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Predicting Growth Rate of Students' Achievement in Mathematics Using Mathematical Growth Model

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ABSTRACT

Students' mathematics achievements in secondary school have an influential effect on their performance in university and their future careers. There are numerous innovative teaching strategies which are promising and have shown to improve students' achievement in mathematics; one will wonder if there will be corresponding achievement growth in mathematics. Therefore, this paper predicts growth rate of students' achievement in mathematics using mathematical growth model. A developmental survey design was employed where students' WAEC mathematics achievement results from 1991-2016 were collected from the Development Division, West African Examination Council (WAEC) Lagos, Nigeria. The Logistic growth mathematical model was used to compute predicted achievement growth rate. The Statistical Package for Social Sciences (SPSS) was also used to plot the graph of the achievement data from 1991 to 2016. The result indicates that the predicted growth rate of students' achievement in mathematics is approximately increased by 25.3% per year. This shows a weak upward trend in predicted rate of achievement at credit level and consequently indicates unsteady sustainability in mathematics achievement as well as economic growth. It is recommended among others that there should be investigation into factors responsible for the weak and unsteady achievement growth in WAEC general mathematics in Nigeria. Also, effective teaching and learning of mathematics at all levels of schooling should be ensured by all concerned in other to sustain the achievement growth in the subject.

INTRODUCTION

Mathematics is a very desirable tool in virtually all spheres of human endeavour, be it science, engineering, industry, education, technology and even the arts. It is the key to a modern technological society. Its versatility is on the increase as it provides newer insights and finds wider application with the passage of time. So proficiency in mathematics is a major advantage in industrialized nations. Every individual needs mathematical knowledge to function intelligently and efficiently in his or her world. Mathematics is one subject that is an integral part of everyone's life and affects virtually every field of human endeavour.

Knowledge of mathematics is crucial to educational and financial success in contemporary society and is becoming ever more so. An average man needs mathematics to survive no matter how rudimentary (Anaduaka & Okafor, 2013). In a nutshell, mathematics is the pivot around which the whole essence of living revolves and the basis for scientific and technological take-off.

One of the major variables that measure an individual's success or failure in mathematics or school activities is academic achievement. Academic achievement or academic performance is the outcome of education - the extent to which a student, teacher or institution has achieved their educational goals. Academic achievement is generally regarded as the display of knowledge attained or skills developed in the school subject (Bossaert, Doumen, Buyse & Verschueren, 2011). The target of every educational program is to ensure that the students achieve a satisfactory outcome. Academic achievement is the expressed outcome of individuals or institutions effort geared to achieve a desired educational goal. This is often cognitive based and measured by examination or continuous assessment but there is no general agreement on how it is best tested or which aspects are most important - procedural knowledge such as skills or declarative knowledge such as facts (Yeung, 2015).

Students' mathematical achievements in secondary school have an influential effect on their performance in university and their future careers. High school students' mathematics achievement predicts college matriculation and graduation, early-career earnings, and earnings growth (Murname, Willett, & Levy, 1995; National Mathematics Advisory Panel (NMAP, 2008). Having a solid background in mathematics helps students develop sophisticated perspectives and offers more career options. The importance of mathematical learning has repeatedly been emphasized by educators and government (Wilkins & Ma, 2002). Both teachers and parents have paid attention to students' achievement in mathematics and their progress every year. Governments have also called for improving students' overall performances and closing students' achievement gaps. Until teachers and parents recognize what factors influence their students' mathematics achievement and improvement, they will be unable to help them make substantial academic progress (Ma & Klinger, 2000). However, Ahiakwo (2006) had earlier found that the achievement of various levels of students in mathematics have decelerated over the years with that of Nigerian students quite remarkable. Furthermore, WAEC (2001 – 2010) Chief examiners reports of results of our public examinations had shown markedly a decline in the percentage of passes in mathematics. Useni, Okolo and Yakubu (2012) also reported less than 50% pass rate at credit level when they carried out the analysis of students' performance in WASSCE mathematics in selected secondary schools in Awe L.G.A of Nasarawa state from 2002-2011.

Now that there are numerous innovative teaching strategies which are promising and have shown to improve students' achievement in mathematics, one will wonder if there will be corresponding achievement growth in mathematics. Previous studies have neglected to address the issue of achievement growth rate of students in detail. Growth rates refer to the percentage change of a specific variable within a specific time period, given a certain context. For investors, growth rates typically represent the compounded annualized rate of growth of a company's revenues, earnings, dividends and even macro concepts such as GDP and the economy as a whole. Expected forward-looking or trailing growth rates are two common kinds of growth rates used for analysis. At their most basic level, achievement growth rates are used to express the annual change in achievement as a percentage. For example, an achievement's growth rate in mathematics is derived as the annual rate of change at which a student's achievement in mathematics increases or

decreases. In fact, this rate of growth is used to measure a mathematics achievement's growth or decay. If the mathematics achievement of a student within a school declines for two consecutive terms, it is considered to be in decay. Conversely, if the school has a growth in mathematics achievement for two consecutive terms, it is considered to be growing. In addition to mathematics achievement growth rates, specific schools also have growth rates. Each school has a unique benchmark number for rates of growth that its performance is measured against. For instance, schools on the cutting edge of high academic standard are more likely to have higher annual rates of achievement growth compared to other low standard schools.

Mathematical Growth Model

The use of mathematical growth model to predict students' mathematics achievement growth rate is an innovative introduction of mathematical modeling in investigating some of the educational phenomena. Specifically, this paper will resort to population growth model in ecology to predict students' achievement growth rate in mathematics. A fundamental population growth model in ecology is the logistic model due to Verhulst. In one respect, logistic population growth is more realistic than exponential growth because logistic growth is not unbounded. We can write the logistic model as,

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) \qquad (1)$$

Solving the logistic model equation, we use separation of variable technique. We obtain

$$\frac{dP}{P(1-\frac{P}{K})} = rdt \Rightarrow \int \frac{dP}{P(1-\frac{P}{K})} = r\int dt$$

Integrating by partial fraction, we have

$$\ln|P| - \ln\left|1 - \frac{P}{K}\right| = rt + c$$

Easy algebraic manipulations give

$$\ln\left|\frac{P}{1-\frac{P}{K}}\right| = rt + c \Rightarrow \frac{P}{1-\frac{P}{K}} = ce^{rt}$$, where c is constant. Solving for P, we get

$$P = \frac{Kce^{rt}}{K + ce^{rt}}$$

If we consider the initial condition $P(0) = P_0$ (assuming that P_0 is not equal to both 0 or K), we get

$$c = \frac{P_0 K}{K - P_0}$$

which, once substituted into the expression for $P(t)$ and simplified, we have the required solution as

$$P(t) = \frac{P_0 K}{P_0 + (K - P_0)e^{-rt}} \qquad (2)$$

where $P(t)$ is the population size at time t (assume that time is measured in days), P_0 is the initial population size, K is the carrying capacity of the environment, defined as the maximum population size an environment can support, and r is a constant representing the rate of population growth or decay. The use of the logistic growth model is widely established in many fields of modeling and forecasting. First order differential equations govern the growth of various species (Banks, 1994). Now, solving equation (2) for the rate of population growth or decay, we have that

$$P(t) = \frac{P_0 K}{P_0 + (K - P_0)e^{-rt}}$$

$$P(t)[P_0 + (K - P_0)e^{-rt}] = P_0 K$$

$$P_0 + (K - P_0)e^{-rt} = \frac{P_0 K}{P(t)}$$

$$(K - P_0)e^{-rt} = \frac{P_0 K}{P(t)} - P_0$$

$$e^{-rt} = \frac{1}{K - P_0}\left(\frac{P_0 K}{P(t)} - P_0\right)$$

$$\Rightarrow -rt = \ln\frac{1}{K - P_0}\left(\frac{P_0 K}{P(t)} - P_0\right)$$

$$\Rightarrow r = -\frac{1}{t}\ln\left[\frac{1}{K - P_0}\left(\frac{P_0 K}{P(t)} - P_0\right)\right] \qquad (3)$$

This equation (3) gives the rate of students' mathematics achievement growth or decay (r).

METHOD

A research is best understood as a process of arriving at dependent solutions to the problems through the systematic collection, analysis and interpretation of data. In relation to this paper, developmental survey design was employed where students' WAEC mathematics achievement results from 1991-2016 (inclusive) were collected from the Development Division, West African Examination Council (WAEC) Lagos, Nigeria. The Logistic growth mathematical model (equations 2 and 3) was used to compute predicted achievement growth rate. The Statistical Package for Social Sciences (SPSS) was also used to plot the graph of the achievement data from 1991 to 2016 (inclusive).

RESULTS AND DISCUSSION

Based on the number of students who had credit pass (C6-A1) in WAEC mathematics examinations from 2004 to 2016 (inclusive) in Table 1 below, let $t = 0, 1, 13$ correspond

to the years 1991, 1992 and 2004 respectively. Then P_0, P_1, $and\ P_{13}$ are respectively 32727, 79026 and 565570.

Table 1. Actual number of students who had credit pass (C6-A1) in WAEC mathematics examinations from 1991 to 2016.

t	*Year*	P_0	*t*	*Year*	P_0
0	1991	32,727	14	2005	388,122
1	1992	79,026	15	2006	472,979
2	1993	53,559	16	2007	198,441
3	1994	83,192	17	2008	314,903
4	1995	76,080	18	2009	425,633
5	1996	51,587	19	2010	453,447
6	1997	47,252	20	2011	587,630
7	1998	70,587	21	2012	819,390
8	1999	57,858	22	2013	555,726
9	2000	173,816	23	2014	529,732
10	2001	350,746	24	2015	544,638
11	2002	142,589	25	2016	597,310
12	2003	237,377	26	2017	-
13	2004	565,570	27	2018	-

Source: Test Development Division, West African Examination Council (WAEC) Lagos, Nigeria.

Table 1 shows the number of students who had credit pass (C6-A1) in WAEC mathematics examinations from 1991 to 2016 (inclusive). This observation suggests increase in the rate of student' achievement in mathematics. The graphical representation of this observation via SPSS is shown in figure 1 below.

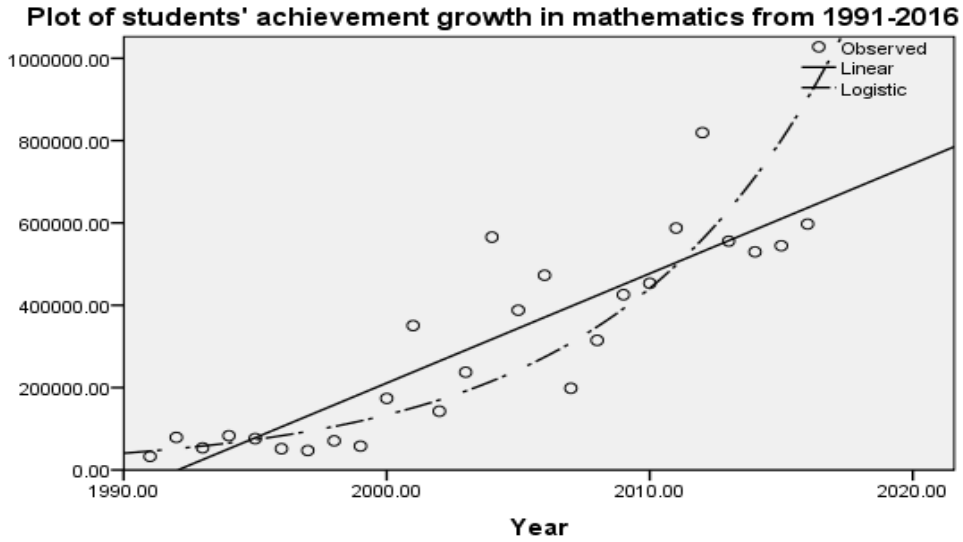

Figure 1. Plot of actual number of students who had credit pass (C6-A1) in WAEC mathematics examinations from 1991 to 2016 against time in years. This shows increasing mathematics achievement growth (linearly and logistically) by year.

Now, to calculate the predicted growth rate of students' achievement in mathematics, we use equation (3) which is given as:

$$r = -\frac{1}{t}\ln\left[\frac{1}{K - P_0}\left(\frac{P_0 K}{P(t)} - P_0\right)\right] \qquad (3)$$

Using the population projection of 1,544,234 of students who sat for WAEC mathematics in 2016, as the carrying capacity with the conditions of $P_0 = 32727$ and $P(13) = 565570$, and from equation (3), we have that

$$r = -\frac{1}{13}\ln\left[\frac{1}{1544234 - 32727}\left(\frac{(32727)(1544234)}{565570} - 32727\right)\right] = 0.253$$

$r = 0.253$

This implies that the predicted growth rate of students' achievement in mathematics is approximately increased by 25.3% per year.

From figure 1, it can be seen that the predicted growth rate of students' achievement in mathematics is approximately increased by 25.3% per year. This corroborate with the study of Zalmon and Wonu (2017) who showed that after twenty six (26) years ago, 27.3% of students in Nigeria obtained credit and above (A1- C6) while 72.7% had pass and below (D7-F9) in the May/June WASSCE in general mathematics. They also showed that there was significant difference in the percentage of students in Nigeria that obtained credit and above (A1 - C6) and pass and below (D7 - F9) in May/June WASSCE in general mathematics from 1991 to 2003 and 2004 to 2016 years. This result indicates little increase in students' achievement in mathematics for the periods considered. This finding is consistent with WAEC chief examiners reports that candidate's performance in mathematics for the May/June 2013 and 2014 examinations appeared to have improved when compared to previous years (WAEC, 2013 & 2014). This improvement over time could be attributed to mathematics curriculum reforms, availability and utilization of innovation instructional materials, strategies and practices for teaching and learning, investment in education and hard-work on the part of the students (Zalmon & Wonu, 2017). Although poor achievement of students in mathematics was observed in the past twenty-six years but there was a little improvement in students' mathematics achievement over time. It is interesting to note that the finding of the present study is not different from the findings of Useni et al (2012) who reported less than 50% pass rate at credit level when they carried out the analysis of students' performance in WASSCE mathematics in selected secondary schools in Awe L.G.A of Nasarawa state from 2002-2011.

CONCLUSION

This is not encouraging for the future of the nation in the aspects of human capacity development in Science, Technology, Engineering and Mathematics (STEM) Education which is a critical aspect of Nigeria vision 20:2020. The implication of the weak upward trend in predicted rate of achievement at credit level is that of sustainability in mathematics achievement as well as economic growth. This suggests that mathematics achievement upward growth in the next few years is in doubt, especially at credit level. This paper enjoined others in recommending that there should be investigation into factors responsible for the unsteady achievement in WAEC general mathematics in Nigeria. Also, effective

teaching and learning of mathematics at all levels of schooling should be ensured by all concerned in other to sustain the achievement growth in the subject.

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Memprediksi Tingkat Perkembangan Prestasi Belajar Matematika Siswa Menggunakan Model Pertumbuhan Matematis

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ABSTRAK

Makalah ini memprediksi tingkat perkembangan prestasi siswa dalam matematika menggunakan model pertumbuhan matematis. Penelitian ini menggunakan *developmental survey design* Data diperoleh dari hasil prestasi siswa WAEC matematika dari 1991-2016 yang melalui Divisi Pengembangan, Dewan Ujian Afrika Barat (WAEC) Lagos, Nigeria. Model matematika pertumbuhan logistik digunakan untuk menghitung prediksi tingkat pertumbuhan pencapaian. Paket Statistik untuk Ilmu Sosial (SPSS) juga digunakan untuk memplot grafik data pencapaian dari tahun 1991 hingga 2016. Hasil menunjukkan bahwa tingkat pertumbuhan yang diprediksi dari prestasi siswa dalam matematika meningkat sekitar 25,3% per tahun. Hal ini menunjukkan tren peningkatan yang lemah dalam memprediksi tingkat pencapaian pada tingkat kredit dan akibatnya menunjukkan keberlanjutan yang tidak stabil dalam pencapaian matematika serta pertumbuhan ekonomi. Disarankan antara lain bahwa harus ada investigasi terhadap faktor-faktor yang bertanggung jawab atas pertumbuhan prestasi yang lemah dan tidak stabil dalam matematika umum WAEC di Nigeria. Juga, pengajaran dan pembelajaran matematika yang efektif di semua tingkat sekolah harus dipastikan oleh semua pihak yang terkait untuk mempertahankan pertumbuhan prestasi dalam mata pelajaran.

Kata kunci: Matematika, prestasi, laju pertumbuhan, model pertumbuhan matematis

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