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A comparision of freshman and senior mathematics student teachers' views of proof concept

Davut Köğce[a*], Cemalettin Yıldız[b]

[a,b]*Karadeniz Technical University, Fatih Faculty of Education, Trabzon,61100,Turkey*

Abstract

This study was conducted to determine and to compare freshman and senior mathematics student teachers' views of 'proof' concept. Case study method was used in this study. The data of the study were obtained by conducting a questionnaire which consists of 4 open-ended questions to total 200 freshman and senior mathematics student teachers studying in Department of Elementary Mathematics Education, in Fatih Faculty of Education, in 2010-2011 academic year. At the end of the study, it was determined that senior mathematics student teachers' definitions of proof were more similar to those reported in the literature than freshman mathematics student teachers' ones.

Keywords: Mathematics education; mathematical proof; levels of proof; mathematical reasoning; student views

1. Introduction

One of the most important goals of mathematics education is to ensure the development of mathematical thinking and reasoning by answering questions of "why?" and "how?". A person whose mathematical thinking and reasoning ability is advance can talk about a problem or activity which he / she studies on, make predictions and hypothesize, prove the accuracy of conclusions, make generalizations. In addition to this he / she can express his / her proofs and generalizations mathematically, isolate with a formal language, interpret the information presented by written or visually (Baki, 2008). In this context, the mathematical proof has a great role in the development of mathematical thinking and reasoning ability (Knuth, 2002; Stylianides, 2007; Tall 2002). Proof is made up of some universally accepted methods. Proofs can be made mainly either by induction or deduction. Deduction type of proof involves several methods such as direct proof, proof by contra positive, and proof by contradiction (Baki, 2008; Moralı, Uğurel, Türnüklü & Yeşildere, 2006). Proof has several other purposes such as *explanation, systemization, communication, discovery of new results, justification of a definition, developing intuition, providing autonomy* (Weber, 2003).

[*]Davut Köğce. Tel.: +90-462-377-72-62
E-mail address: d_kogce@yahoo.com

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With the increasing importance attached to proof in mathematics, the thinking processes and development of students from diverse age groups have become the subject of many studies (Knuth, 2002; Stylianides, 2007). However, doing a proof is considered to be a challenging, fearsome, and unlovely process by many students at all levels including the elementary, secondary and higher education (Almeida, 2003; De Villiers, 1990; Jones, 2000; Raman, 2003). Looking at the literature, it is stated that students have many difficulties and lack of information about the concept of proof and proofing. It is observed that there are a lot of things to cause this difficulties such as; lack of knowledge about the definitions of proof and how to use them (Edwards & Ward, 2004; Knapp, 2005; Moore, 1994; Weber, 2006), not to understand the nature of proof, mathematical rules and proof strategies (Gibson, 1998; Weber, 2006) and not to use mathematical language correctly (Baker & Campbell, 2004; Edwards & Ward, 2004; Knapp, 2005; Moore, 1994).

By studies of Anapa and Şamkar (2010) and Jones (2000); it's been determined that student teachers have lack of self-trust on proofing and have difficulties to understand proof theorems. In a study by Moralı et al. (2006), most pre-service teachers were found to have either no or insufficient views about doing proofs. In another work by Özer and Arıkan (2002), it was found that high school students could not use proof methods and techniques sufficiently and could not do proof at an expected level. Similar results were reached, in another study by Almedia (2001). In a study conducted by Köğce, Aydın and Yıldız (2010) it has been identified that in a large part of the 10th grade students believe the necessity of the proofing because of their perception of proofing as to show the accuracy of a statement given, *to see the origin of mathematical knowledge, facilitating understanding, providing continuity on learning and to see the truth and inaccuracy*.

To determine the opinions and perceptions of teachers of future (student teachers) about the concept of proof has great importance**,** as teachers' perceptions and experience of proofing is effective in the process of gaining proofing abilities (Almeida, 2003). Considering the literature, in terms of teaching profession, there is not any study which compares freshman mathematics student teachers with senior mathematics student teachers in terms of the concept of mathematical proofing. When the role of doing proofs in mathematics education is taken into account, it's obvious that the number of studies is scarce in this field in Turkey. For this reason, the main purpose of this study is to determine and to compare freshman and senior mathematics student teachers' views of 'proof' concept.

2. Method

This is a descriptive study using case study method. Although case studies are widely used in both qualitative and quantitative inquiries, in the case of qualitative research they enable in depth investigation of a single or a number of cases, phenomena or events with a limited sampling (Çepni, 2007).

2.1. Participants

The study group consists of total 200 randomly selected freshmen and senior mathematics student teachers studying in Department of Elementary Mathematics Education, in 2010-2011 academic year. 170 mathematics student teachers' answers were taken into consideration because 30 mathematics student teachers did not answer questionnaire items. 99 of them are freshman student teachers and 71of them are senior student teachers.

2.2. Data collection tools

The data of this study were collected using a questionnaire consisting of 4 open-ended questions. Mathematics student teachers' answers to the first 2 questions have been used as a data source in this study. These items used questionnaire are as follows:

Item 1: What's mathematical proof according to you? Can you define it briefly?

Item 2: Is there a need for mathematical proof? Why?

2.3. Data analysis

All student teachers' responses to these items were thematically classified in regard to their similarities and differences by the authors (Merriam, 1988; Yin, 1994). In this process, the authors marked their responses individually, and then, all disagreement points were solved by negotiation. Finally, the codes have been created by referring to expert opinions.

3. Results and Discussion

The views of the students related to the definition of proof were coded and the percentages and frequencies of these codes with an example student answer were given in Table 1.

Table 1. Frequencies, percentages and a sample student response of the codes generated for Item 1*

Codes of Students' Responses

C1: to show the accuracy of a result	**C7:** the logical explanation
C2: to show how to practice mathematical operations	**C8:** to provide an expression for all values
C3: to show a result numerically	**C9:** to show an expression as algebraic
C4: to show the accuracy of a proposition or theorem	**C10:** to reach unknown by the help of known values
C5: to show the accuracy of a formula or rule	**C11:** create a formula or generalization
C6: to show the accuracy or inaccuracy of a given statement	**C12:** to explain an information on the basis of previous information

Codes	Freshman Student Teachers			Senior Student Teachers		
	f	%	A Sample Student Response	f	%	A Sample Student Response
C1	9	9	*Solving a problem or question to verify the result*	1	1	*The whole actions taken to ensure the accuracy of the results obtained in mathematics*
C2	11	11	*to show how to find the result of an operation*	2	3	*The set of operations provides a better understanding of mathematical expressions which are accepted as correct*
C3	1	1	*to show the accuracy of the expression by values taken by arbitrary*	0	0	---
C4	29	29	*to search the accuracy of a theorem or proposition with mathematical language*	18	26	*to show the precision of a proposition or theorem by mathematical terms and symbols*
C5	33	33	*to search the accuracy of a our information and formulas by using some of the methods*	4	6	*to show why and where mathematical formulas and information is coming from*
C6	3	3	*to put forward accuracy or inaccuracy of a problem*	8	11	*Basic element of mathematics which help us to understand accuracy or inaccuracy of a promotion*
C7	16	16	*to reach a logical conclusion while problem solving*	10	14	*to show how the mathematical expressions form by various methods in the framework of logic*
C8	4	4	*to search the accuracy of any judgement for all values*	7	10	*to show the validity of a theorem or proposition in each case*
C9	14	14	*to explain a formula or equation without depending on numbers (i.e. algebraic)*	6	9	*to show that a theorem is provided for which they're given a variable value for each case*
C10	5	5	*to explain a mathematical expression by the help of knowledge and acceptance acquired previously*	0	0	---
C11	7	7	*to put forward a formula without depending on numbers*	2	3	*to reach a generalization or formula by converting indefiniteness and contradictions to definite conclusions*
C12	0	0	---	26	37	*to answer question of why, without doubt, concerning theorems and formulas*

*: *Since students' responses can be labelled under more than one code, the percentage may exceed 100%.*

It's seen that student teachers' views on the concept of proofing are collected under 12 different codes. The codes **C1, C4** and **C5** which refers to verification, **C2** which takes attention to process of mathematical operations, **C7** which emphasis on the logical side of proofing, **C9** which refers to mathematical language, **C11** which emphasis on abstraction, seem more among freshman mathematics student teachers. In addition, **C3** which emphasis the first step of inductive reasoning method and **C10** which emphasis to process of proofing were only expressed by freshman mathematics student teachers. In contrast, it's been seen that senior mathematics student teachers were more mention the codes **C6** and **C8**. In addition, **C12** which is important to the development of mathematical thinking and reasoning were only expressed by senior mathematics student teachers. These definitions made by student teachers about mathematical proofing has similar findings in studies done by De Villiers (1990), Köğce, Aydın and Yıldız (2010), Stylianides (2007) and Weber (2003).

The views of the students related to the necessity of doing proof were coded and the percentages and frequencies of these codes with an example student answer were given in Table 2.

Table 2. Frequencies and percentages of students' responses for Item 2*

Codes regarding the Necessity of Proof						
Yes Necessary						
CYN1: to provide to see the origin of mathematical expressions				**CYN6:** to develop a new perspective giving meaning to mathematics		
CYN2: to facilitate comprehension understanding rather than memorization				**CYN7:** improving cogency		
CYN3: providing retention				**CYN8:** providing to see the formation of formulas and rules		
CYN4: to provide seeing accuracy or inaccuracy				**CYN9:** to develop the power of thought and comment		
CYN5: providing to see accuracy				**CYN10:** to provide the production of new information		
No Necessary						
CNN1: to trust mathematical generalizations				**CNN2:** to confuse student's minds		
Codes	**Freshman Student Teachers**			**Senior Student Teachers**		
	f	**%**	**A Sample Student Response**	**f**	**%**	**A Sample Student Response**
CYN1	15	15	*... If the basis of mathematical formulas are proved, it will understand rather than memorization…*	*16*	*23*	*Proofing helps us to search mathematical information in deep and to learn underlying causes.*
CYN2	39	39	*... We can learn better by proofing instead of memorization…*	*33*	*47*	*... Proofs help to understand mathematics. It frees students from memorization.*
CYN3	15	15	*Proofing plays a very important role in learning ... to increase the permanence of formulas in mind.*	*12*	*17*	*... By proofing, where and which stages the information came will be seen.*
CYN4	6	6	*Proofing precisely shows the correctness of theorems or rules us. Therefore, it is the indispensable element of mathematics.*	*2*	*3*	*... Proofing sometimes reveals inaccuracy of a theorem which is seen correct and sometimes nonexistence of a theorem...*
CYN5	29	29	*We have to prove an idea to show its accuracy.*	*13*	*19*	*... It is important for everyone to accept mathematics as a set of rules …*
CYN6	5	5	*... Maths which is based on memorization is meaningless and forgotten easily. If it prove, the basis of mathematics can understand and carried out ideas more easily.*	*0*	*0*	---
CYN7	18	18	*... We can use mathematical proof to explain any statement or opinion, or persuading someone.*	*15*	*21*	*Proofing is made for to convince and persuade someone to our claims. Therefore, it is sine qua non of mathematics.*
CYN8	28	28	*Proofing is important to learn where a formula or expression comes from.*	*31*	*44*	*Proofs can show how a mathematical theorem or idea occurs and the stages which a mathematical theorem or idea passes…*
CYN9	6	6	*Proofing helps to think and make comments on question, to understand the questions and subjects better…*	*7*	*10*	*Proofing help us to understand facts better in mathematics. It can increase the power of commenting and problem solving.*
CYN10	4	4	*Proofing can lead us to new discoveries. We can implement these discoveries in our lives…*	*11*	*16*	*If the indefinitenesses haven't been proved, the science and technology wouldn't be at this point.*
CNN1	2	2	*The formulas used in mathematics or the accuracy of generalizations are certain.*	*0*	*0*	---
CNN2	3	3	*I think there is no need proofing. Because it is more confusing for students.*	*0*	*0*	---

**: Since students' responses can be labelled under more than one code, the percentage may exceed 100%.*

Mathematics student teachers' answers to questions about the necessity of proof have been collected under 12 different codes. **CYN1, CYN2, CYN3, CYN5, CYN7** and **CYN8** codes which is about necessity of mathematical proofing appears intensely in freshman and senior mathematics student teachers. **CYN4** code is low in both grade, **CYN9** and **CYN10** codes are intense among senior mathematics student teachers. **CYN6** code is only said by freshman mathematics student teachers. These codes which are obtained about the necessity of mathematical proofing are similar to the codes in study which is made by Köğce, Aydın and Yıldız (2010). In addition, freshman mathematics student teachers who claim the unnecessity of mathematical proofing express their reasons in **CNN1** and **CNN2**. In a study, Anapa and Şamkar (2010) have determined that student teachers do not trust themselves and cannot understand proofing of theorem. This case may be the result of acceptance of the information presented without filtering logically.

4. Conclusion and Recommendation

Majority of freshman mathematics student teachers express proofing as showing accuracy of an expression and senior mathematics student teachers express as showing both accuracy and inaccuracy of expression, being applicable to all conditions and to explain information by relationship of cause and effect. This case can be interpreted as senior mathematics student teachers look mathematical proofing more wide-angle than freshman mathematics student teachers. Even if some of the definitions about proofing of student teachers are superficial; it is recommended to academic staff to take into account of the potential of proofing in learning and teaching of mathematics and to organize activities according to it.

Looking at the findings about necessity of the proofing, it has been identified that while senior mathematics student teachers have believe the necessity of mathematical proofing, a part of freshman mathematics student teachers have views that there is no need proofing. It is recommended that teachers and academic staff should question the problems which have seen with students and to organize activities which improve thinking and reasoning skills of students to overcome this problem.

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