Pilot Study on the Impact of In Situ Spaced Professional Learning on Teachers' Mathematical Knowledge of Multiplicative Thinking

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This paper reports on a pilot study that incorporated an alternative professional learning model that was school-based and focused an identified area of need: multiplicative thinking. The shift to multiplicative thinking can be challenging for both students and teachers due to its multifaceted nature. The study involved the delivery of six modules of learning related to multiplicative structures, pedagogical approaches to learning and subsequent between session activities to staff in 14 participating primary schools. Our findings suggest such a model of professional learning that includes enactment and reflection supports change in teachers' pedagogical content knowledge.

In the current political climate, there is increased pressure on teachers to improve student-learning outcomes in mathematics education. Within Australia and New Zealand major initiatives such as, 'Count Me In Too' in New South Wales, the 'Victorian Early Numeracy Research Project', and the 'New Zealand Numeracy Development Project' were implemented to improve the professional capabilities of teachers and subsequently raise student achievement in mathematics (Bobis et al., 2005). Key components of these initiatives included the development of learning frameworks for teachers designed to identify student learning and inform planning; professional learning programs for teachers; the use of one-to-one interview assessment tools; and the appointment of consultants or numeracy coaches to support teachers in their planning and teaching. The aim of these projects was to link professional learning to students' learning and classroom practice through off-site professional learning with the support of a numeracy leader, or external mentor (Bobis et al., 2005).

This paper presents evidence of an alternative professional learning model that was offered in situ (based in individual schools); required a whole school commitment; and targeted at a specific area of need, namely multiplicative thinking. We argue that such a model that involves teachers as learners in a learning community, and directly relates to classroom practice, has the potential to impact on teachers' mathematics content knowledge (MCK), pedagogical content knowledge (PCK), and subsequent student learning.

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Theoretical Background

The research literature drawn on to inform this study included teacher professional learning models and the importance of multiplicative thinking, and the difficulties of teaching and learning this content.

Recent studies highlight the need to situate professional learning for teachers in realistic contexts as part of the on-going work in schools, in contrast to one-off models of professional development (Bruce, Esmonde, Ross, Dookie, & Beatty, 2010). Teachers are seen as learners and schools as learning communities (Clarke & Hollingworth, 2002). Bruce et al., (2010) support Clarke and Hollingworth's notion of professional learning being embedded in classroom experiences and practices within the school context, and argue that such professional learning is characterised as occurring in sustained and iterative cycles of planning, practice and reflecting. Furthermore, Desimone (2009) suggested that professional learning should be over an extended period of time, because significant change in teacher practice, and subsequently student learning can take up to five years. Timperley, Wilson, Barrar and Fung (2007) suggested that, "professional development that led to sustained better practice, had a focus on developing teachers' pedagogical content knowledge in sufficient depth to form the basis of principled decisions about practice" (p. xivi). Cobb, Wood and Yachel (1990) suggested an effective motivator for change could be to create "cognitive conflict" in the teachers' minds, by challenging their approach prior to them attempting to modify their classroom practice.

Clarke & Hollingsworth (2002) developed an Interconnected Model of Teacher Professional Growth that elaborated on a linear professional learning model proposed earlier by Guskey (1986). This model indicates a shift in emphasis in relation to professional learning and teacher change, from perceiving teachers as passive participants in a deficit model, to seeing change as a complex process that involves learning and growth. Within such a model teachers are considered "active learners in shaping their professional learning through reflective participation in professional learning programs and in their practice" (Clarke & Hollingsworth, 2002, p. 948).

The Interconnected Model of Teacher Professional Growth highlights four domains (external, personal, practices, and consequences) within a change environment. Each of the domains is connected so that change in one domain leads to changes in other domains through processes of 'enactment' and 'reflection'. Enaction is the process of interpreting and acting on a set of beliefs and pedagogy. In other words enaction is putting new ideas or new beliefs into practice. Reflection works with enaction to ensure that the implemented action is actively and carefully considered over time. The model focuses on two reflective practices: reflecting on the changes in teacher beliefs; and reflecting on the implementation of the new knowledge or new pedagogy. It also emphasises the change environment, and the impact the environment in which the teachers' work, has on teacher change.

Others (e.g., Clarke, Clarke, & Roche, 2011; Sowder, 2007) emphasised the importance of understanding how students think about and learn mathematics. Sowder purported that students provided an interpretive lens that "helps teachers to think about their students, the mathematics they are learning, the tasks that are appropriate for the learning of that mathematics, and the questions that need to be asked to lead to better understanding" (p. 164). Clarke, et al. (2011) argued that the use of task-based one-to-one interviews builds teacher expertise through "enhancing their understanding of individual and group understanding of mathematics" (p. 901) and thus building teachers' PCK and their MCK related to particular content matter.

Multiplicative Thinking

A recurring theme in the literature is that multiplicative thinking is central to students' mathematical understanding is the basis of proportional reasoning, and a necessary prerequisite for understanding algebra, ratio, rate, scale, and interpreting statistical and probability situations (e.g., Hurst & Hurrell, 2014). Some researchers argue that the difficulties associated with students' lack of proportional reasoning are related to their limited experiences of different multiplicative situations (e.g., Greer, 1988) or to their reliance on additive thinking when multiplicative thinking is required (e.g., Hurst & Hurrell). Sullivan, Clarke, Cheeseman, and Mulligan (2001) argued that teachers' reluctance to engage students in problems that gradually remove physical prompts and encourage students to form mental images of multiplicative situations is possibly why students do not make the transition to abstracting. Greer (1988) suggested three ways to overcome a reliance on additive thinking: first to include more complex number combinations in word problems so that the appropriate operation cannot be intuitively grasped; second to provide multi-step word problems, rather than single operation word problems; and third to experience the different multiplicative situations (Equal Groups, Rate, Multiplicative Comparison, and Rectangular Area/Array). Given the complexity and importance associated with developing multiplicative thinking, teachers need to have a sound mathematics content knowledge and pedagogical content knowledge for developing multiplicative thinking in their students.

Informed by the research literature this pilot study was situated within the teachers' own school and was directly related to their practice. Multiplicative thinking was identified by the teachers as a current concern, and that they were struggling to move students from additive to multiplicative thinking. The professional learning (PL) was spaced across three terms, and focused on developing teacher content knowledge relating to multiplicative thinking and pedagogical practices. Part of the PL required participants to conduct one-to-one interviews with students to assist the teachers to understand how their students are developing multiplicative thinking- thus building the teachers' PCK (Clarke et al., 2011).

In this paper we address the research questions: What is the impact of an in situ, spaced, professional learning on teachers' pedagogical content knowledge for developing multiplicative thinking in their students? What challenges do teachers experience when planning for and teaching multiplication and division?

Methodology

The following study is a pilot study of professional learning involving 14 schools out of a possible 57 Catholic primary schools in a New South Wales Catholic Education System. The results of this study will inform a larger scale study to include more system primary schools over a period of four years (2016 -2019).

The approach taken in this pilot study involved five 90-minute professional learning modules, delivered to each school across three school terms. The focus of the PL was to increase teachers' pedagogical content knowledge relating to multiplicative thinking and how students develop multiplicative thinking. The PL targeted Stage 2 (Year 3 & Year 4) teachers. However, each school had identified in their School Action Plans a high proportion of other students at their schools who were still reliant on additive strategies for multiplication and division. The pilot schools wanted the PL to be delivered to all teachers from Foundation to Year 6. The PL included between session activities that required participants to administer a multiplicative thinking interview with a sample of students from each grade and for teachers to trial tasks with their whole class.

Professional learning structure. The research team, led by a university academic, developed a series of modules with Teaching Educators (TEs) from a New South Wales Diocese. All the PL modules followed the same structure and included:

1. a professional reading about the multiplicative structure;
2. opportunities in Modules 2-5 to analyse student data (student work samples) after the tasks had been completed in the mathematics lesson;
3. opportunities for teachers to reflect on these student work samples - the observations of multiplicative thinking in student responses were recorded in teacher reflective journals;
4. opportunities for teachers to solve a series of learning tasks focused on each multiplicative structure that they would then plan to teach to their students after each module; and,
5. teaching the tasks from each module as a between module activity.

Professional learning modules. Module 1 was an overview of multiplicative structures and introduced teachers to a new multiplicative thinking interview; Module 2 examined the use of arrays as a multiplicative structure; Module 3 examined the multiplicative structure 'times as many'; Module 4 examined the multiplicative structure of allocation and rate; and, Module 5 involved an analysis of interview data and teacher reflection of student learning. Within each module there were challenging tasks related to the content and ways to adapt and extend tasks. Throughout each module important ideas about learning mathematics with understanding (exploring, reasoning, questioning, justifying and reflecting) were included. The elements of a lesson structure (problem solving, sharing solution methods and discussing the effectiveness of solution methods) and planning lessons were also considered in each module. Each module also included a resource pack for teachers.

Seven TEs facilitated the PL at participating schools they were aligned to across three terms (Term 2-4) as part of each schools' regular after school mathematics PL. Some TEs facilitated the PL in two or more schools. Pre- and post-surveys were completed by the teachers and leaders within the first and last modules. The facilitators' materials consisted of:

- a multiplicative thinking interview given to a sample of three students from each grade at two points in time (pre- and post- the teacher professional learning modules);
- a teacher survey of mathematical content knowledge about multiplicative thinking and how students learn to think multiplicatively in problems situations at two points in time (pre- and post- the professional learning);
- four professional readings for teachers about each multiplicative structure;
- a series of tasks focused on each multiplicative structure; and,
- work samples of students' solutions to the mathematics tasks taught after each module.

Participants. The participants for this pilot study included all classroom teachers lead teachers, Assistant Principals and Principals at each participating school (N=230). The school principals, teachers and students involved in the pilot study agreed to be part of the research. The Diocese Catholic Education Office gave permission for the research to be undertaken and University ethics approval was received.

Data collection. The data reported in this paper is from two of the nine open response questions in the teacher pre- and post-surveys. These questions were designed to gain insights into teachers' understanding of how students develop the ability to think multiplicatively in problem situations and the perceived challenges they face when planning and teaching multiplication and division. Two open response questions reported here include:

- How do you believe students develop multiplicative thinking?
- What are the main challenges you experience when planning and teaching multiplication and division? Why do you think this?

Data analysis. The teachers' responses to open response items were entered into a spreadsheet, coded then categorised through the analysis of data using a grounded theory approach (Strauss & Corbin, 1990). If a teacher shared multiple ideas or themes, each was coded as a separate response. The first two authors independently coded the teachers' responses using open coding to identify key themes. In collaboration, these authors conducted a further cycle of coding to derive 10 agreed categories. These ten categories were further refined to create eight categories with the input of the other authors. The frequency of responses for each category was collated and patterns identified across pre/post data.

Results and Discussion

This section presents the results relating to the open response questions, including teachers' perceptions of how students develop multiplicative thinking, and teachers' main challenges experienced when planning and teaching multiplication and division. There were less responses for the question related to challenges associated with planning and teaching as some of the participants were not classroom teachers.

Table 1 shows the pre- and post-responses related to the eight categories developed from the analysis of the data. The categories relating to multiplicative data (1-5) are presented in order of the percentages of responses in the pre survey data, then the general categories.

Table 1
Percentage of Responses Relating to How Students Develop Multiplicative Thinking

Category	Pre (n=244)	Post (n=236)
Multiplicative categories		
1. Using arrays, partial arrays and visualising the structure	11	28
2. Moving from additive moving to multiplicative thinking	8	2
3. Use of multiplicative language and recognising the relationship between multiplication and division	7	8
4. Being challenged to use more efficient strategies	4	15
5. Experiencing multiplicative structures	0	17
General categories		
6. Materials and representations moving to abstract thinking	30	11
7. Engaging in real life problems and open tasks	27	12
8. Teacher demonstration and practice	13	7

Prior to the PL approximately 70% of responses related to general pedagogical approaches to mathematics, compared to 30% post the Pl. The percentage of responses relating to multiplicative thinking (70%) was more than double that prior to the PL (30%), which appears to suggest that the professional learning program had a positive impact on how they considered students develop multiplicative thinking. The TEs indicated that they emphasised the use of materials and representations, moving to abstract thinking and real-life problems and open tasks in their work with schools, so it is not surprising to see the higher proportion of responses for these two categories. More responses focused on the need for students experiencing different multiplicative structures, being challenged to use more efficient strategies and the use of arrays, after the professional learning.

The data related to the second survey question is presented in Tables 2 and 3. The teachers' responses relating to planning are presented in Table 2. The categories are ordered according to the responses for the pre-survey data related to multiplicative thinking first and then general pedagogical issues.

Table 2
Percentage of Responses (Pre/Post) Relating to Challenges when Planning

Category	Pre (n=217)	Post (n=196)
Multiplicative thinking issues		
1. Incorporate multiplicative language in planning and teaching	6	10
2. Provide experiences of both aspects of division	5	7
3. Provide experiences of the different multiplicative structures	2	12
4. Moving students from using additive to multiplicative strategies	2	8
General pedagogical issues		
5. Catering for diversity: Planning open-ended task	47	32
6. Writing enabling and extending prompts	1	8
7. Using relevant real life contexts and problems that challenge students	25	16
8. Moving students from materials and representations to abstract thinking	6	4
9. Making links between assessment data and syllabus expectations	4	2
10. Time to plan as a team	3	1

The main challenges teachers had post the PL were still predominantly related to catering for diversity, and using relevant real-life problems that challenge students. Planning open-ended tasks and problems relating to real life context is challenging for teachers because generating such task requires sound understanding of the key ideas underpinning the mathematics content and knowledge of their students' conceptual understanding. Although the PL engaged teachers in these aspects they are still areas of concern for teachers.

While their knowledge of the components of multiplicative thinking has increased, so has the challenge of incorporating them into their planning as they are still assimilating some of this new learning. As Desimone (2009) indicated, change takes time, so ongoing support from Lead Teacher and TEs would be encouraged. These results also indicate that aspects relating to planning need to be incorporated into the full implementation of this project.

Table 3
Percentage of Responses (Pre/Post) Relating to Challenges when Teaching

Category	Pre (n=217)	Post (n=196)
Multiplicative thinking issues		
1. Encouraging students to use efficient methods to solve problems	14	30

2.	Encouraging students to make links between multiplication and division	10	25
3.	Encouraging the use of multiplicative language	12	25
General pedagogical issues			
4.	Finding the balance between explicitly teaching strategies and allowing students to generate their own.	19	4
5.	Predicting challenges or possible questions students might have and knowing how to respond in the moment	11	6
6.	Allowing students enough time to struggle before intervening	10	5
7.	Parental pressure to teach the "times tables" and long division	10	2
8.	Lacking knowledge to teach this content well	14	3

As reported in Table 3 the responses prior to the professional learning suggest more of a tension between getting the balance right between explicit teaching and student generated strategies, knowing when to hold back from telling, and how to respond in the moment. While these aspects were still present in the post-survey responses they were not the main challenges. The majority of post-survey responses (80%) related to challenges associated with teaching related to multiplicative thinking compared with 36% of responses before the professional learning. In particular, encouraging students to use efficient methods to solve problems, make links between multiplication and division, and to use multiplicative language, were aspects of their practice they found challenging. Having greater awareness of the importance of each of these components in assisting students to move from additive to multiplicative thinking is encouraging.

The post professional learning survey data relating to challenges in planning and teaching for multiplicative thinking highlight a need to place greater emphasis on task selection, catering for diversity, links between multiplication and division, and multiplicative language around planning and teaching in the revised professional learning program.

Concluding Comments

This PL was informed by earlier research (Clarke & Hollingsworth, 2002: Clarke et al., 2011; Timperley et al., 2007) in that it was design around an identified area of need; teachers were active participants in the learning; and involved enactment, reflection and between session tasks. Enacting new mathematical practices and reflecting on the impact on student learning has an impact on change in teachers' understanding of how students learn. As one Year 3 teacher (Sophie) reflected in her diary:

> The language of times-as-many was challenging for students initially but once they had more experience with tasks like this, I saw a shift in the strategies they used and they were using multiplicative language and making connections between multiplication and division.

Noticing such a shift illustrates Sophie's own growth in understanding how students develop multiplicative thinking.

In relation to the research questions, the findings suggest that providing in situ targeted professional learning across a year that includes teachers conducting one-to-one task based interviews has potential to improve teachers PCK and CK with respect to multiplication and division. Although not reported here, several teachers indicated that conducting the one-to-one interviews provided a greater understanding of the complexity associated with how students develop multiplicative thinking, highlighting the value of including interviews as part of the professional learning. The findings indicated areas to explore in more depth in the full implementation of this PL model. These include making the links between multiplication and division more explicit, ways to differentiate learning, task selection, and planning. We conclude that the teachers appreciated the opportunity to engage in PL in their school, related to an identified area of need, and to build collective understanding and work collaboratively within and across year levels.

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