‘I SENSE’ AND ‘I CAN’: FRAMING INTUITIONS IN SOCIAL INTERACTIONS

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In this article we examine intuitions as they emerge in groupwork activities. We provide a framework and a methodology to code various aspects of the activity, social and mathematical. Focusing mostly on students’ gazes, we explore how affective moves give rise to, and determine, students’ interactions and thoughts. We argue that intuition does not take place in the mind of the individual, it is not a matter of ‘I think’, but it arises from actions and reactions, in relationships with others and with artefacts. Data from a 50 minutes groupwork activity of four grade-9 students allows us to further discuss our framework.

INTRODUCTION AND BACKGROUND

Dewey (1938) states that intuitions and illuminations are not "part of the theories of logical forms" (p.103). Illumination is the phenomenon of sudden clarification arriving in a flash of insight and accompanied by feelings of certainty (see Liljedahl, 2012, and references therein). Intuition, as well, is a form of thinking that provides the learner with a sense of certainty (Fischbein, 1987): it is perceived as global (rather than analytical), coercive and self-evident. Sometimes intuitions from everyday experience contrast with mathematical knowledge and can impede learning: misconceptions are such kind of intuitions (Fischbein, 1987).

Andrà & Santi (2013) underline that intuitions are a way of establishing a relationship between the learning subject and the object of knowledge, they are a mode of existence of the consciousness which intertwines with perception, sensorimotor activity, emotions (which provide the learner with a sense of likelihood of success, see Roth & Radford, 2011), and mathematical generalization. They conclude that intuitions can start in a private, individual moment, but it is in the communitarian self (Radford, 2012) that they develop towards mathematical generalizations. If so, which is the relationship between the individual moment of illumination (see also Liljedahl, 2012) and the emerging of shared intuitions in the communitarian self, which can develop into mathematical deductive forms of thinking and proving? In order to answer to this question, we have developed a methodological framework (Liljedahl & Andrà, in press) that helps us capturing and decoding the turbulent undercurrents of groupwork mathematical activities. After briefly presenting the framework that informs our research, we apply it to the analysis of an episode in a grade-9 class working on basic concepts in probability. We will discuss illuminations that emerge and develop in the

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social interactions, as well as how they inter-relate with other modes of existence of the consciousness.

FRAMEWORK, THEORETICAL AND METHODOLOGICAL

Groupwork activities in the classroom have gained more and more attention in the last decades. In such activities, communication plays a primary role. Sfard (2001) points out that "communication may be defined as a person's attempt to make an interlocutor act, think or feel according to her intentions" (p.13). Following this view, thinking is thus subordinated to and informed by the demand of making communication effective. Within this domain (called interactionist or participationist) learning is seen as becoming participant in a mathematical activity. Activity is sensitive to context and allows the growth of mutual understanding and coordination between the individual and the rest of the community. Accordingly, each activity has its roots in our cultural heritage and can be shaped and re-shaped by the group of practitioners. It is within this framework that thinking is conceptualized as a case of communication, since interactionist research postulates the inherently social origin of all human activities (Sfard, 2001).

Sfard (2001) suggests that in learning processes, seen as initiations to become skillful participant in mathematical discourses, two key factors need to be considered: the tools that mediate the communication and the meta-discursive rules that regulate it. The focus of this paper is on the latter.

Meta-discursive rules have an implicit nature, they are tacit, and it is within the system of such rules that culturally-specific norms, values and beliefs are encoded (Sfard, 2001). According to Merlau-Ponty (2002), awareness is not a matter of 'I think that' but of 'I can': before the reflective, the positing thought, there is an act ('I can do this'). Specifically, since learning "occurs in and through relations with others in the pursuit of collectively motivated activity" (Roth & Radford, 2011), motivation is the orientation of the activity. Emotions express the student's current state with respect to the motive of the activity, they express her sense of likelihood of success in realizing such motive (Roth & Radford, 2011). Given the social environment in which the students act, interact and determine the moves of the activity on the ground of their emotions, we methodologically exploit the idea of interactive flowchart.

Interactive flowcharts were introduced by Sfard and Kieran (2001) as a way to capture "two types of speaker's meta-discursive intentions: the wish to react to a previous contribution of a partner or the wish to evoke a response in another interlocutor" (p.58). A conversation can be coded as being comprised of a series of invisible arrows aimed at specific people and/or specific utterances. The scheme follows two basic structures: (a) a vertically or diagonally upward arrow is called a *reactive arrow* and points towards a previous utterance; (b) a vertically or diagonally downward arrow is called a *proactive arrow* and it points towards the person from whom a reaction is expected. Add to this a distinction between arrows that are on task or mathematical in nature (solid) and off-task or non-mathematical in nature (dashed). Sfard and Kieran

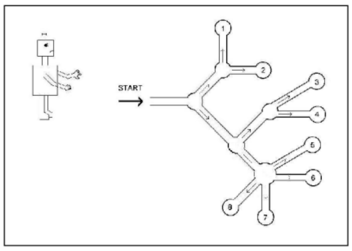
(2001) developed this scheme to code conversations between two people. Ryve (2006) extended this scheme to account for more than two people by assuming that a proactive utterance is meant to address each of the other participants. Table 1 in our example is read as follows: M in 1 makes a proactive statement to L and D, D reacts in 2, and so on. In our earlier research (Liljedahl & Andrà, in press), we found it was necessary to consider the flow of conversation, but also who the participants are looking at. As such, we introduce new set of arrows, meant to represent where someone is gazing during each utterance. We use red arrows to represent the speaker and blue arrows to represent non-speakers. In Table 1, for example, M looks at the paper in 1, D looks at the paper in 2.

METHODOLOGY

At the core of the research presented here is a 45 second video clip of a group of four students working on a mathematics problem.

The problem was inspired by the work of Iversen and Nilsson (2005), who used a similar task to see how students make sense of random phenomena. The problem is:

> A robot walks along a corridor, it turns right with probability 1/3 and it turns left with probability 2/3. The map shows the labyrinth where the robot has to move. Compute the probability for the robot to be in each of the rooms.



Iversen and Nilsson (2005) asked the students to say which is the room with the highest probability. Our problem was crafted so as to use the representation provided by the task in order to introduce the concepts and the algorithms related to the tree diagram: why should one multiply subsequent branches? Why and when should one add? The task was presented like a game, and the students seemed willing to work on it as such.

The task was used as part of a series of four lessons on probability in a grade-9 (14-15 year olds) class in Bologna, Italy. The task formed a significant portion of the second lesson. Four students, Luca (L), Fabio (F), Davide (D), and Marco (M) were selected to be videotaped while they worked on the task as a group. They worked on the task in a separate room and were filmed by a grade-12 student from the same school. The entire session lasted 50 minutes. The first 5 minutes of this video were transcribed. From this, the first 45 seconds were selected to constitute the data for the research being presented here. This subset of the data was selected because it exemplified some very interesting and turbulent undercurrents of group interactions. We also introduce a new interlocutor to the interaction – the paper (P) with the problem on it. This paper holds the gaze of the participants at different times of the conversation (we do not code blue arrows when the students are looking at P). Unlike the arrows representing utterances all of the gaze arrows are diagonally downward to represent the passage of time.

READING DATA

Table 1 presents the transcript and interactive flowchart with the blue-red gaze arrows. Figure 1 shows some snapshots from the video overlaid with some gaze arrows (for ease of reading, each student has assigned a color: yellow for L, blue for F, green for D and red for M); the arrows help the reader to focus on gazes and do not follow the blue-red coding used in Table 1. We first present the data codified according to our methodological framework, then we analyze the codified data.

			L	D	M	P
00:00	M:	To the left two thirds, to the right one third.	o	o	o	o
00:01	D:	Yes, I don't remember. (*speaks over M*)	o	o	o	o
00:03	M:	Then it goes two thirds, two thirds.	o	o	o	o
00:06	M	Can you give me a pen, please?	o	o	o	o
00:07	L:	No, let's do the first case, which is the one where it goes always …	o	o	o	o
00:10	M:	… left. You have two thirds here …	o	o	o	o
00:11	L:	That is the most probable one. (*speaks over M*)	o	o	o	o
00:13	M:	…and here is one third.	o	o	o	o
00:15	L:	Should you erase?	o	o	o	o
00:16	M:	Yes, bravo!	o	o	o	o
00:17	D:	I'm cute!	o	o	o	o
00:19	M:	Two thirds and here one third, hence these two thirds…	o	o	o	o
00:21	F:	... they g ... they go ….	o	o	o	o
00:22	M:	Two thirds of two thirds.	o	o	o	o
00:25	D:	But … but what are you saying? Then no …	o	o	o	o
00:27	M:	Of these two thirds you should do …	o	o	o	o
00:28	D:	We have … but what do we have to compute? (*speaks over M*)	o	o	o	o
00:30	L&M:	The probability that the robot will arrive in each one …	o	o	o	o
00:34	M:	of these rooms.	o	o	o	o
00:35	D:	In the meantime, let's see …	o	o	o	o
00:36	L:	Why don't we first compute how many probabilities there are in all?	o	o	o	o
00:37	M:	To me this is the room with the highest probability.	o	o	o	o
	D:	Why?	o	o	o	o
00:42	L:	There are 8 in all.	o	o	o	o
	M:	Because here there are the highest number of probabilities, and then …	o	o	o	o
00:45	D:	Of course	o	o	o	o
	M:	… the probability is higher.	o	o	o	o

Table 1: Interactive Flowchart with Gaze Arrows

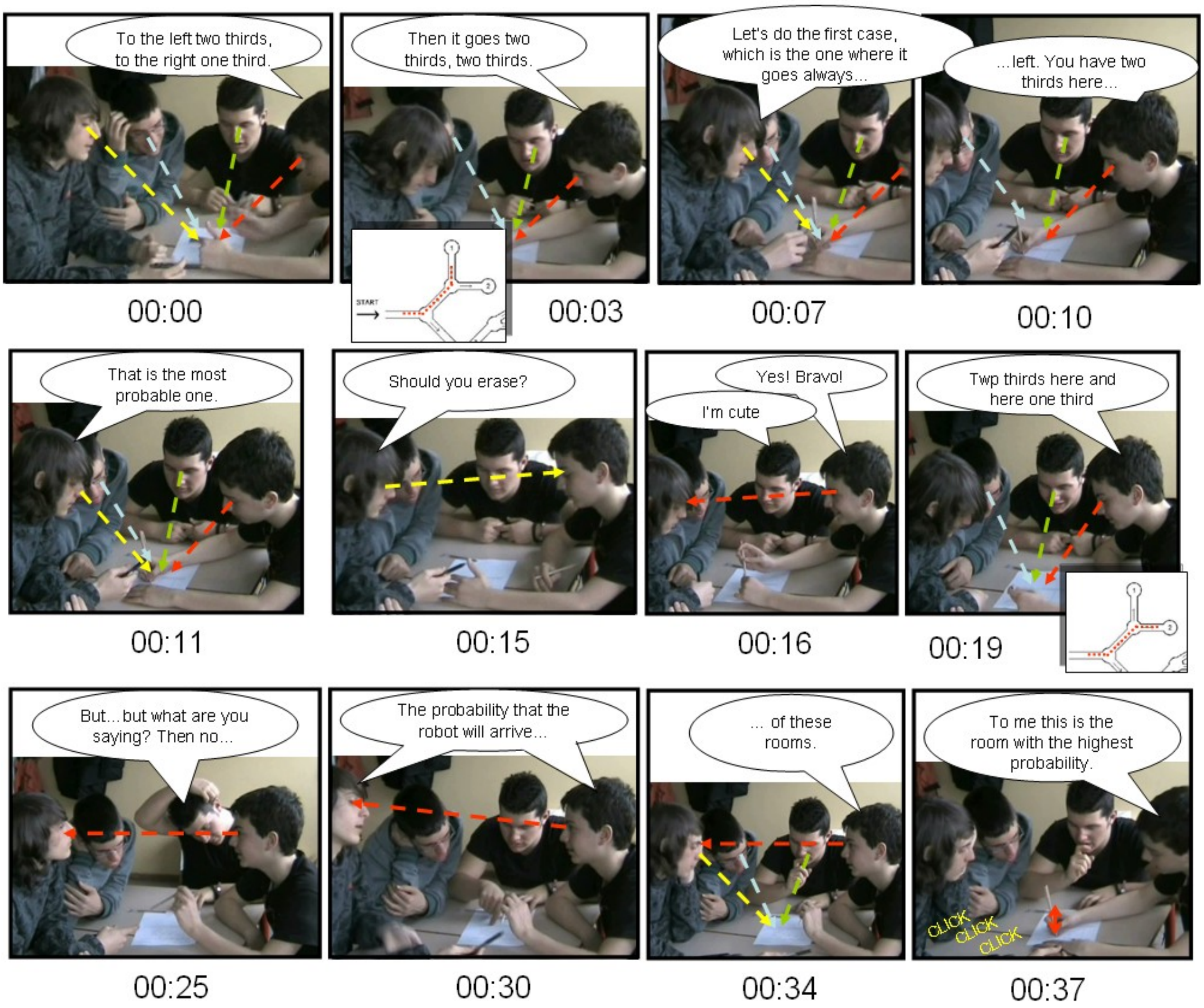


Figure 1: Some snapshots from the video overlaid with gaze arrows.

Data codified with our methodology

If we look at the verbal transcript, we see that the students are making sense of the task. Both L (00:11) and M (00:37) come to notice that the highest probability is related to the first room, an observation (coming from the students) which is in line with the original formulation of the task by Iversen and Nilsson (2005).

The interactive flowchart shows that M is contributing the most proactive statements (n=7) as opposed to L (n=3) or D (n=0). M and D responds to the most number of proactive statements (each n=5) as compared to L (n=1 not counting the self-talk as a reaction). Finally, there is a marked difference in the number of proactive statements that each person makes that are reacted to – M (n=6), D (n=3), and L (n=1, not counting the self-talk).

The gaze arrows show that D never looks at L. D doesn't look at anyone – he only looks at the paper when he is speaking. Figure 1 tells us that the students spend a lot of time looking at the paper, indeed. M, on the other hand, spends more time looking at L (n=6 in Table 1) than at the paper (n=5). At 00:25 D is asking a question while gazing at the paper. But M is not looking at D – he is looking at L. Then, while M responds to D's question at 00:27 he continues to look at L. This happens again at 00:34. At the same time L only looks at M three times. Once at 00:15, then again at 00:25 while D is asking a question, and finally 00:36 while M is looking at the paper.

ANALYSIS

Gaze arrows in Table 1 and Figure 1 tell us that, as much as M is attending to L, L is ignoring, maybe even avoiding, M. Why is M so intent on L and why is L ignoring M? We can see something interesting happening at 00:25. While D is asking the question, L and M are looking at each other. But these are not looks of equal intensity. In the video M is clearly more intense in his gaze upon L, who, after a while, glances away from M (see also Figure 1). From that moment on M continues to be very intensely focused on L. L seems to sense this and diverts his gaze from M, only looking back at him while M is looking at the paper (00:36). Clearly there is an affective aspect to the interaction between L and M. There are emotions, efficacy, will, and motivation in how L and M are interacting with each other.

True, all the students express their will to solve the task: D's questions aimed at letting him follow M's reasoning, his posture, his repeated and attentive gazes at the paper speak to D's will to be part, to contribute to the solution. On the side of both M and D there are many attempts to make their interlocutors act, think or feel (Sfard, 2001). M addresses mostly L, D prompts M. Power relationships are established: *power to do*. We see that an 'I can' and an 'I sense' intervene in this groupwork activity: M's and L's ones, respectively. M is working with fractions, he is interested in the procedure. We see that an 'I can' ('I can deal with this kind of computations', 'I can do this kind of math') emerge in his speech, in his interactions with his classmates. L, instead, seems more interested in understanding the overall sense of the activity ("Why don't we first compute how many probabilities are there in all?" 00.36). We rather see an 'I sense' in L's words. We have already commented that both L (00:11) and M (00:37) come to notice that the highest probability is related to the first room, but seemingly from different standpoints: L makes his conclusion based on the fact that room 1 is arrived at by always going left, which has a higher probability than right. We can say that L has an illumination, a rapid coming to mind of the features of the room with the highest probability, coming out of the blue, few seconds after the beginning of the activity. M, on the other hand, arrives at the same conclusion much later, by means of computations. Only after considering fractions can he say that room 1 has the highest probability.

There is a tension between L and M, between conceptual 'I sense' and operational 'I can'. Moreover, we see that each of these stances prevents each student from seeing the other's point of view. 'I can' might be inclusive: in our example, M is trying to pull L in. On the other hand illumination ('I sense') is rather individual and private, it does not need to pull others into it: after the moment of illumination, in fact, there is a distinct phase of validation—aimed at put such an 'I sense' into sharable, communicable, terms (see Liljedahl, 2012). Communication takes place in order to stimulate a reaction: L's illumination at 00:11, in fact, takes the form of a rather self-thought, and it is not reacted. L's illumination is as sudden as private.

M's intensive gazes on L speak to M's 'I can': he can go on with his reasoning if L is with him. L's avoiding, expressed by (absence of) gazes to M, tells us that L is avoiding this kind of 'I can': L 'cannot' use fractions, he prefers to reason at another level, more theoretical. In Figure 1, at 00:37, M taps with his pencil on the paper, pointing at room 1. M is sharing his 'I can', his claim about the room with the highest probability. L is reacting to M, neither verbally nor with gazes, but with his own pencil, opening and closing it repeatedly (CLICK CLICK CLICK in Figure 1). Interaction is taking place at another level: M is expressing his 'I can' while L is again expressing his avoidance of fractions, his 'I can't use fractions'. At the same time, we see the will to participate, to solve the task, expressed by all the students—in different manners.

DISCUSSION AND CONCLUSION

Stemming from findings in interactionist research (Sfard, 2001), we have explored how affective moves give rise to and determine groupwork activity. Affective moves are meant as meta-discursive rules that shape actions, motivation, and interactions of students, thus directing learning (see also Roth and Radford, 2011). Participation in a groupwork activity is social, but it is also mathematical: we can distinguish the social and the mathematical in our analysis, but we cannot separate them. Many moves of the activity we have analyzed are both social and mathematical in nature.

According to our framework, we can also say that even L's 'I sense' originates from an 'I can'. In other words, we can see that it is from L's 'I can see a structure' that the illumination about room 1 at 00:11 starts, and it is from M's 'I can use fractions', 'I am good with fractions', that all his proactive statements arise. L's 'I can', more conceptual in nature than M's one, is expressed by an 'I sense' at 00:11 ("That is the most probable one"). The initial 'I sense' at 00:11 mirrors another 'I can', more operational, at 00:36 ("why don't we count how many probabilities are there in all?"). M also expresses an 'I sense', which is rather procedural and it is linked to the fractions involved: M's 'I can' is thus operational. Following Merleau-Ponty (2002), we can say that intuition is first an 'I can', it is originated by will and power to do. Intuition is socially communicated, expressed, as an 'I sense'. In social interactions, sometimes there emerge mostly the 'I can' (which is also more involving, as we have argued), other times the 'I sense' is predominant.

'I can' is conveyed by gazes in our methodological framework: M, in fact, expressed his 'I can' by looking intensively to L, and L's avoidance of fractions is mirrored by his avoidance of glancing at M. Also D's absence of gazes to M and L speaks to a consonant absence of 'I can': D is not good in math, while M and L are (we know this from the teacher). Looking at the paper expresses D's need to adhere to the task. His prompts to M express his need to go slow.

The 'I can', might become shared with others when the nature of this 'I can' is involving. For example, when it entails actions (operations with fractions, in our example). Illuminations of different nature need a subsequent moment to become sharable. Our findings also confirm the unavoidably central role of emotion and motivation in learning processes—especially in interactionist researches.

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