

## Historical Problems in the History of Mathematics: Examples for the Classroom

Cemalettin YILDIZ\*, Adnan BAKI\*

**Abstract:** Teachers often look for good sources to find problems motivating students and stiffen the concepts they are going to teach and use in the classroom. This causes the question: "Where can we find good problems that can be used in classrooms?". History of mathematics presents many historical problems which can enrich the classroom environment and be a self source of learning. By historical problems students can notice that mathematical problems may be solved in different ways and they can understand some methods different societies used in history are more practical. Because of these reasons knowledge about historical problems that have an important place in mathematics teaching are presented in this study. Document analysis method is used in study. Some studies about historical problems and some examples of historical problems presented in textbooks are given in this study. Also some historical problems and usage of these problems on lessons are mentioned.

*Keywords:* Mathematics education, history of mathematics, historical problems.

Most of the students search for good problems to use on their lessons. These kind of problems can be searched in historical development process of mathematics. Because History of Mathematics (HM) is an excellent source of interesting historical problems stimulating problem solving skills of students (Wilson & Chauvot, 2000). Historical problems show the kind of mathematics ancient people interested in (grain storage, taxation, business start up etc.). Even if they are old problems, solution method of them acts as an intuitive work in developing more modern solution methods (Swetz, 1994a). Also historical problems give us knowledge about past and they show the present situation of nowadays (Swetz, 2000).

Historical problems importantly contributes to motivation of students and getting lessons more enjoyable. Besides, historical problems contribute to development of mathematical thinking of students (Liu, 2003). It is a fact that historical problems take interest of students and contribute to sampling of the subjects too (Kar & İpek, 2009). Historical problems also say many things to us about applications, importance and basics of mathematics. Generally these facts come out as exciting student inventions (Swetz, 1994a).

Since they are real, historical problems carry many historical properties inside them. These kind of problems take students to another time, place and most important of these to another cultural environment. Students suddenly find themselves busy with interesting and motivating innovations such as dowry price calculation or defining necessary meadows for the farm animals. For example they can change the Babylon problems given in base 60 to the base 10. Similar to this, we can change the ancient measuring units to the modern ones we use today (Swetz, 1994a).

Historical problems must be in a form that makes students think logically and strengthens their mathematical skills. That's why aims of using problems taken from HM must be identified well (Kar & İpek, 2009). According to Swetz (1994b) historical problems can be used in a class for 5 different

\* Karadeniz Technical University, Fatih Faculty of Education, Trabzon, 61100, Turkey, email: cemalyildiz61@gmail.com

\* Karadeniz Technical University, Fatih Faculty of Education, Trabzon, 61100, Turkey

purposes: 1.To be able to take attention of the readers and allow discussion of periodical, 2.To improve the reasoning skills of students with examples taken from different cultures and time periods, 3.To support teaching, 4.To discover situations, 5.To illustrate the growth of mathematical proficiency.

Except the aims of historical problems written above, historical problems can be used to show students the ancient problem solving methods and techniques too. Because solutions of historical problems may be useful for their own problems of students and may give new ideas to them. Also historical problems may be used to increase self confidence of students (Savizi, 2007). Because of these reasons it can be said that historical problems have an important place in mathematics teaching. That's why in this study some studies about historical problems, historical problems taken from textbooks and historical problems of different societies are included too.

### **Method**

In this compilation type research, document analysis method is used. This method depends on collection and examination of present recordings and documents (Balci, 2006).

#### ***Historical Problems***

##### ***Studies about historical problems***

Ofir and Arcavi (1992) give examples to falsification method used historical problems and they show how these problems are solved by ancient Egyptians and modern method. They state their purposes as more than getting students specialize about trying the false, their aims are speaking with students about mathematics and evaluating their development process. Solution of the 24<sup>th</sup> problem on Rhind papyrus is shown both with the modern method and the trying the false method in this study. Also in this study problems of 16<sup>th</sup> century Italian mathematicians Tartaglia and Calandri are included and solutions of these problems with both the modern method and the trying the false method are given.

Swetz (2000) made a study about how the historical problems can be converted to suitable forms for using in the modern classrooms we have today. That's why in his book he gave place to more than 25 historical problems about mathematical expressions and geometry which are intended to use mathematical intelligence. At the end of the study it is stated that HM is full of many rich problems that can be moved to modern classrooms we have today.

Meavilla and Flores (2007) made a teaching practice for using historical problems on mathematics lessons. They expressed the aim of the teaching practice they prepared as getting students notice usage of some methods students used for solution of mathematical problems in historical process and make them see some different solution methods of problem solving. Also solutions of some historical problems are given in this teaching application too.

Oliver (2007) showed solutions of three sample historical problems, one taken from Moscow papyrus and the other two from Babylon papyrus showing how arithmetical and geometrical problems were solved at past. In the study, the periods are separated into three as the period of changes in spellings of algebraic expressions past to now is called as verbal period, using abbreviations period and the period that symbols used are expressed and examples of algebraic expressions used in each period are given.

Savizi (2007) searched practical historical problems. As an example to these problems he explained the method of Biruni that he measured the perimeter of the world. Researcher stated that historical problems take interest of students and show students the applications of mathematics to solve problems in real life. Also in the study it is indicated thinking about solution and modelling methods of a historical problem show students ancient ways and techniques of problem solving. Next to these in the research it is also expressed solution of historical problems may be useful for own problems of the students and may give new ideas to them.

When the studies about historical problems summarized above are examined, HM is seen to be full of rich problems that can be moved to classrooms. Also it is stated that with the historical problems students can notice mathematical problems may be solved in many different ways and some methods ancient cultures used may be more handy. As well as historical problems are said they might be used to help in taking interest of the students and solving the problems they meet.

### Examples of historical problems prominent in textbooks from past to now

In Turkey HM took place in textbooks by the elementary mathematics instructional program renewed in 2005. That's why textbooks of 6<sup>th</sup>-8<sup>th</sup> classes published after 2005 are examined. As the result of these examinations it is found that historical problems are included only in the 6<sup>th</sup> class books prepared by Göğün and Demir (2008) and Durmuş (2008). The historical problems took place in these textbooks are given in Figures 1, 2 and 3 respectively.



Figure 1. Problems of increasing the volume of a cube to twice of it and dividing an angle into 3 co-pieces (Göğün & Demir, 2008)

Hindistan'da M.S. 570-600 yıllarda devrin şahı, satranççı bulan kişiye ödül olarak ne istediğini sorar. Bu kişi "Bulduğum bu oyunun birinci karesi için 1 buğday, 2. karesi için 2 buğday, 3. karesi için 4 buğday istiyorum. Böylece her karede, bir önceki karede aldığım buğdayım 2 mislini almış olacağım." der. Şah ise bu isteği küçümser. Sizce bu istek gerçekleştirilebilir mi? Neden?

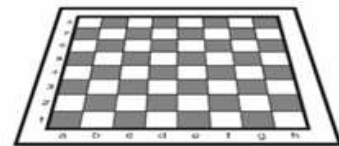
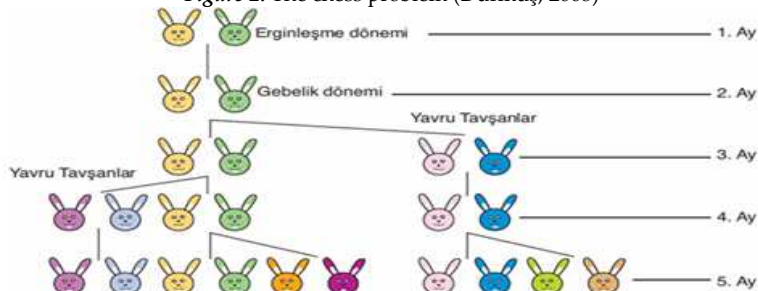


Figure 2. The chess problem (Durmuş, 2008)



Biri erkek, diğeri dişi olan bir çift yavru tavşan bir ayda erginleşiyor. Her çift tavşanın bir çift yavru tavşan doğurabilmesi için, erginleştikten sonra bir ay geçmesi gerekiyor. Hiçbir tavşanın ölmediğini ve her dişi tavşanın ayda bir erkek ve bir dişi tavşan doğurduğunu düşünürsek, bir yılın sonunda kaç çift tavşan olur?

Figure 3. The rabbit problem (Durmuş, 2008)

When Figures 1, 2 and 3 are examined it is seen that historical problems in textbooks are given only in question forms and there are no activities about them. Also it is very remarkable that there is no historical problem in 7<sup>th</sup> and 8<sup>th</sup> class textbooks.

### *Problems of different cultures*

Some historical problems chosen from studies of Swetz (1994a) and Grugnetti (2000) are given in the following.

**Babylonia-Tablet Collections (2000-1000 B.C.):** Thousands of Babylonian clay tablets have been discovered. Most have been unearthed at the sites of the ancient cities of Nippur and Susa. About 400 of these tablets are known to contain mathematical material, but few been deciphered.

1. I have added the area and two thirds the side of my square, and it is  $35/60$ . What is the side of the square. **Answer:**  $1/2$  unit.

**Egypt-Rhind Papyrus (1650 B.C.):** Rhind Papyrus was written nearly in 2000 B.C. It is a source that gives knowledge about ancient Egyptian mathematics. This papyrus includes 87 problems and their solutions.

2. Divide 100 loaves among 10 men including a boatman, a foreman, and a doorkeeper, who receive double portions. What is the share of each? **Answer:** Regular share,  $100/13$  loaves; special share,  $200/13$  loaves.

**Egypt-Cairo Papyrus (250 B.C.):** A collection of 40 problems, 9 of which concern right triangles.

3. An erect pole of 10 cubits has its base moved 6 cubits. Determine the new height and the distance the top of the pole is lowered. **Answer:** 9 cubits; 2 cubits.

**China-Nine Chapters on the Mathematical Arts (100 B.C.):** A collection of 246 problems and their solutions. The problems all pertain to the bureaucratic needs of the Chinese empire. The problems are divided into nine chapters, with each chapter focusing on a specific application.

4. A tree is 20 feet tall and has a circumference of 3 feet. Three is a vine that winds seven equally spaced times around the tree and reaches to the top. What is the length of the vine? **Answer:** 29 feet.

**Greece-Greek Anthology (A.D. 500):** A collection of 46 problems assembled by the Metrodorus. The problems are set in a witty or riddle form.

5. Demochares has lived a fourth of his life as a boy, a fifth as a youth, and a third as a man, and has spent 13 years in his old age. How old is he? **Answer:** 60.

**Europe-Propositions for Sharpening the Wits of the Young (A.D. 800):** A collection of 53 puzzle problems compiled by the monk Alcuin of York.

6. A ladder has 100 steps. On the first step sits 1 pigeon; on the second, 2; on the third, 3; and so on up to the hundredth. How many pigeons in all? **Answer:** 5050.

**Islamic World-The Algebra of al Khwarizmi (A.D. 820):** Khwarizmi wrote an algebraic book named "*El' Kitab'ül-Muhtasar fi Hisab'il Cebri ve'l-Mukabele*". Khwarizmi combined mathematical knowledge of Greeks and simple equations of Indians with this book.

7. You have two sums of money, the difference of which is 2 dirhams; you divide the smaller sum by the larger and the quotient is equal to  $1/2$ . What are the two sums of money? **Answer:** 4 dirhams and 2 dirhams.

**India-Lilavati (A.D. 1150):** A text written by the mathematician-astronomer Bhaskara II and named after his daughter. This work is a summary of 500 years of Hindu mathematics tradition.

8. One fourth of a herd of camels was seen in the forest; twice the square root of that herd had gone to the mountain slopes; and 3 times 5 camels remained on the riverbank. What is the numerical measure of that herd of camels? **Answer:** 36 camels.

**Italy-Liber Abaci (A.D. 1200):** This book is written by Italian mathematician Fibonacci. The book consisted of 15 chapters devoted to techniques of arithmetic and simple algebra.

9. Two towers, the heights of which are 30 paces and 40 paces, have a 50 paces distance. Between the two towers there is a font where two birds, flying down from the two towers at the same speed will arrive at the same time. What is the distance of the font from the two towers? **Answer:** 18 paces.

**America-The New Normal Mental Arithmetic (A.D. 1873):** Written by Edward Brooks, mathematics professor of Pennsylvania University. The Brooks series of arithmetics were widely used in Pennsylvania and Maryland during the second half of the 19<sup>th</sup> century.

10. Suppose that for every 4 cows a farmer has, he should plow 1 acre of land, and allow 1 acre of pasture for every 3 cows; how many cows could he keep on 140 acres? **Answer:** 240 cows.

**England-A Treatise of Arithmetic (A.D. 1880):** A standard British text of the time, written by J. Hamblin Smith.

11. If 12 horses can plow 96 acres in 6 days, how many horses will plow 64 acres in 8 days? **Answer:** 6 horses.

These historical problems can be used as a worksheet in class environment after they are arranged to the problem solving steps of Polya.

### Conclusion and recommendation

This study shows historical problems may be very useful for the students when they are used in classes, historical problems aren't given much place in the present textbooks we use today and there are many historical problems that can be used on lessons. These are suggested according to the research:

1. Number of historical problems on 6<sup>th</sup> class textbooks may be increased and historical problems may be given place in 7<sup>th</sup> and 8<sup>th</sup> classes textbooks. Also activities about historical problems may be added to 6<sup>th</sup>-8<sup>th</sup> classes textbooks.

2. HM includes many historical problems inside. These problems can be used for educational studies during the education process. Using historical problems may be better for a teacher who wants to write the problem of the week on blackboard (Swetz, 2000).

3. Historical problems can be used as exercise studies or for looking at the mathematical subjects from a different point of view. Also historical problems may act as excellent homework problems for the students or as challenging problems for advanced level students too (Marshall, 2000).

### References

- Balci, A. (2006). Research methods, techniques and principles in social science. Ankara: Pegem Publications.
- Durmuş, S. (Editor) (2008). Elementary 6 mathematics textbook Ankara: İmpress İmaj Domestic and Foreign Trade Co. Ltd.

- Göğün, Y., & Demir, A. (Editor) (2008). Elementary 6 mathematics textbook. Ankara:Özgün Printing.
- Grugnetti, L. (2000). Ancient problems for the development of strategic thinking. In J. Fauvel & J. Van Maanen (Eds.) History in mathematics education: The ICMI study (pp. 78-81), Dordrecht, Boston, London: Kluwer Academic Publishers.
- Kar, T., & İpek, A. S. (2009). The usage of visual representations in solving word problems in history of mathematics. *Journal of Qafqaz University*, 28, 138-147.
- Liu, P. H. (2003). Do teachers need to incorporate the history of mathematics in their teaching?. *Connecting Research to Teaching*, 96(6), 416-421.
- Meavilla, V., & Flores, A. (2007). History of mathematics and problem solving: A teaching suggestion. *International Journal of Mathematical Education in Science and Technology*, 38(2), 253-259.
- Marshall, G. L. (2000). Using history of mathematics to improve secondary students' attitudes toward mathematics. Unpublished Ph.D. Thesis, Illinois State University, USA.
- Ofir, R., & Arcavi, A. (1992). Word problems and equations: An historical activity for the algebra classroom. *Mathematical Gazette*, 76 (475), 69-84.
- Oliver, J. (2007). How our methods of writing algebra have evolved: A thread through history. *Australian Senior Mathematics Journal*, 21(2), 12-17.
- Savizi, B. (2007). Applicable problems in the history of mathematics: Practical examples for the classroom. *Teaching Mathematics and its Applications*, 26(1), 45-50.
- Swetz, F. J. (1994a). Learning activities from the history of mathematics. J. Weston Walch Publisher.
- Swetz, F. J. (1994b). Using problems from the history of mathematics in classroom instruction. In F. Swetz, J. Fauvel, O. Bekken, B. Johansson & V. Katz (Eds.) *Learn from the masters* (pp. 25-38), Washington DC: The Mathematical Association of America.
- Swetz, F. J. (2000). Problem solving from the history of mathematics. In V. Katz (Ed.). *Using history to teach mathematics: An international perspective* (pp. 59-68), Washington DC: The Mathematical Association of America.
- Wilson, P. S., & Chauvot, J. B. (2000). Who? How? What? A strategy for using history to teach mathematics. *Mathematics Teacher*, 93(8), 642-645.