Revisiting Friedrich Froebel and his Gifts for Kindergarten: What are the Benefits for Primary Mathematics Education?

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This paper provides historical insights and educational background of Froebel's Gifts, hands-on materials developed in the early 19th century. Based on an explorative study with 54 German children (aged 5 to 10) in 2016, we first took steps to explore how these materials meet the demands for early mathematics learning of primary children. Starting with a brief introduction into the work and life of Friedrich Froebel, we outline how using Froebel's Gifts can stimulate the acquisition of mathematical knowledge and abilities, then conclude by considering future research directions within Australian schools.

Introduction

Knowledge of geometrical shapes and solids is a core element of geometry and mathematics education in primary schools. Classroom activities often focus on sorting and naming (prototypical) representatives of three-dimensional objects. However, recent international studies suggest that there is a need to shift focus from naming and sorting shapes to learning experiences that develop primary children's spatial reasoning, including "working on the composing/decomposing, classifying, comparing and mentally manipulating both two- and three-dimensional figures" (Sinclair & Bruce, 2015, p. 319).

Block play has been a popular activity in early childhood for decades. Block-building activities provide children with opportunities to classify solids, manipulate shapes, and develop spatial sense (Reinhold et al., 2013). In this paper, we describe block play activities invented by Friedrich Wilhelm August Froebel, recent studies that have explored the mathematical potential when using blocks in German schools, and conclude with suggestions for future research within Australian schools.

Historical Background – Froebel at his Time

Friedrich Wilhelm August Froebel (1782-1852)

Friedrich Wilhelm August Froebel, one of the most famous European philanthropists and reform pedagogues, was born in Germany in 1782. He lived during a historical period marked by social and political turbulences with feudalism turned to capitalism in most parts of Europe. Froebel, the son of a priest, completed a forester's apprenticeship after finishing school. This was followed by uncompleted studies in science at the University of Jena (Germany). After a short employment as a private secretary, Froebel moved to Frankfurt/Main (Germany) to apply to do architecture, but changed his plans to train as a teacher. He taught in schools in Frankfurt and Yverdon-les-Bains (Switzerland) that used the pedagogy of Johann Heinrich Pestalozzi (1746-1827). Froebel also worked as a private tutor in a nobleman's family and returned to Germany (University of Göttingen) to study languages and natural sciences. In 1816, Froebel finished his academic studies and founded

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a correctional institution in Griesheim (Germany). During this phase (1817-1830), he focused on establishing his ideas of pedagogy for early childhood and released one of his most in-depth publications: *Die Menschenerziehung* (*Education of Mankind*; Fröbel, 1826). His philosophical and ideological views were further expanded in the publication, *Die Menschenerziehung* providing insights into Froebel's understanding of educational development beginning with early childhood (Fröbel, 1832/1982).

Based on this concept, Froebel opened the *Anstalt zur Pflege des Beschäftigungstriebs für Kindheit und Jugend* (institute for the care of the children's desire for occupation) in 1837, situated in Bad Blankenburg (Germany). Thereby, he established what he called the first "kindergarten" (children's garden) – where the children were supposed to raise and bloom like flowers in the metaphorical sense. Friedrich Froebel died in 1852, aged 70.

Children's Play and Play Gifts Invented by Froebel

Froebel's influence on early childhood pedagogy (including early mathematics learning) went beyond establishing childcare institutes. In doing so, he developed standards for childcare that were compatible with a humanistic and democratic approach to education. No doubt, his approach was influenced by the philosophical discussion during the period of Enlightment in the 18th century in Europe. Froebel's opinions of how children engaged at kindergarten differed from his contemporaries' concepts that were often limited to ideas of adult supervision and instruction (Heiland, 1991).

Similar to Pestalozzi, Froebel was convinced that children gain a deeper understanding of the world around them when given opportunities to interact with the world via concrete activities, including manipulation of prepared hands-on materials. In addition, Froebel stressed the key role of children's play for their educational development (Fröbel, 1832/1982). He recommended providing children with well-chosen materials during play with the intention to stimulate their understanding of the world. Hence, Froebel's constructive approach (in the sense of discovering the world, e.g., spatial relationships or properties of geometrical solids by constructing with blocks) forms an important part of the work in "kindergarten" (Frey et al., 2006, p. 169).

Pursuing his intentions, Froebel developed special educational toys for his "kindergarten", namely the so-called *gifts* ("Spielgaben"). Two meanings are associated with this term: first, the idea of 'presenting the child a gift to play with', and second that each child possesses innate human 'gifts'. The gifts were embedded in a variety of learning settings, and kindergarten educators were encouraged to guide the children during their reconstruction of geometrical arrangements.

In total, Froebel developed six gifts, encompassing the growing complexity of the world around us: Gift 1 includes six balls made of different materials whereas Gift 2 includes a variety of solids (cube, cylinder, sphere). In contrast, Gifts 3, 4, 5, and 6 represent the idea of decomposing the cube into smaller units like small cubes, small cuboids/rectangular prisms, triangular prisms of different sizes. For example, Gift 3 consists of a threefold divided cube which (after being cut orthogonally in each direction), produces eight congruent smaller cubes that can be assembled into a variety of (new) arrangements while playing (see, Figure 2). Gift 4 (Figure 2) is made of eight smaller cuboids (relation of side lengths is 1:2:4). This gift and Gifts 5 and 6 (including prisms) invite children to explore mathematical concepts such as measuring heights and surfaces and encourage discoveries of the idea of balance or part-whole relationships (Henschen & Tescher, 2013). Furthermore, Gift 5 resembles Gift 3, however instead of eight cubes (Gift 3), Gift 5 appears to consist of 27 cubes. By cutting three of the 27 cubes into two congruent triangular prisms, and cutting another three cubes into four smaller triangular prisms Gift 5 offers another expansion of possible constructions (Hebenstreit, 2003). Here, the ratio of volume to single part is 1:2:4 (Figure 1). Gift 6, resembles the less complex Gift 4 comprising the division of 27 cuboids, making a composition of 36 elements of different shapes (cuboids of the same size as in Gift 4 and divided cuboids).

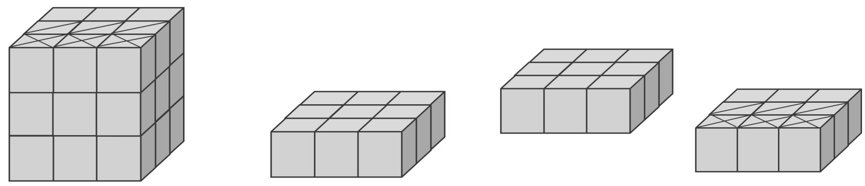

Figure 1. Froebel's Gift 5 (parts arranged as a cube on the left, decomposed into layers on the right).

An analysis of the elements of the Froebel's Gifts 3, 4, 5, and 6 reveals the growth in structural complexity. The gifts have the potential to encourage children to explore and discover geometric properties of three-dimensional objects, including spatial relations among parts of the gift pieces. In addition to arranging the pieces of a gift as a cube, Froebel suggested to arrange all parts of a gift in new arrays. While arranging the pieces in these ways, a child explores what Froebel called *forms of knowledge* (dividing the entire cube into smaller, equal pieces)*, forms of life* (arrangements that resemble objects from children's environment like a sofa or staircase) and *forms of beauty* (symmetrical arrangements of smaller parts). This element of Froebel's concept aligns with Pestalozzi's philosophy, reflecting three different approaches of life and learning: *Forms of knowledge* – our head and mind; *forms of life* – what we touch with our hands; and *forms of beauty* – what touches our heart (Frey et al, 2006; Heiland, 1991).

Exploring Froebel's Gifts in Primary Mathematics – First Insights

A Traditional Hands-on Material Seen from Today's Perspective

The original setting for using the Froebel's Gifts with children was in early education of Friedrich Froebel's first kindergarten, established in the early 19th century. Having a closer look at the structural properties of the gifts, it could be argued that from today's mathematics educational perspective the gifts have the potential to foster young children's understanding of geometry, deepen their geometric knowledge and spatial skills.

Geometry and spatial thinking is viewed as children's foundation knowledge needed for learning mathematics in the early years (e.g., Horne, 2003). Guay and McDaniel (1977) found that mathematically high-achieving children in elementary school outperform mathematically low-achieving children not only in terms of their arithmetic skills and knowledge but also concerning their visualization skills. This connection is shown specifically for the ability to mentally rotate or mentally compose/decompose spatial arrangements (Linn & Petersen, 1985). In addition, activities in the first years of learning arithmetic are most often based on the use of hands-on materials (e.g., depicting an arithmetic operation with counters placed in rows of tens, each of the rows partitioned in two lots of five counters laid down with a blank space). Children have to identify the geometrical structure of these hands-on materials before developing mental representations, based on the intentional manipulation of these materials (Radatz, 1990). In line with these arguments, Clements and Sarama (2011) have indicated that geometry and spatial reasoning are important to the development of mathematics. However, they also indicated that these areas of mathematics are often neglected, or receive little attention in the early years' classrooms. This is a concern as geometry forms a fundamental dimension of the mathematics curriculum.

Meeting Australian and German Standards in Primary Education

The National Council of Teachers of Mathematics (NCTM, 2000) recommends that spatial reasoning and geometric modelling includes creating mental images of geometric shapes, recognizing and representing shapes from different perspectives. Similarly, within the Australian Curriculum (AC; Australian Curriculum and Reporting Authority, 2017), children develop geometric reasoning skills when describing, analysing, and understanding structures in their world. However, geometric reasoning skills as suggested within the AC are introduced to children at Grade 3 (aged eight) rather than when commencing school.

Geometry has been in the German curriculum for primary education since the late 1960s (Franke & Reinhold, 2016), with a strong focus on shape recognition and drawing skills, especially in the Deutsche Demokratische Republik (DDR, communist part of Germany until 1990). During the 1990s, the mathematics education discussion on the importance of geometric activities and visualization influenced a shift from shape recognition, drawing and preparation for geometry at secondary level, to geometrical reasoning and opportunities to develop spatial abilities. Today's German national standards for mathematics in primary (KMK, 2004) outline a wide range of geometrical competencies children should develop by the end of Grade 4. These refer to three (among) five key content areas, namely *space and shape* (Raum & Form), *pattern and structure* (Muster & Struktur), and *measurement* with length and volume (Größen & Messen). Regarding *space and shape*, the standards stress the importance of visualization skills (e.g. creating mental images of geometric shapes, mentally manipulating them, orientation in space, changing perspectives). The standards also indicate that children should be able to identify, name, depict, produce and systematically investigate the geometric properties of representatives of geometrical shapes and objects.

Raising New Questions Concerning Traditional Hands-On Material

Studies with German preschool and primary school children's use of Froebel's Gifts provided evidence of geometric reasoning and strategies when solving construction problems (Reinhold, 2015; Reinhold & Wöller, 2016). In these studies, the tasks using the gifts were initially used as a tool to investigate the development of young children's geometrical concept knowledge. However, experiencing the gifts in these studies raised the question: "To what extend are the gifts suitable to pursue current objectives in primary mathematics education (Grades 1 to 4, aged 5-10)?" This broad and challenging question motivated an explorative project involving Froebel's Gifts in German schools.

In 2016, six complementary studies conducted by Master of Education students contributed to the exploration and use of Froebel's gifts in German primary classrooms (see also Friedl et al., 2017). Each master's student developed an individual research focus and question, including various methods for data collection and analysis. They collected and interpreted data with a common aim to empirically support the hypothesis that learning environments encompassing Froebel's Gifts contributes to children's development of a mathematical issue. The chosen research topics included geometry and arithmetic. The master's students observed the children working in small groups or with larger groups of children during mathematics lessons. All activities were video-recorded and mostly accompanied by one-to-one interviews with individual children after their work with the Froebel's Gifts. Altogether, 54 German children (Grades 1 to 4) in different primary schools participated in the project and complementary studies. The following examples from these explorations report insights into the results.

Discoveries in Geometry

Previous studies within German schools focused on how analyses of differences in children's block construction processes and products contribute to a deeper understanding of their conceptual knowledge of geometrical solids (e.g. Reinhold & Wöller, 2016). For younger (preschool) children, we also reported on their difficulties in (re)constructions of cube arrays for purposes of enumeration (Reinhold et al., 2013). However, there was some evidence that children's fine motor function and their general kinaesthetic competence when assembling single blocks or components of three-dimensional arrays was usually developed at the beginning of school. Therefore, we asked primary children to articulate their conceptual knowledge of geometrical solids when constructing with Froebel's Gifts. These investigations revealed that primary children (aged eight to nine) faced difficulties when demonstrating their geometrical understanding of cuboids and concepts of a cube. During their construction of cubes and cuboids in the interviews, children explained why they considered their construction was a cube by naming properties or offering other comments such as: "Because it has equally long sides", "It looks the same from all sides", or "All surfaces are the same and it's three-dimensional."

These research findings were the starting point for the design of classroom activities with Froebel's Gifts 3 and 4 for Grades 1 and 2. Wiesner (2016) included explorations of the blocks that led to a range of constructions – with some children discovering (without further explanation) that two of the rectangular prisms in Gift 4 had the same volume as two cubes in Gift 3. Further group activities included the use of several sets of Froebel's Gifts led to the (re)construction of bigger cubes or cuboids (Figure 2). These activities were accompanied by exercises that required the children to name the shapes of the blocks they used. During the discussion at the conclusion of each lesson, the teacher (a master's student) stressed the special relationship between cube and cuboids. Individual interviews before and after this intervention supported the hypothesis that the activities with the gifts had a positive influence on the development of children's conceptual geometrical knowledge of cubes and cuboids. For example, children were likely to correctly name shape properties by making connections to cubes or cuboids (see also Friedl et al., 2017).

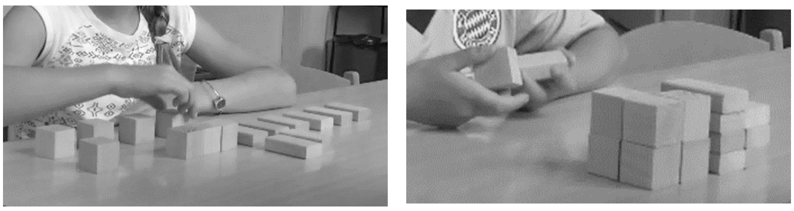

Figure 2. Exploring the properties of cubes and cuboids with Froebel's Gifts 3 and 4 (Wiesner, 2016).

Daniel (2016) conducted a study with a pre-post-test design and an intervention using the Gifts 3 and 4. He designed activities according to the concept of natural differentiation (e.g., Krauthausen & Scherer, 2013) for a group of Grade 4 students. For example, providing children with a construction plan (Figure 3) challenged them to construct various possible solutions using the pieces of Gifts 3 or 4.

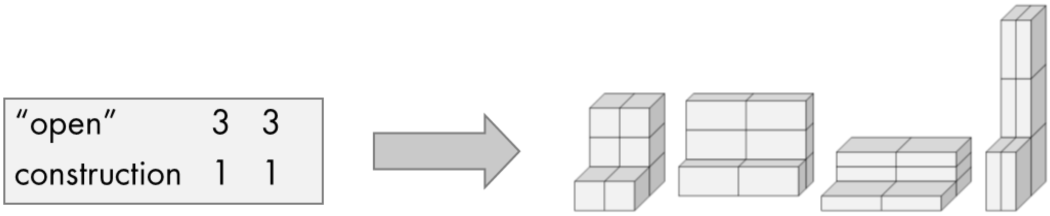

Figure 3. Construction plan and solutions using the blocks of Froebel's Gifts 3 and 4 (Daniel, 2016).

Another challenge was to analyse a complex array made from pieces of Gift 4, that required children to visualize (mentally move and compose pieces) how many and what type of different birds-eye views can occur if all pieces of Gift 4 (made of eight congruent rectangular prisms) are constructed as a cube (Figure 4). The analyses of the pre- and post-test revealed that children demonstrated a more analytic approach when distinguishing between cubes and cuboids. Terms such as "surface" or "volume" were used accurately.

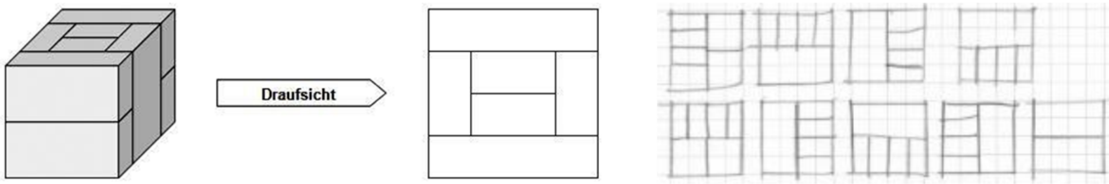

Figure 4. Task and student's drawings of (possible) top views (Daniel, 2016).

Tackling Arithmetic Issues

In addition to using Froebel's Gifts to explore geometric topics, there were investigations relating to arithmetic issues. For example, Laußmann (2016) focused on part-whole relationships and used the pieces of Gift 3 in her work with first-graders. After a lesson that provided opportunities for children to explore free constructions with Gift 3, Laußmann asked the children (in an interview) to find as many different partitions (groups) into three (four, five, or six) subsets as possible (Figure 5). All children found at least one possible solution, some found a range, while others worked systematically and tried to find *all* combinations. Sullivan (2015) described a task designed to find multiple solutions as having the potential for all students to find a solution, as 'low floor' benefits, and when students generalise having found all solutions, as 'high ceiling' benefits. This activity not only addressed the arithmetic idea of part-whole-relationships, but challenged the children's problem-solving, argumentation skills and ability to justify their argument.

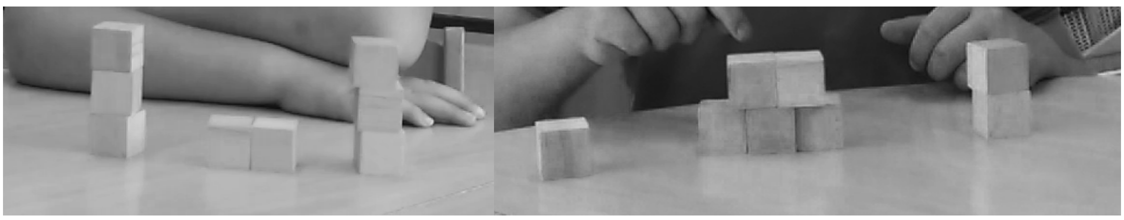

Figure 5. Dividing the number of cubes of Froebel's Gift 3 into three subsets (Laußmann, 2016).

Von Lucke (2016), explored Grade 1 children's arithmetic knowledge when using Gift 6 and focused on number concepts. Her study explored children's knowledge of sorting and classifying objects and counting representations of the same objects. In her interviews, Von Lucke asked the children to compare the number of single units in two sets, of same cardinality (e.g., six smaller and six bigger cuboids), exploring the concept of invariance, or one-to-one correspondence, respectively. As a result, activities of construction and re-construction with the pieces appeared to positively influence children's understanding of the cardinality as most children were able to demonstrate one-to-one correspondence by rearranging the pieces (Figure 6).

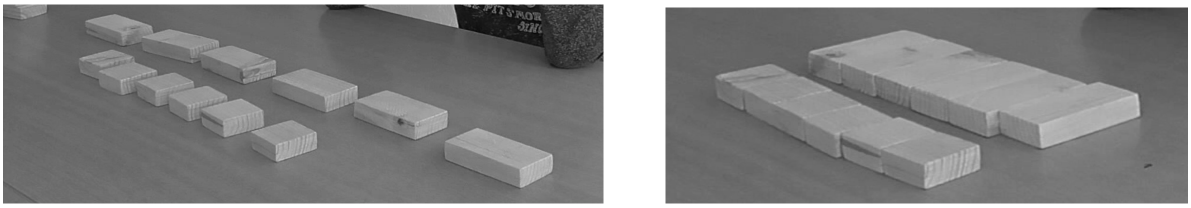

Figure 6. Invariance of the number of single units in two sets of the same cardinality (Von Lucke, 2016).

Conclusion: Research Desiderata and Ensuing Studies

The focus of our paper was to provide a historical overview of Froebel's Gifts and to present examples of the mathematical attributes of the gift sets. In previous studies on children's geometrical concept knowledge (e.g., Reinhold & Wöller, 2016), activities with the gifts were informative for exploring children's knowledge of geometrical objects. Moreover, several explorative studies by master's students suggest that using the gifts in German primary classrooms contribute to meeting today's international standards for mathematics education in Grades 1 to 4 – aligning the German and the Australian curriculum, and providing opportunities for natural differentiation in tasks with "low floor and high ceiling" (Sullivan, 2015). However, a review of literature suggests that Froebel's Gifts have yet to be used in Australian research studies. Therefore, we identify three possibilities for prospective research based on the results of these (explorative) studies with the gifts, namely using Froebel's Gifts (i) as a *tool to investigate Australian children's geometrical concept knowledge* on cubes and cuboids, (ii) in *pre- and in-service teacher education*, and (iii) *in the Australian primary classroom*. The outcomes will contribute to knowledge for improving educational opportunities and outcomes in education, with a focus on the early years of primary mathematics education, teaching and learning, and ultimately develop a theoretical framework for international comparisons.

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