TEACHERS' PRODUCTIVE MATHEMATICAL NOTICING DURING LESSON PREPARATION

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This study uses lesson study to investigate what mathematics teachers notice about students' mathematical reasoning during the planning of a lesson on fractions. Most research examines teaching noticing during or after a lesson, and focuses on the specificity of what teachers notice as a characteristic of noticing expertise. In this paper I propose a new notion of productive noticing, and apply it to analyse two vignettes of teachers' mathematical noticing during lesson preparation. Findings suggest that teachers' noticing is most productive when it goes beyond the specificity of what teachers notice to include justification based on what they have noticed about students' thinking. The study also demonstrates the usefulness of this construct in analysing what mathematics teachers notice when planning lessons.

INTRODUCTION

Mathematics teacher noticing—what mathematics teachers see and how they understand instructional events or details they see in classrooms (Mason, 2002; Sherin, Jacobs, & Philipp, 2011)—is central to mathematics teaching practices, and is needed for improving teaching (Mason, 2002). Most researchers who study mathematics teacher noticing do so by examining what teachers observe from video clips of lessons (Star, Lynch, & Perova, 2011; van Es, 2011); while others (Sherin, Russ, & Colestock, 2011) capture what teachers notice in-the-moment during lessons. In this paper, I extend the notion of productive noticing to enable investigation of what mathematics teachers notice during the planning of mathematics lessons. The key research questions addressed in this paper are: What do mathematics teachers notice about students' mathematical thinking during lesson preparation? More importantly, what distinguishes teachers' productive noticing from less productive noticing?

THEORETICAL CONSIDERATIONS

Mathematics teacher noticing

According to Mason (2002), noticing is a set of practices that work together to enhance teachers' awareness to new responses in classroom situations. These practices include "reflecting systematically; recognising choices and alternatives; preparing and noticing possibilities; and validating with others" (Mason, 2002, p. 95). Many researchers view noticing as consisting of two main processes: "attending to particular events and making sense of events in an instructional setting" (Sherin, Jacobs, et al., 2011, p. 5), but Jacobs, Lamb, and Philipp (2010) also include how teachers decide to respond to instructional events in order to link the intended responses to the two main

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processes of noticing. This triad view of noticing—attending to; making sense of; and deciding to respond—ties in with Mason's (2002) idea that noticing should bring to the mind of teachers a different way to respond.

However, it can be very challenging to notice salient mathematical details in a classroom setting. Marking and discerning instructional events that are critical and useful can be difficult for teachers. In a video-club study involving 30 pre-service teachers, Star et al. (2011) found that they had problems attending to specific mathematical details of lesson tasks. Vondrová and Žalská (2013) also found that the pre-service teachers in their study did not notice mathematics-specific details, even when they were shown short video clips with prominent mathematical incidents.

Developing teachers' ability to notice

Approaches to develop teachers' noticing often centre around the use of video clips of teaching—where teachers are shown clips of classroom teaching and asked to notice certain features of the instruction (Sherin, Russ, et al., 2011; Star et al., 2011; van Es, 2011). These approaches tend to focus largely on noticing instructional details after lessons are conducted. In order to examine teachers' in-the-moment noticing, Sherin, Russ, et al. (2011) asked teachers to record short segments of video clips of what they noticed during lessons, using a wearable camera, before they discussed these recorded segments. Even though this approach gave researchers improved access to teachers' in-the-moment noticing by triangulation with teachers' reflections on the recorded segments, the researchers acknowledged that the sense-making and decision-making processes may not be fully captured (Sherin, Russ, et al., 2011).

One issue with this approach of developing teachers' ability to notice is the lack of focus on preparation to notice. As Mason (2002) put it, "noticing is an act of attention, and as such is not something you can decide to do all of a sudden. It has to happen to you, through the exercise of some internal or external impulse or trigger" (p. 61). More specifically, Mason (2002) highlights advanced preparation to notice, and the use of prior experience to enhance noticing in order to have a different act in mind. In this paper, I propose a development of teachers' noticing ability through explicit preparation during the planning of a mathematics lesson.

Productive mathematical noticing—focusing on the 'Three Points'

Most research focuses on the specificity of what teachers notice, but specificity is not sufficient for noticing to be productive. In a study involving seven prospective secondary school mathematics teachers, Fernandez, Llinares, and Valls (2012) found that most were unable to relate the strategies used by students to the characteristics of the problem, even though they were all able to describe the specific strategies at the beginning of the study. In the context of lesson planning, one possible approach is to support teachers' ability to notice mathematical features by directing their attention to key mathematical ideas and students' learning difficulties related to these concepts.

In a previous paper (Choy, 2013), I proposed a characterisation of productive noticing using Yang and Ricks' (2013) Three-point framework—key point; difficult point; and critical point. According to Yang and Ricks (2013), the *key point* refers to key mathematical concepts or ideas of the lesson; the *difficult point* refers to cognitive obstacles encountered by students when they attempt to learn the key point; while the *critical point* refers to the approach taken by teachers to help students overcome the difficult point. I propose that teachers' productive mathematical noticing occurs when they are able to:

- attend to specific details related to the key point, difficult point or critical point that could potentially lead to new responses;
- relate these details to prior knowledge and experiences to gain new understanding for instruction (key point and difficult point);
- combine this new understanding to decide how to respond (critical point) to instructional events.

This characterization of productive mathematical noticing uses the 'three points' not only to direct teachers' attention to specific details of what they notice, but also to highlight the need to connect the critical point to the key point and difficult point.

METHODOLOGY

This paper uses data from a seven-week lesson study cycle situated in a Singapore primary school. Lesson study, as a collaborative inquiry approach, provides a means to make teachers' thinking during lesson planning "more visible" (Lewis, Friedkin, Baker, & Perry, 2011, p. 171). There are five key tasks in lesson study—developing a research theme; working, discussing and anticipating student thinking through mathematics tasks; developing a shared lesson plan; collecting data during observation of research lesson and conducting a post-lesson discussion (Lewis et al., 2011). In this paper, I report results drawn from the first three tasks corresponding to the lesson preparation phase of the lesson study.

Six mathematics teachers formed the lesson study group that explored the teaching of 'fraction of a set' for Primary Four students (aged 10). Five of the teachers have more than 10 years of teaching experience and the other has at least five years.

To facilitate productive mathematical noticing during lesson preparation, I introduced Yang and Ricks' (2013) Three-Point Framework to teachers and encouraged them to focus their discussion for each lesson study task on the specifics of these three points. The teachers discussed explicitly the key mathematical ideas they wanted to teach, and the associated "difficult points" from their readings, prior experience or observations of their own students. Next they focussed their discussion on possible approaches (critical points) that could help students overcome the difficulties and learn the key ideas, before they agreed on a teaching approach. Teachers then designed the main task and anticipated students' possible responses to the task in relation to the points raised.

Finally, the team prepared a shared lesson plan containing the lesson sequence, key tasks, anticipated students' responses and planned teachers' responses to students.

The researcher primarily took on the role of observer during the seven lesson study sessions, and served as a resource person for the mathematical knowledge for teaching, while Ms Kirsty (a pseudonym), the team leader, was facilitator. Data were collected and generated through voice recordings of the lesson study sessions and video recording of the lesson. The recordings were parsed and segmented into episodes, as determined by the goal of the conversation. The findings were developed through identifying categories, codes and themes related to what teachers noticed in the episodes. The episodes were then classified as more or less productive using the framework above. Noteworthy episodes were further developed into vignettes to highlight the characteristics of more and less productive mathematical noticing.

RESULTS AND DISCUSSION

Focussing on the data drawn from the first four sessions on lesson preparation, the language of the 'Three-Point Framework' seemed to have helped teachers attend to specific key points, difficult points, and critical points related to the topic.

Less productive noticing

During the second session, Mr Anthony went through how the textbooks present a diagrammatic representation of 2/3 + 1/4 by showing two diagrams with 12 equal parts each. Mr Anthony then highlighted that the reason for the 12 parts was not obvious to the students.

> Mr Anthony: So the children will ask, why do you give me 12 equal parts? Why didn't you give me 6 or 18 equal parts? So, Ah… we look at the multiples of 3, 6, 9, and so on… at the end, we have 4, 8, 12… Coincidentally, we find just the lowest common multiple, so we have to use 12.

Here, Mr Anthony attended to a specific mathematical detail (key point) that might present new possibilities in the approach. He was also very specific with regard to students' difficulties—that they did not understand why 12 parts were used in the fractional representation (difficult point).

When asked how he would helped them to bridge this gap, Mr Anthony recounted:

> Mr Anthony: No choice… Because they are not in the same family, we want them to do some transaction, or you want to mix them together, we need to do something alike.

Furthermore, he highlighted that students often just latch on to the procedure:

> Mr Anthony: They will tell me this: My teacher tells me this… you multiply me and I multiply you. [Laughter] So, if the question is not that big, some times they are given 5/6 and then 4/9. They start to multiply 9 with 6 and 6 with 9…Yeah! That's right! And the numbers get bigger and bigger… Then they don't know how to do.

The rest of the teachers in the team also agreed with Mr Anthony that the problem was common among students. However, the teachers did not explore this difficulty further and attributed the difficulty to a lack of procedural competency in finding the lowest common multiple:

Ms Kirsty: Because they fail to understand the factors and multiples well. They don't know the least common multiple.

Ms Regina: They don't know how to list and find the lowest.

Mr Jeff: This is like the easy way out.

The teachers thought that students could not find the lowest common multiple, but did not suggest why this was the main issue. It seemed that the crux of the problem was the reason behind the 12 equal parts instead of finding the multiple 12. However, they attended to specific key and difficult points, even though they did not reason and make sense of the difficult point to arrive at a possible approach (critical point).

Productive noticing with reasoning and justification

When discussing students' difficulties in learning about a fraction of a set, Mr Jeff highlighted that students' difficulty in understanding fraction of a set could be due to a 'met-before' (Tall, 2004) of the notion of fraction as 'part of a whole':

> I think the objective for fraction of a set is for students to see, to interpret fraction as part of a set of objects. Previously, the fraction [concept] they learnt is more of part of a whole. They are very used to thinking about part out of a whole. Now that we give them a lot of whole things, they cannot link that actually these fractional parts can refer to a set of whole things also. So I think, to me, I feel that the connection that is missing, is that, how this fraction concept—which is part of one whole, which they have learnt so far—can be linked to whole things. For example, previously we used to teach fractions as parts of a cake or pizza. From that, how can it be that we have many pizzas, we don't cut out the pizza, there is a fraction of the pizzas. I think they cannot make a link there.

Mr Jeff elaborated further what he meant:

> For me, the main difficulty is to relate part of a whole into items that are "whole" but you take a fraction out of it. So, I think that's where the confusion comes.

He went on further to give a more concrete example:

> For example, if you say ¾ of the cats are… [Imitating the students] Ah… you cut the cat into three quarters? [Laughter] Cut each cat into four parts. So, yeah, but based on what they learnt so far, that may be the first thought they might have. To them, fraction could still be cutting up into parts. Whereas, fractions of a set, we leave the things as a whole entity but we look it as a collection of things. So out of these five things, how many are blue etc… For me, that would be the main difficulty.

In this short exchange, Mr Jeff clearly identified the need to extend the notion of fraction to a set of items (key point). He was also able to attend to the expected difficult point with a good level of specificity. Mr Jeff linked students' difficulty with a met-before of fraction—'part of a whole'—and suggested how students' image of fractions as ‘parts’ might conflict with the concept of fractions referring to a subset of ‘whole’ objects. However, unlike the first vignette, Mr Jeff used two examples—pizza cutting and cat cutting—to illustrate students’ difficulties. His use of specific examples strengthened what he attended to, and how he made sense of his prior experiences with students. Therefore, the link between the key point and difficult point was made explicit for the other teachers, and this provided an impetus for other teachers to notice students’ thinking. Hence, Mr Jeff’s noticing of students’ possible difficulty had productive potential for enhancing students’ thinking because it helped other teachers to focus their attention during the design of the task.

Besides directing teachers’ attention to the three points, Mr Jeff’s productive noticing also heightened other teachers’ sensitivities to students’ thinking when they were teaching. For example, Ms Kirsty became more cognisant of her students’ difficulties in grasping the concept and related what she attended to during another planning session.

> Ms Kirsty: And I think what we said is very right. They are not equating this concept of fraction as being the relationship between the part and its whole.
>
> Mr Jeff: As in, the object being the whole, right?
>
> Ms Kirsty: Not the fraction… part… and… what.
>
> Researcher: Part of a whole?
>
> Mr Jeff: … the number of whole things?
>
> Ms Kirsty: Part of a whole… not as relationship between a part and its whole… but as part of a whole. They are still with the impression of ‘part of a whole’.
>
> Mr Jeff: Actually the item that we use must be something that we cannot cut out one… like cars… tables… chairs

Ms Kirsty’s observations resonated with Mr Jeff’s noticing of student thinking about fraction of a set, and this later advanced the design of the task. Noticing is “validated” when others recognise that what is being noticed corresponds to their own experience (Mason, 2002, p.93). This validation heightens one’s sensitivity to notice, and promotes the possibility of improving practice (Mason, 2002). Mr Jeff’s reasoning based on his noticing also seemed to provide some justification for the proposed approach or response to students’ difficulties: Mr Jeff suggested using items that cannot be “cut” to help students get over the ‘part of a whole’ image of fractions. Moreover, when Mr Jeff was asked about a possible approach to help students understand the concept, he suggested an approach that made explicit links between the key point, difficult point and critical point:

> I think the confusion part also comes when… for example… this example here… we tell that … ¼ of the cups are yellow and then the answer is 4 cups. Huh… ¼ and then why got 4 in the 1/4? They cannot link between the… the ¼ in their mind is still ¼ of a whole… and then there is this four cups, four whole things… and so they cannot link… I was thinking whether we can put it into… something more familiar because… eh… they have learnt

> models, how to represent questions in model also, so, I was just looking at this… instead of just doing this, could we box the whole thing up instead. And to them, they are familiar with the part-whole model… a whole box is a whole… so while keeping the items inside and we draw the box… and… and… yes… we tell them that this looks familiar, and it looks like the model as a whole, right? These lines can be the partitioning of the whole model. While doing that… they can still see that the 4 items are still inside the parts. I don't know whether that can help them to make the connection that if this one box [partition] is ¼ of the whole, inside that box, I have four things. And this is where the 4 came from?

Mr Jeff's suggested approach (critical point) was directly linked to students' image of ¼ as 'part of a whole' (difficult point). Mr Jeff attempted to use the part-whole model, which the students were familiar with, as a scaffold to help students see that there could be 'whole items' inside a 'part'. This provided a bridge for students to extend their notion of fractions by emphasising fraction as a means to express the relationship between a part and its whole (key point).

What distinguished Mr Jeff's noticing as more productive was not the workability of the approach suggested, but rather the justification that reinforces the alignment between the three points. Justifying based on what was noticed not only helped the teachers maintain their attention on specific key and difficult points, but also lessened the likelihood of generating a critical point that does not provide opportunities to enhance students' reasoning.

CONCLUSION AND IMPLICATIONS

Productive mathematical noticing brings to the minds of teachers different ways to respond during teaching, and this can potentially improve the teaching of mathematics. This study highlights how processes of noticing can be incorporated into lesson planning. The findings suggest that the construct of productive noticing can be used to analyse teachers' noticing during lesson preparation. Moreover, teachers' noticing seems to be more productive when it goes beyond the specificity of what teachers notice about the three points, to include justification as a means to strengthen the linkages between the three points. The ability to notice productively during lesson preparation is important because it sensitises teachers to think about what to teach, students' possible misconceptions, and ways to deal with these problems. Further research is needed to characterise productive noticing more rigorously, and more work is needed to show how this construct can be applied to teacher noticing during and after instruction. Nevertheless, this study brings out the value and potential of productive noticing to improve teachers' practice.

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