EXPLORING MATHEMATICS TOGETHER: FIGURING THE WORLDS OF TEACHERS AND PROSPECTIVE TEACHERS

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This paper reports on an intentionally constructed hybrid space, the Odyssey, as an approach to address the gap between theory and practice in teacher education programs. In the Odyssey, prospective teachers and mentor teachers engaged in joint explorations of mathematics. We analyzed the interactions among participants using the concepts of figured worlds and positional identities (Holland, Lachicotte, Skinner, & Cain, 1998). Findings point to the potential of experiences such as the Odyssey to challenge the power differential that often exists between mentor and prospective teachers. Furthermore the act of engaging in mathematical activities together may encourage prospective teachers to elaborate on their mathematical explanations as well as allow mentor teachers the opportunity to (re)visit mathematical ideas.

INTRODUCTION

Teacher preparation approaches often include university courses and field experiences in nearby schools, but the connections between these two settings are not always made explicit. As a result, a rift is sometimes created that juxtaposes the theoretical aspects of teaching learned in university classes and the reality that prospective teachers (PTs) experience in their school placements. PTs are often left on their own to mediate potentially conflicting messages that they get from university faculty and the mentor teachers (MTs) in the local schools.

The study presented here is part of a larger project that was designed to bridge these different settings (university courses and field experiences) through several joint activities. These included MTs visiting the methods courses, PTs working in the MTs' classrooms, MTs and PTs jointly interviewing children on mathematical thinking, and the Odyssey, a summer institute where PTs and MTs engaged in the practices of mathematics and science. Many of these common experiences are likely to reflect an expert-novice differential, particularly in terms of pedagogy, where the MTs are the experts and the PTs the novices. The Odyssey, however, with its focus on participants doing mathematics and science, offers a very different kind of experience, one in which the expert-novice differential is not based on an MT-PT distinction. Hence, in this study we explore the potential of environments such as the Odyssey to make connections between field experiences (practical) and university courses (theoretical) through a learning experience that is more egalitarian in nature.

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THEORETICAL FRAMEWORK

The design of the overall project is grounded on a third-space framework (Moje, Collazo, Carrillo, & Marx, 2001) where prospective teachers, mentor teachers, university faculty and content specialists work together in common spaces to discuss issues of content and pedagogy. The third-space framework in this work attempts to define new, hybrid spaces where various perspectives on teaching converge to create new understandings on the part of all of the participants about what it means to teach. Operating in the third space serves to bring academic and practitioner knowledge together in ways that are less influenced by traditional power relationships that divide these discourses. This in turn opens up new learning opportunities for prospective teachers (Zeichner, 2010). In order to interpret the social interactions displayed in these hybrid spaces, we have turned to Holland's idea of figured worlds (Holland, et al., 1998) to describe within group interactions.

Figured Worlds, Local Spaces of Practice, and Positional Identities

A figured world is "a socially and culturally constructed realm of interpretation in which particular characters and actors are recognized, significance is assigned to certain acts, and particular outcomes are valued over others" (Holland et. al., 1998, p. 52). They are "as-if" or virtual realms in which persons become indoctrinated to the norms of the figured world through continual participation with other actors within the realm. Figured worlds are continuously defined and redefined by the everyday actions that occur within them. While one might describe typical behaviors within these worlds, it is important to recognize the behavioral variations that can occur. It is also important to know that multiple figured worlds are often present, though not necessarily all at the same time.

Given that multiple figured worlds are often present at any given time, it is critical to investigate the way they interact. Holland and Lave (2000) refer to the social context of this interaction as the local space of practice. This space is socially and historically situated, a real-world setting that exists in a particular place and time. As such, this local space of practice at least partially determines the presence and magnitude of particular figured worlds within a space. In the study presented here, the Odyssey was the local space of practice.

Since individuals may, and often do enact multiple figured worlds within a local space of practice, there exists the likelihood that these figured worlds may come into conflict given a particular setting. In the case of the Odyssey, a number of figured worlds came into play. The university faculty asked PTs and MTs to engage in Odyssey activities as "doers of mathematics", hence invoking the figured world of the discipline of mathematics. Throughout the Odyssey, PTs and MTs also enacted the figured worlds of the elementary school classroom as well as that of university courses.

In each of the different figured worlds that are present, there exist interpersonal power relationships that affect one's participation in the local space of practice. Holland et al. (1998) refer to these differences in position relative to other group members as

positional identities. Positional identities have to do with "the day-to-day and on-the-ground relations of power deference and entitlement, social affliction and distance – with the social interactional, social-relational structures of the lived world" (p. 127). In essence, positional identity refers to the awareness of a person's social position within a figured world. The local space of practice combined with the prominence of the various figured worlds evident within, allow for participants to take up and assert different positional identities.

In mathematics education, positionality has been looked at as a way of interpreting power relationships between students in the classroom. Esmonde and Langer-Osuna (2013) investigate differences in engagement in mathematical activities as students are given the opportunity to take up different positions within a local space of practice that contains multiple figured worlds. In the Odyssey, we have extended this investigation to interactions between PTs and MTs within the context of a teacher education program. As the participants navigate the various figured worlds present, they often modify the way they position themselves as they negotiate what it means to practice mathematics within these figured worlds. In what follows we look at these shifts in position as opportunities for participants to make connections between the theoretical and the practical aspects of teaching mathematics.

METHOD

In the mathematics portion of the Odyssey (which is the focus of this study), participants were encouraged to think mathematically about the problems presented in the seminar as "doers of mathematics" rather than thinking about teaching considerations such as the way that children might take up these tasks. The problems focused on pattern exploration and on generating and justifying general rules (one set of problems led to $n(n-1)/2$ and the other to 2^n). Data for this study consist of video recordings of four groups of PTs and MTs as they worked on the different problems. There were more MTs than PTs at the Odyssey, but each of the four groups of four in our study had at least one PT. One camera was placed at one end of the table and a flat microphone was placed in the center of the table to record as much of the interaction as possible. In addition to the video recordings, participants' notebooks were collected. Researcher field notes were also referred to at times to clarify the context of the interactions when needed. Using Powell, Francisco and Maher's (2003) model for video analysis, we identified critical events relating to our research goal. The research team watched the videos specifically looking for instances where we noticed changes in either how group members were being positioned, or how they were positioning themselves. These critical events became clips and were transcribed. The transcriptions are the data for the analysis. A first pass through the data led to two codes related to the form of engagement with the task: doers of mathematics and teachers. Codes for this study were developed around the idea of Holland et al. (1998) of positional identities.

Within these broadly defined categories of doers of mathematics and teachers, several positions emerged in our analysis. Within the category of doers of mathematics, we identified: 1) *mathematical expert*: when a participant defers to another for a mathematical explanation or for help with a problem; (2) *sense maker*: engages in the activity in ways that go beyond simply obtaining an answer. The sense maker is not only hoping to accomplish the task, but also gain insight and understanding of underlying processes; 3) *rule oriented*: engages in the task in a formulaic or procedural nature. The rule oriented focuses on coming up with the general expression but is less concerned with explanations and understandings; 4) *resistor*: pushes the other members of the group to revisit or make sense of the work they are doing, hence resisting the status quo and often instigating a new direction in the exploration of the problem.

Similarly, within the category of teacher, we identified three different positions: 1) *pedagogical expert*: most often enacted by a MT in interaction with a PT. In these interactions MTs draw upon their classroom experience and share their pedagogical knowledge with the PT(s); 2) *professional colleague*: most often enacted between multiple MTs (but could involve PTs). Participants make observations and connections related to elementary school teaching practices; 3) *teacher-to-be*: a position taken up by PTs in interactions with MTs around possible connections between the task at hand and the elementary classroom (e.g., around manipulative materials).

RESULTS

This study sought to explore the potential of spaces such as the Odyssey to address the disconnect that often exists between university-based and field-based experiences in teacher preparation. We first give an overview of the main findings and then illustrate some of them in more detail. Throughout the Odyssey, MTs and PTs engaged in the problems as colleagues thereby invoking a symbiotic relationship. For MTs the Odyssey allowed them to explore and learn content as learners of mathematics, an opportunity that is often more difficult to achieve in the figured world of classroom teaching. In this context, MTs seemed to turn to PTs as mathematical experts, arguing that PTs had had more recent experiences with mathematics through their university courses. PTs were actively engaged in the mathematical tasks, often starting in a *rule oriented* position but switching to *sense makers* in the course of their interaction with group members. MTs as *resistors* seemed to facilitate this switch. PTs were pushed to explain the "whys" of the mathematics behind the formulas to assist MTs in making sense of the group's work. Hence, PTs seem to have gained valuable experience as mathematical explainers.

Through this symbiotic relationship, we argue that participants are afforded various opportunities to navigate through different figured worlds. Figure 1 (below) shows the potential path that participants might take through an Odyssey interaction. We focus on three figured worlds: the elementary school classroom, the university courses, and the discipline of mathematics. Participants may travel along a meandering path in and out

of the various intersections of figured worlds. They may also travel outside these three figured worlds altogether into any one of a number of figured worlds that are less immediately present all the while attempting to make sense of what it means to understand mathematics.

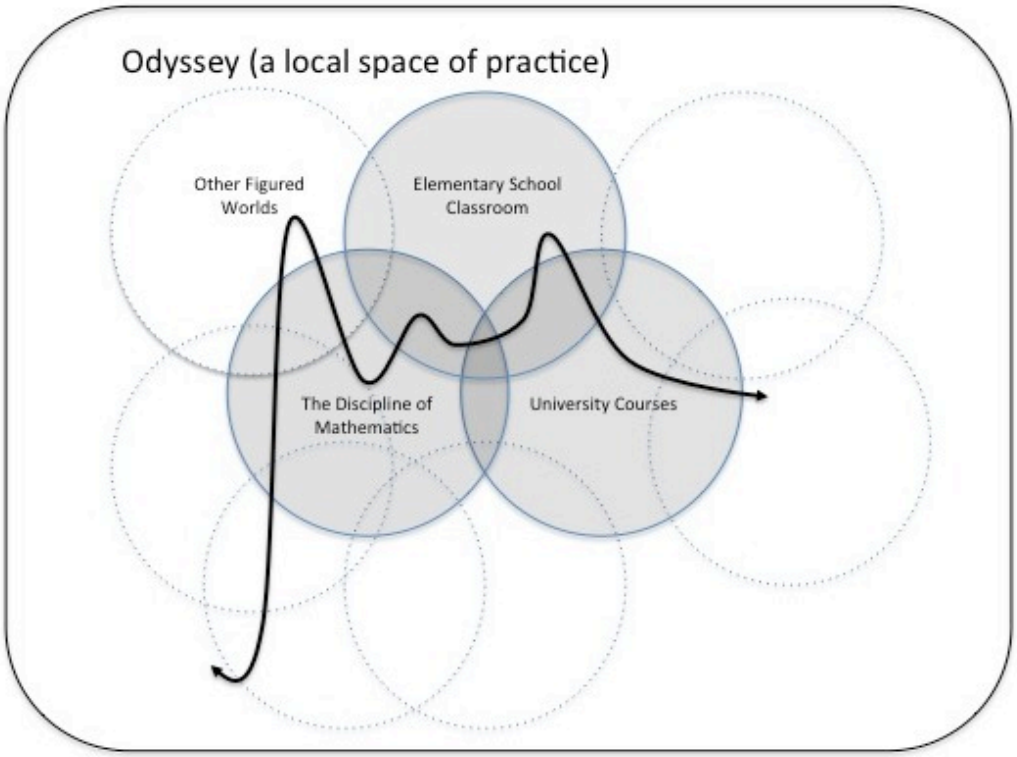

Figure 1: Potential trajectory of participant positioning through various figured worlds in the Odyssey.

PTs as Mathematical Experts

One of the problems involved making trains with colored rods of different lengths and finding all possible combinations for a given length (e.g., if the length is 3, there would be 4 possible trains: 3; 2+1; 1+2; 1+1+1). Odette (MT), in talking to one of the university facilitators, says:

> One of these guys [pointing to the 2 PTs in her group] had the **bright idea** to start with one white and see how many we can make… And then we started thinking, "is there any that we kind of missed?" And we filled in a couple that we had missed. But these guys [gestures to the PTs] are trying to **figure it out mathematically with combinations**. So, they might have some input for you.

Odette is positioning the PTs as *mathematical experts* in that they are the ones who may be coming up with the general statement. Indeed, the PTs tended to want to come up with formulas right away, often in what university faculty labeled as a procedural approach, hence positioning them as *rule oriented*. However, the interactions with MTs could serve as catalysts for switching to *sense maker*, as we illustrate next.

From Rule Oriented to Sense Maker

In the handshakes problem (how many handshakes are possible in a room full of people assuming that each person shakes hands with all of the other people in the room), Beatrice (PT) has come up with an expression to find the number of handshakes as being (n-1) + (n-2) + (n- 3)…, where n is the number of people in the room. Tonya (MT) does not understand why for 3 people it is 3 handshakes (she thinks it is 6):

Tonya: I don't get that. Three people, three handshakes.

Beatrice: Yeah.

Tonya: No.

Beatrice: Yeah.

Odette (MT): Yeah. Just draw it out.

Beatrice: Because 3 minus 1 is two, plus 3 minus 2 is one... two plus one is three.

Beatrice's answer does not address Tonya's question. Instead Beatrice shows Tonya how using her formula gives an answer of 3 and that seems to be her evidence for why it is 3. Odette's comment of "just draw it out" is not picked up by Beatrice or Tonya. Shortly after, Celine (MT) draws it out and shows Tonya why it is 3. As they move to 4 people, Beatrice is still focused on her algebraic expression, while Odette and Celine are talking through the process to try to come up with a general expression. But then all of a sudden Beatrice turns to Tonya (who has been gesturing handshakes to try to visualize the case for 4 people) and says:

> If you draw it like this (pointing to a drawing Beatrice has made in her notebook), it's like there... they shake, and then... they'll shake and then these last two shake. And then you just count one, two, three, four, five, six.

In this interaction with Tonya, Beatrice switches from a focus on using her formula to a sense making approach similar to Odette's and Celine's in making a drawing and showing Tonya why there are six handshakes with four people. This points to the potential of interactions such as these to connect the PTs' knowledge of mathematics with the MTs' pedagogical knowledge. The MTs bring to these mathematical tasks their experiences as teachers asking students to explain their work as well as more exposure through their years of professional development to conceptual approaches to teaching mathematics. Hence, they may be drawing on these backgrounds to push for a sense-making approach to doing mathematics.

Engaging as a Teacher

Although the university facilitators viewed the Odyssey as an opportunity to engage as doers of mathematics, some MTs engaged primarily as teachers. Zelda (MT) is one such case. Throughout the investigation of the trains' problem, Zelda contributes minimally to the group's mathematical work. She keeps track in her notebook of the different combinations that the group is mentioning, and on a few occasions interjects combinations that they may have missed. But she does not engage in the group efforts

on trying to find a pattern and eventually a general expression. Most of Zelda's comments relate to connections to her experience as a classroom teacher (e.g., in terms of the kinds of affordances that different manipulative materials may offer). Zelda's positionings were mostly as *pedagogical expert* or as *professional colleague*. For example, in the excerpt below Zelda, as *pedagogical expert*, connects the trains' problem to an early grades activity where children are to find different ways to make up a number, and proceeds to explain this to Norma (PT):

> [Looking at Norma and gesturing as if she had interlocking cubes] They [children] will take the numbers apart, it's like when they say "the number is 12, how many different ways can you make 12?"; it's the same idea, you give them interlocking blocks and they are always breaking them apart [gesturing as if she had the interlocking cubes in her hands].

The positioning of one member of the group as a *pedagogical expert* in the context of a mathematical activity provides a potentially fruitful space in which we might bridge theory and practice. In some cases, the mathematical task provided an opportunity for MTs to share their pedagogical experience, as well as for PTs to bring up questions about classroom applications.

CONCLUSION

In teacher preparation efforts, experiences that bring PTs and MTs together often center on pedagogy and therefore are likely to reflect a power differential, where MTs are seen as the experts. The Odyssey was unique in that it purposefully constructed a local space of practice in an off-site setting that was neither the domain of the PT (the methods classroom) or the MT (elementary classroom). This space brought PTs and MTs to do mathematics together and in so doing acknowledged different kinds of knowledge. In the Odyssey context, MTs often displayed vulnerability with regards to their content knowledge that might not be evident in other settings. Through this vulnerability and their call for explanations, MTs seemed to encourage the PTs to reposition themselves in the group as *sense makers*. Within this space of practice, MTs could investigate mathematical challenges that they might not have seen in some time, while at the same time, PTs gained an awareness of the need to make sense of the mathematics and gained pedagogical insights through observing how the MTs interacted. The following participants' reflections capture the egalitarian potential of spaces such as the Odyssey.

> It was fun to work together on math and bring our experience / expertises together. It didn't matter what we knew, but we all worked together. It was also nice to get to know all the mentor teachers…. Math really focused on everyone and was extremely beneficial. [Olivia (PT)]

> I enjoyed meeting the PTs in a more informal setting; it seemed we were able to bond more, share ideas, be on more equal. [Zelda (MT)]

> Comfortable learning together. [PTs] see us as "learners" also, not just teachers. [Tonya (MT)]

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