for various reasons beyond exams: as a solid foundation for a structural basis of calculus at school level, others see the tool-box aspect as a means to an end in order to enhance their students´ competencies to solve optimization tasks. The actual classroom interaction makes teachers aware that the full spectrum of student ability (& success) needs to be considered. Whereas Mr. G2 takes the impact of these normative aspects for granted, other teachers articulate a negative attitude towards schema-orientation due to the determining factors of centralized exams. It is apparent though that for all teachers in our sample the preparation of the final exam does indeed play a certain role in their system of goals.

DISCUSSION

In this report we exclusively focused on aspects of the structure teachers´ beliefs systems. Since we expect differences among a teacher's belief systems referring different mathematical disciplines, our focus was on calculus at upper secondary level.

Firstly, we tried to identify how a teacher's central goals (beliefs) are correlated and, in some sense, why these goals are correlated. Based on Green's (1971) distinction of primary and derivative beliefs, we proposed the distinction of coordinated goals and subordinated (or super-ordinated) goals. In this distinction, a goal X is subordinated to another goal Y if X is a means to an end to (potentially) achieve Y. For example, an application-orientation for Mr. A is a means to an end for achieving students' motivation. Regarding a system of goals (or beliefs) as hierarchically arranged the subordinated goal of application-orientation of Mr. A is on a lower level than the goal of students' motivation. In contrast two coordinated goals, e.g. the process-orientation and the application-orientation in the case of Mr. P, are on the same level referring his hierarchically arranged system of goals concerning calculus teaching.

Further, we identified relations between goals that are insufficiently described by coordination or subordination, i.e. a contradiction between goals. For example, although goals representing the formalist view are central for Mr. G2, he does not intend to enact these goals since he expects to impede students' learning when enacting these goals. Thus, different goals sometimes match each other, but sometimes the system of goals seems to have a quasi-logical structure and include contradicting goals representing conflicts of goals.

Finally, a possible distinction of teaching goals refers to the derivation of these goals. For example, although for a teacher like Mr. G2 goals representing the instrumentalist view are at most peripheral, these goals play a certain role in his teaching. However, enacting these goals is not primarily a means to an end for his own central goals, but for his students' central goals referring to their final exams.

We suggest two reasons for researching the relations of teachers' goals or beliefs in detail. Firstly, our results facilitate a deeper understanding of relations between goals or beliefs beyond statistical correlations. For example, in our sample the empirical independence between an application-oriented view and a formalist view (Figure 1; $r \approx 0$) could be based on different relations between these views. Actually, some teachers value formalist goals high and neglect application oriented goals. However, for other teachers (like Mr. A) both formalist and application oriented goals are central although application oriented goals are subordinated to formalist goals. In turn, other teachers like Mr. G1 value formalist goals high, but do not intend to enact these goals.

Further, as illustrated by the above examples, the teachers' beliefs about teaching calculus can be seen as a multiple-layered hierarchical system of goals that each teacher tries to make sense of individually. This sense making could possibly throw some light on the relationship between teachers' espoused beliefs or goals and their enacted beliefs or goals, which is a difficult, but crucial relationship in educational research (Skott, 2009; Furinghetti & Morselli, 2011). For example, the teachers mentioned above show that e.g. an instrumentalist view is not a central part of their belief system though it seems to be a significant part of their classroom practice taking into account students´ learning. This somehow confirms findings of Skott that research on beliefs and their enactment needs to consider a multiple set of factors involving the inclusion of a social perspective on belief-practice relationships (Skott, 2009, p.29). Further, the distinction of central and peripheral beliefs or goals, as well as the distinction of relations between beliefs or goals – e.g. in terms of coordination and subordination – could serve as an explanation of reported inconsistencies or consistencies between espoused and enacted beliefs or goals (Skott, 2009; Eichler, 2011).

However, the mentioned relationship between teachers' espoused and enacted beliefs as well as the relationship between a teacher's classroom practice and his students' learning still needs further research to contribute to the ongoing research on mathematics-related affect.

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