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Collaborative Gestures Among High School Students Conjointly Proving Geometric Conjectures

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Abstract: Collaborative gestures in the mathematics classroom occur when multiple learners coordinate their bodies in concert to accomplish mathematical goals. Collaborative gestures show how cognition becomes distributed across a system of dynamic agents, allowing for members of groups of students to act and gesture as one. We explore ways high school students gesture collaboratively while proving geometric conjectures in the sociotechnological context of an embodied video game in their classrooms. We find that students use their bodies and joint action to reason together and establish intersubjectivity. Gestures can both allow learners to access mathematical ideas and exclude peers. We also find that gestures generally, and collaborative gestures specifically, are associated with powerful forms of mathematical reasoning about generalizable properties of space and shape.

Gestures – learner-generated movements of the body that often accompany speech to convey ideas or emphasis – have been identified as important to the learning of mathematics (Alibali & Nathan, 2012). Theories of gesture production examine these behaviors as both individual cognitive processes (e.g., Hostetter & Alibali, 2008), and as a significant aspect of collaborative processes, as learners work together to solve mathematical and spatial problems (Enyedy, 2005; Hall, Ma, & Nemirovsky, 2015). Extending notions of depictive gestures that use iconicity to invoke mathematical entities and relations (Streeck, Goodwin, & LeBaron, 2011), Walkington and colleagues (in press) created a taxonomy for different ways gestures can be used collaboratively – including one learner *echoing* a gesture that another learner has previously made, one learner silently following along and *mirroring* another learner's gesture as it happens in real time, and *joint gestures* where two learners place their hands together in the same gesture space to formulate a single mathematical object or system. However, this taxonomy was created from gestures used by math teachers enrolled in courses at a selective university – which is arguably a very specialized population when it comes to gesture usage. In the present study, we extend this research to examine collaborative gestures among high school students in a diverse, urban school. We examine how these students use gestures across a variety of geometric tasks and engage in multi-modal analysis of two cases where gesture is used in collaborative ways. With this work, we seek to understand how gestures among learners can support collaborative problem solving.

Theoretical Framework

The gestures formed by collaborators engaged in sociotechnological contexts have long been integral to the study of computer-supported collaborative learning (Koschmann, 1996; Roschelle, 1992; Stahl, 2003). The theory of Gesture as Simulated Action (GSA; Hostetter & Alibali, 2008) posits that gestures arise during speaking when pre-motor activation, formed in response to motor or perceptual imagery, is activated beyond a speaker's current gesture threshold. This threshold can vary depending on factors such as the current task demands (e.g., spatial imagery, task difficulty), individual differences (e.g., level of spatial skills), and situational considerations (e.g., social contexts). Nathan (2017) proposed an extension of GSA where learners' actions and movements serve as inputs capable of driving the cognition-action system toward associated cognitive states through a bi-directional process. In other words, in addition to cognitive states giving rise to physical actions, directing learners to engage in certain physical motions may guide people toward certain cognitive states, prompting them to achieve intuitions and insights relevant to understanding and solving tasks.

Although these theories forefront the individual's cognitive processes, practice involves the coordination of many different inscriptions and representational technologies by differently-positioned actors whose actions occur across a range of social and physical spaces, which allows cognition to become *distributed* (Goodwin, 1995; Hutchins, 1995) and *extended* (Clark, 2008). Distributed, or extended, cognition observes that joint activity can be best understood with the system as the unit of analysis and accentuates how representations that exist inside and outside of the minds of individuals (like gesture) are used and transformed, as the system works to achieve a shared goal or plan. Students also often rely on gestures to establish and maintain intersubjectivity in order to operate conjointly in an effective manner (Nathan & Alibali, 2011). We define *collaborative gestures* as gestures that are physically and gesturally taken up by multiple learners. In this way, collaborative gestures hold a meaning that is explicitly dependent upon the gestures that have been enacted by interactional partners. These gestures represent an important case of distributed cognition in that they demonstrate how body actions can influence and extend over multiple learners' mathematical reasoning in a way that is fundamentally different from traditional modes of exchanging mathematical information (i.e., speech and written work) (Walkington et al., in press; see also Flood, 2018). Proof is an important area for investigation because it is a complex and abstract area of math (whereas many studies of embodied math address primary grade topics). We examine collaborative gestures in the context of students' proofs of geometric conjectures, a task that naturally integrates individual cognitive processes and collaborative processes. Students must ascertain for themselves the generalized properties of space and shape through investigations and logic (Harel & Sowder, 1998). Further, proof is socially mediated in that the goal is to persuade others to accept one's principled reasoning and justifications. It appears, then, that proof is "a richly embodied practice that involves inscribing and manipulating notations, interacting with those notations through speech and gesture, and using the body to enact the meanings of mathematical ideas" (Marghetis, Edwards, & Núñez, 2014, p. 243). Our primary research questions are: How do high school students use gestures collaboratively when proving geometric conjectures in groups? How are collaborative gestures associated with mathematical problem solving?

Method

Participants included 51 high school students (20 female) from an urban Career and Technical Education high school in the Southern United States. At the school, 83% of students were economically disadvantaged, with 57% Hispanic, 18% African American, 21% Asian, and 2% Caucasian. Of the 51 participants, 22 were in ninth grade and 29 were in tenth grade; 13 had not yet taken a high school geometry course. Participants were offered a \$20 gift card and were pulled from class to participate in the study.

Participants were placed in groups – there were 18 groups of 2 and 5 groups of 3. Groups played *The Hidden Village* – a motion capture video game – on a laptop hooked up to a Kinect™ camera. Following the introductory setup and calibration (Figure 1), players are introduced to the game narrative, and learn their player, lost and seeking to return home, will encounter tribal members whose actions that perform in order to earn amulets and advance their progress for a safe journey. The game then cycles through these encounters with a new character for each mathematical conjecture (8 in the current set up), following a Latin square design (see Table 1): Each new character performs poses that players must match closely enough (by trained Kinect standards) multiple times (3 times in this set up). Players then are presented with a new conjecture to which they respond True or False, and provide a (recorded) justification for their answers. They then select a justification among 4 multiple-choice options to advance to the next character/conjecture.

Table 1: Eight game conjectures, with average performance metrics and gesture metrics (all in percentages)

Conjecture	Proof	Insight	Intuition	Any Gesture	Collab. Gesture
1. Given that you know the measure of all three angles of a triangle, there is only one unique triangle that can be formed with these three angle measurements	0	0	27	82	14
2. If one angle of a triangle is larger than the second angle, then the side opposite first angle is longer than the side opposite the second angle.	5	33	48	83	32
3. If you double the length and the width of a rectangle, then the area is exactly doubled.	35	39	52	74	14
4. Reflecting any point over the x-axis is the same as rotating the point 90 degrees clockwise about the origin.	18	45	55	86	33
5. The area of a parallelogram is the same as the area of a rectangle with the same base and width.	5	50	55	91	29
6. Diagonals of a rectangle always have the same length.	4	61	65	96	32
7. Opposite angles of two lines that cross are always the same.	4	74	78	96	41
8. The sum of the length of two sides of a triangle is always greater than the length of the third side.	5	33	29	73	14
TOTALS	**9.6**	**42.4**	**51.4**	**85**	**26**

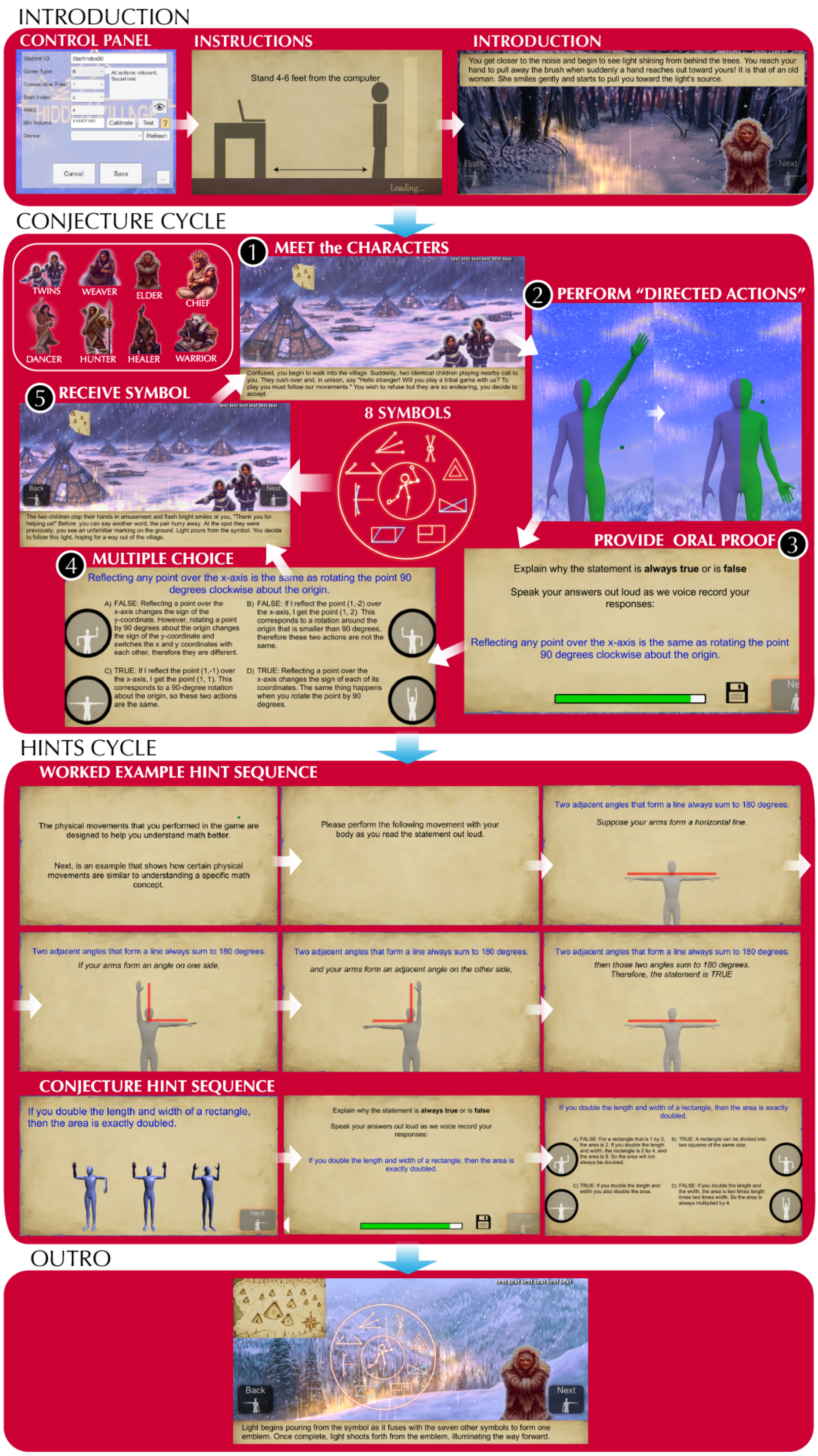


Figure 1. Flow chart of gameplay for *The Hidden Village* cycles through each of 8 mathematical conjectures.

After completing the final conjecture, players in this version of the game receive hints for 4 of the previous conjectures (ordered in a 4x4 Latin square) explicitly relating prior conjectures to in-game movements, at which time players are re-prompted for True/False and justification responses The student controlling the game rotated through each group.

Videotapes of groups of participants' informal proofs while playing *The Hidden Village* provide us with rich, multimodal, presentations of students' collaborative mathematical reasoning. The primary unit of analysis was the video clips, each of which was made of one group proving one conjecture. Clips were coded for (1) whether participants made any gestures while justifying the conjectures that represented or indicated mathematical objects or relationships, and (2) whether participants made any such *collaborative gestures* (defined earlier) while justifying the conjectures. Clips were also coded according to whether students were able to: correctly determine the truth value of each conjecture during their initial snap judgment (our measure of *intuition*); show understanding of key mathematical ideas and relationships underlying each conjecture (*insight* measure); and construct a valid, logical deductive mathematical argument to justify or disprove each conjecture (*proof*; see Harel & Sowder, 1998; see example in Table 2). Collaborative gestures were coded by type (mirror, echo, addressee, joint, and alternate; Table 3). Finally, all collaborative gesture clips were re-viewed to select clips for further analysis that pushed the boundaries of and offered worthwhile extensions for the collaborative gesture taxonomy previously developed using data of math teachers proving conjectures. Multimodal analysis (see McNeill, 1992) was used to examine these clips. We present two of these clips in the next section.

Table 2: Example coding criteria for conjecture performance measures

Conjecture	Intuition	Insight	Proof
Diagonals of a rectangle are always congruent	TRUE	TRUE because opposite sides are congruent and/or are parallel and/or the angles are right angles and/or the rectangle is symmetric.	TRUE because diagonals form two right triangles that are congruent (by SAS), since opposite sides of a rectangle are congruent, so their hypotenuses must be congruent.

Table 3: Collaborative gesture coding criteria

Category	Description
Mirror	One student makes a gesture, and then a second student makes the same (or similar) gesture at the same time. Only the second gesturer is coded as collaborative -mirroring. Must be evidence that the person doing the mirroring was looking at the original gesturer.
Mirror and Build	One student mirrors the gestures of another student, but then before dropping their hands builds upon the mirrored gesture with a new gesture that the original student is not making.
Mirror-Anticipation	One person is gesturing, and then a second person anticipates a gesture they are about to do (correctly or incorrectly) and makes that gesture.
Echo	One student makes a gesture, and then a second student makes the same gesture (or a very similar gesture) afterwards. Must be evidence that the person doing the echoing was looking at the original gesturer when the original gesturer originally made the gesture.
Echo and Build	One student makes a gesture, and then a second student makes part or all of the same gesture (or a very similar gesture) afterwards. However, the second student also changes or adds to the gesture in some way, in addition to echoing the original gesture.
Addressee	One person is speaking (may or may not be gesturing), and another person gestures to show what that person is talking about while they are still talking.
Joint Gesture	Multiple people manipulate, point to, or formulate a single mathematical object or a unified set of interacting mathematical objects. Each gesturer who is included in the joint gesture is coded as doing a collaborative gesture.
Alternate & Build	One learner gestures their understanding, and then another learner follows up, building on their reasoning using a different gesture, but generally agreeing with their viewpoint
Alternate & Redirect	One learner gestures their understanding, while another learner shows how their reasoning is different using a different gesture, potentially refuting the original person's reasoning

Results

Participant groups gave the correct intuition 51.4% of the time, had the correct insight 42.4% of the time, and gave a valid oral proof 9.6% of the time (Table 1). They made any gestures on 85% of attempts to prove or disprove conjectures and made collaborative gestures specifically on 26% of proof attempts (Table 1). Of the 45 instances of collaborative gestures, 25 were echoing gestures, 11 were mirroring gestures, 9 were alternating gestures, and 7 were joint gestures. Table 1 shows how these metrics varied by conjecture.

Figure 2 shows the relationship between gesture and outcomes. Performance measures varied by how student groups gestured. Student groups had the correct intuition 27% of the time when they did not gesture, compared to 56% of the time when they made any gesture, and 70% of the time when they made collaborative gestures. Similarly, student groups had the correct insight 12% of the time when they did not gesture, compared to 48% of the time when they made any gesture, and 72% of the time when they made collaborative gesture. For the proof outcome, however, accuracy was low enough overall that there was little differentiation (8% no gesture, versus 10% any gesture and 9% collaborative gesture). These relationships do not imply causation between collaborative gesture and successful reasoning, but show how gestures, especially collaborative gestures, have a powerful association with students' mathematical intuition and insight. Student groups who made no gestures were least likely to give a correct intuition or insight, while groups who made collaborative gestures specifically were most successful.

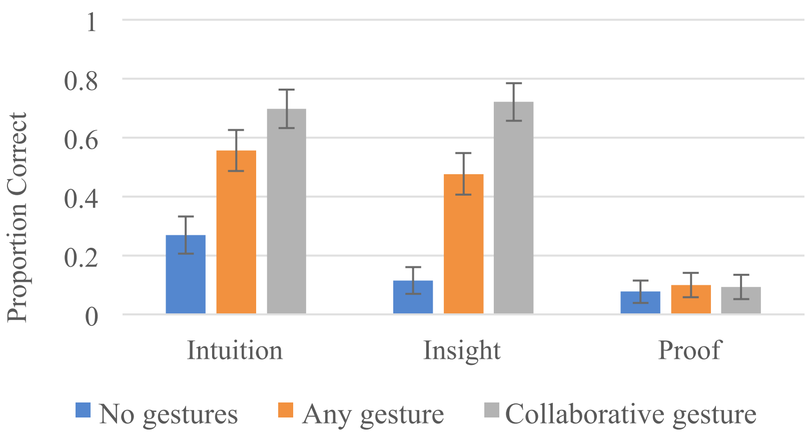


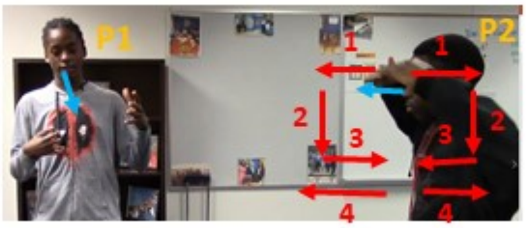
Figure 2. Association between outcomes (intuition, insight, proof) and gesture categories. Error bars represent standard error of the mean.

Our first transcript (Figure 3) shows two male students proving the conjecture that diagonals of a rectangle have the same length. The student on the left (P1) is controlling the game and gives an oral proof for the conjecture, which includes gesture. One interesting thing about this exchange is that P2 is literally silent the entire time, but continuously communicates significant mathematical information to P1 only using gesture. P2's gestures initially resemble addressee gestures (Alibali & Nathan, 2012), but then the two of them are gesturing simultaneously, with P1 both echoing and mirroring P2. P1 and P2 then reach intersubjectivity around the idea of diagonals being the same length using gesture as their primary mode of collaborative exchange of ideas. As P1 begins the proof, P2 begins a rhythmic, repeated series of gestures in the air tracing the sides of a rectangle (Lines 1-3). On Line 2, P1 echoes this gesture series, but then adds diagonal gestures instead of strictly horizontal and vertical gestures (Lines 3-4), noting that one diagonal is just a reflection in a different direction of the other. In response to this insight, P2 pauses his gesture sequence, nods at P1, and then makes his own diagonal gesture, echoing P1. P1 attends to this gesture with his gaze as P2 nods again, completing their proof. This transcript demonstrates how complex mathematical reasoning is communicated via gesture, and the power of collaborative gestures for supporting intersubjectivity, even in the absence of collaborative speech.

Our second transcript (Figure 4) shows two males students (P4 and P5) and one female student (P3) reasoning about the conjecture stating that the longest side of a triangle is across from the largest angle. This transcript shows an interesting and extended use of joint gestures to create embodied mathematical objects, and one larger observation from the dataset was that such gestures can open up access to mathematical ideas. Rather than there being someone controlling the computer, writing on the paper, or handling the manipulatives, anyone can make gestures at any time, and they are highly visible to all group members. However, this case shows that collaborative gestures can still exclude. Before the transcript began, P5 made a triangle gesture with his fingers, and tried unsuccessfully to get P4 to attend to his gesture. At the beginning of the transcript, P4 forms a triangle with his fingers (Line 6), and then P5 touches two of the sides of this triangle with his fingers in turn (Lines 7-

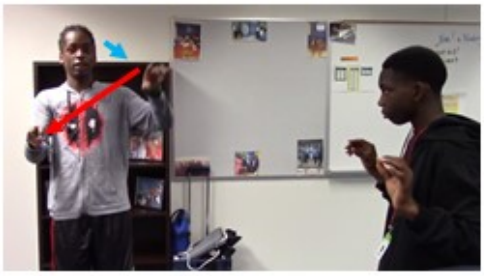
9), referencing the angles as they reason through the relationships together. P5 keeps his finger on P4's triangle gesture for a full 11 seconds (Line 10), and although P4 never looks at the joint gesture they are formulating, he

1 P1: "Um, okay, so the **length is going to stay the same because…**
((P2 traces horizontal and vertical lines in air with two hands, tracing rectangular shapes. P2's gaze is on P1.))

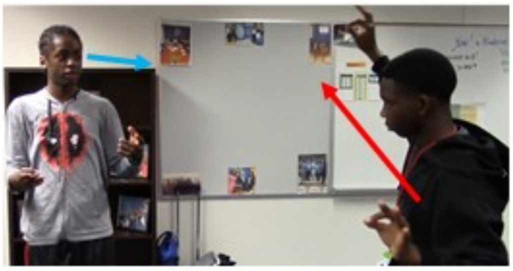


2 P1: **like if you think of it- if you think of it um…**"
((P1 extends two arms above him with pointer fingers extended and touching. He moves both fingers apart, brings them down, and connects them back together back and forth repeatedly. P1's gaze is following hand movements, but briefly flashes to P2. At the same time, P2 continues his pattern of horizontal and vertical lines in air, with gaze on partner.))

3 P1: "**The length,** the length isn't changing…"
((P1 moves hand back and forth from top left to bottom right. P1's gaze is on partner. P2 continues tracing lines in air, and then pauses his gesture and nods at P1. P2's gaze is on partner.))



4 P1: "…. it's more **the direction is changing**.
((P1 moves hand back and forth from top right to bottom left. Gaze stays on hands. P2 pauses and appears to be listening, gaze on P1.))



5 P1: "**You** know? So yeah. They will have the same length you know so yeah."
((P2 places all four index fingers and thumbs together by his stomach, and then draws right arm upward and diagonally. P1's gaze is on P2. P2 nods to P1.))

Conventions: Double parentheses describe gaze and gestures. Bolded text shows speech that co-occurred with gestures. Arrows show movement (red) or direction of gaze (blue). Numbers show the order in which gestures occurred. Method adapted from Goodwin (2003).

Figure 3. Multi-modal transcript of two students proving that diagonals of a rectangle are congruent

gets tactile feedback in that he can feel where on his gestured triangle P5 is pointing. The gesture remains anchored there as a sort of diagrammatic representation that they can continue to refer to for further scrutiny. P3, briefly glances at the joint gesture (Line 7) but remains excluded from P4's & P5's joint embodied reasoning. After the transcript ends, P4 and P5 continue reasoning together using collaborative gestures, excluding P3, who eventually draws on her hand to try to make sense of the relationships by herself.

6 P4: "The first **triangle…**
((P4 holds both index fingers and thumbs out and connects index to index finger and thumb to thumb. Gaze is on screen.))

7 P5: **This one…**
((P5 uses pinky finger to touch P4's index finger on the right. P4 and P5 gaze at screen, P3 briefly gazes at their hands.))

8 P4: **"is larger than the second angle…"**
9 P5: **"This one…"**
((P5 uses pinky finger to touch P4's index finger on the left. Each gaze is on the screen.))

10 P4: **"then the side opposite of the first triangle "… is longer than the side opposite**- wait, what?"
((P4 and P5 hold gesture, and then both drop it.))

11 P3: "I don't get it."

Conventions: Double parentheses describe gaze and gestures. Bolded text shows speech that co-occurred with gestures. Arrows show movement (red) or direction of gaze (blue). Method adapted from Goodwin (2003).

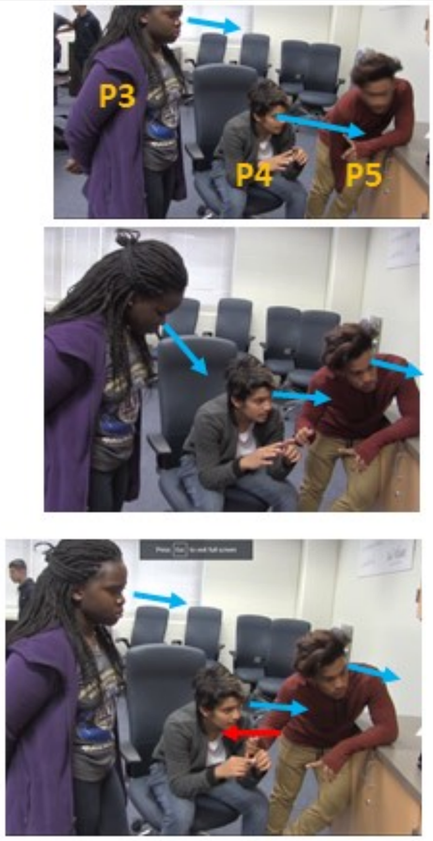


Figure 4. Multi-modal transcript of three students exploring the conjecture that the largest angle of a triangle is across from the longest side.

Educational and Scientific Significance

This study extends research on gesture and embodied mathematical cognition by empirically describing students' collaborative gestures in a sociotechnological context. First, we documented the ways in which high school students use gesture collaboratively and show how gestures and collaborative gestures are each associated with different problem-solving outcomes. Second, we show important ways in which students can use joint, embodied action to explore the truth and generalizability of geometric concepts, establish common ground through shared body-based resources, and build – literally – on each other's reasoning and enacted representations. Students use gestures collaboratively in powerful ways that enhances the quality of their reasoning. Having students sense-make together about mathematics using their bodies has the potential to open up access to mathematical reasoning and argumentation. In the present study, students enlisted their bodies and drew on the bodily actions of their peers in collaborative ways within an embodied gaming environment where typical media – pencil/paper, dynamic geometry software, measurement tools, and manipulatives – were not available. An important question stemming form this work is how such embodied collaboration can be facilitated. Geometric proofs may be a particularly rich domain for fostering collaborative gestures since these activities invoke spatial reasoning and motor systems, as well as socially mediated exploration and persuasion. We believe next steps include exploring task designs and other sociotechnological ways to engage students in joint, embodied reasoning, which activate collaborative gesture as another resource for their extended shared cognition.

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Acknowledgments

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