A COMPARISON OF MIDDLE SCHOOL STUDENTS' ARGUMENTS CREATED WHILE WORKING IN TECHNOLOGICAL AND NON-TECHNOLOGICAL ENVIRONMENTS

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This study characterized and compared the arguments eighth grade mathematics students created while working in technological and non-technological environments. Toulmin's (1958/2003) argumentation model was used to analyze the content and structure of the arguments, including the ways in which the students used the tools (technological and non-technological). Students in both classes were less likely to make their reasoning explicit when using tools, technological or non-technological, which may be related to the task on which the students were working and the use of the tools as a visual referent.

Background

The development of mathematical reasoning and understanding are culturally mediated, through language and through the use of tools (Lave & Wenger, 1991; Vygotsky, 1978). From this socio-cultural perspective, there is a strong connection between students' uses of tools and their learning because tools are the instruments of access to the knowledge, activities, and practices of a classroom community (Lave & Wenger, 1991). Productive activity with the tools and the understanding of them are not separate. Verillon and Rabardel (1995) indicate that tools do not exist as tools until the user is able "to appropriate it for himself and has integrated it with his activity" (p. 84). The use of tools and understanding their significance interact to become a single learning process.

In mathematics classrooms, tools are frequently used. Traditional tools such as compass, straight-edge, ruler, and scale have been used for thousands of years to explore, conjecture, and make meaning of mathematics, especially geometry. With the advent of the microprocessor, technology tools have become more prevalently used in the mathematics classroom. One type of technology tool that has become popular in the teaching and learning of geometry are dynamic geometry environments (DGEs), such as The Geometer's Sketchpad (Jackiw, 2001). Many teachers, researchers, and professional organizations have suggested the use of dynamic geometry environments to teach geometry (e.g. NCTM, 2000).

The use of tools is an important aspect of participating in a community of practice. Similarly, the use of language plays an important role for an individual to participate in the community. Although it is important for individuals to learn from the discussions of other members of the community, it is even more important to learn how to speak in the community of practice (Lave & Wenger, 1991). The affordances of the tools dictate the understandings that one can construct within the community and, in turn, the understandings that can be developed by the classroom community. Thus, the discourse of the classroom, which includes argumentation, is based, in part, on the use of tools. As technological tools become more prevalently used in the classroom, in what ways does the content and structure of the arguments students create differ when using technological and non-technological tools? The purpose of this study is to describe the arguments created by students enrolled in two eighth grade mathematics classes, one in which the students regularly use a DGE and one in which the students use non-technological tools, and compare the content and structure of the arguments created by pairs of students in each class.

Wiest, L. R., & Lamberg, T. (Eds.). (2011). Proceedings of the 33rd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Reno, NV: University of Nevada, Reno.

One model that has been used by several researchers (e.g., Hollebrands, Conner, & Smith, 2010; Lavy, 2006; Stephan & Rasmussen, 2002) in mathematics education to examine students' mathematical arguments is Toulmin's model of argumentation (Toulmin, 1958/2003). Toulmin decomposed an argument into six components: claim (the conclusion whose validity is being established), data (the facts being appealed to as the foundation of the claim), warrant (the link between the data and the claim), backing (circumstances in which the warrant would otherwise be invalid), qualifier (confers the strength of the warrant), and rebuttal (circumstances in which the warrant does not hold). Figure 1 shows how these six components (data, claim, warrant, backing, qualifier, and rebuttal) fit together. Data is provided or constructed and "so" a claim is made based on this data. This claim can be made based on this data "since" the warrant. The warrant is relevant "on account" of the backing. The claim is valid "unless" the rebuttal occurs.

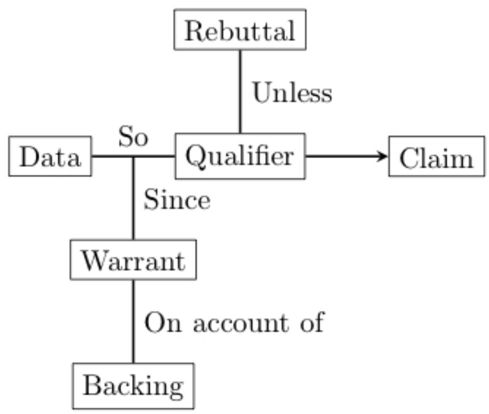

Figure 1. Toulmin's (1958/2003) Model of Argumentation

Methods

For this study, two classes of eighth grade students were selected to participate in a two-week classroom teaching-experiment on triangles taught by the author. I conducted the study with students of varying ethnicities and socio-economic status at an urban public middle school in the southeast United States. For one of the classes, technology played an integral role. Students used a dynamic geometry environment, The Geometer's Sketchpad (Jackiw, 2001), to explore and investigate geometric concepts. The majority of the tasks developed for this unit utilized this technology. For the other class, the students used traditional mathematical tools such as scissors, rulers, compasses, protractors, and snap-cubes. For both classes, the tools provided students with a means to reason and develop new understandings about the geometry concepts. In order to minimize the amount of variation between the two classes, the selection and sequence of tasks for each class were similar. Furthermore, the design and implementation of the majority of the tasks and activities for both classes were similar in nature with the major difference being the tools available to the students. In some instances, the tasks differed to capitalize on the affordances of the tools.

During the study, the students in both classes were placed in pairs. By having the students work in pairs, the students were given the opportunity to have discussions with their partners while working on the tasks. These discussions were the primary focus of the study's analysis. I chose to use pairs rather than larger groups to maximize the opportunities for students to interact with the mathematical task while still having peer-to-peer discourse. Four pairs from each class were purposefully selected to be the focus of the data collection and three pairs were selected for analysis based on the students' attendance. To select the pairings, I asked the regular classroom teacher to identify students that would be willing to verbalize their thinking and work well together. The pairs of students used in the analysis from the technology class were Amy and Judy; David and Erica; and, Heather and Mary[1]. The pairs of students from the non-technology class were Andy and Frank, Bob and Ellen; and, Clair and Jim[1].

Data collection consisted of: video and audio recordings of the two classes both small group and whole class discussions; video-recordings of the computer screen to capture the students' uses of technology; and, artifacts which include copies of students' written work including in-class work, homework, quizzes, and exams. Of each class's eight class meetings, the researcher only analyzed the small group and whole-class discussion for three tasks, although results from the analysis of one task, the triangle inequality, will be presented in this paper. I transcribed the video recordings of the whole class discussions and small group discussions. From these transcriptions, reasoning episodes were established by identifying claims. I then created a description of the argumentation episodes for that claim which included the participants' words (from the transcripts) and actions including the students' uses of the mathematical tools. Then, I diagrammed the argument according to the model developed by Toulmin (1958/2003) including each of the six constructs, data, claim, warrant, backing, qualifier, and rebuttal. For a given argument, some of these components were not specified and I had to make some inferences. In these cases, I noted these inferences and attributed them to a student, the teacher, or a combination of students and/or teacher. In the diagrams, I used a box to outline each of the spoken or known constructs. If I made an inference, I used a "cloud" to note the inference in the diagram.

Results

The results reported in this paper are based on data taken from tasks in which the instructional goal was for the students to develop an understanding of the triangle inequality theorem. The students in the technology class used a pre-constructed sketch to explore whether given sets of segments would form a triangle. The sketch allowed the students to adjust the lengths of the segments using sliders and to drag the segments without changing their lengths. The students in the non-technology class completed the same task using snap-cubes and rulers. While working on the triangle inequality tasks, the three pairs of students from both classes created arguments of various structures and contents. Three themes emerge in the comparison of the content and structure of arguments: the difference in the number of arguments between the two classes and the frequency in the use of the technology/tools; the explicitness of the warrants and the use of technology/tools; and, the content of the additional data.

The Number of Arguments and the Frequency in the Use of Technology/Tools.

The first theme to emerge in the analysis of the arguments is the difference in the number of arguments created by the pairs of students in each class. The pairs of students working in the technological environment created 75 arguments. In comparison, the pairs of students working in the non-technological environment created 56 arguments. This difference is even greater considering the students in the non-technology class had twenty additional minutes of class time to create arguments when they were asked to compare their answers to exercises on the homework with their partners. Removing these arguments, the students in the non-technological class only created 44 arguments. In addition to the discrepancy in the number of arguments, there was also a difference in the proportion of arguments in which the students used the technology/tools.

While working on the triangle inequality task, the pairs of students in the technology class employed the technology in 59 (79%) of their arguments. The pairs of students in the non-technology class created 19 (34%) arguments in which they used the tools. The higher proportion of arguments in which the pairs of students employed technology is most likely due to its use during the opening activity. During this activity, the teacher did not intend for the students to use the technology. However, the students requested to use it to assist them in solving the problems. The students in the non-technology class did not request to use the snap-cubes during a similar opening activity. At times, the pairs of students in the non-technology class used a ruler to make a sketch, but this was infrequent. Perhaps, the students in the technology class viewed the technology as a tool that can assist them in solving problems. The students in the non-technology class may not have viewed the snap-cubes in this manner. Rather, the students may have seen the snap-cubes as a means to create data to make a generalization, but not to solve problems.

The Explicitness of the Warrants and the Use of Technology/Tools

Although the pairs of students in the technology class created more arguments, the content and structure of the arguments in relation to the use of tools, both technological and non-technological, were similar for both classes. When working on the triangle inequality tasks, the pairs of students in both classes were more likely to provide an explicit warrant when not actively using the technology/tools than when the pairs of students created an argument using the technology/tool.

Arguments created by the pairs of students working in the technological environment. Of the 59 arguments in which the pairs of students in the technology class employed technology, they only provided an explicit warrant for 6 (10%) of these arguments. Of the 16 arguments in which technology was not actively employed, the pairs of students provided an explicit warrant for 12 (75%) of these arguments. The explicitness of the warrants may be related to the type of tasks on which the students were working. In general, when the students were using the technology, they were merely attempting to determine whether a triangle could be formed with a set of segments. For example, Judy and Amy attempted to determine whether a triangle could be formed with segments of lengths 3, 4, and 4. Amy adjusted the sliders accordingly and dragged the endpoints of the diagram to form a triangle. Judy claimed that a triangle can be formed by stating, “Possible” and Amy agreed by stating, “Yep.” This argument is illustrated in Figure 2a.

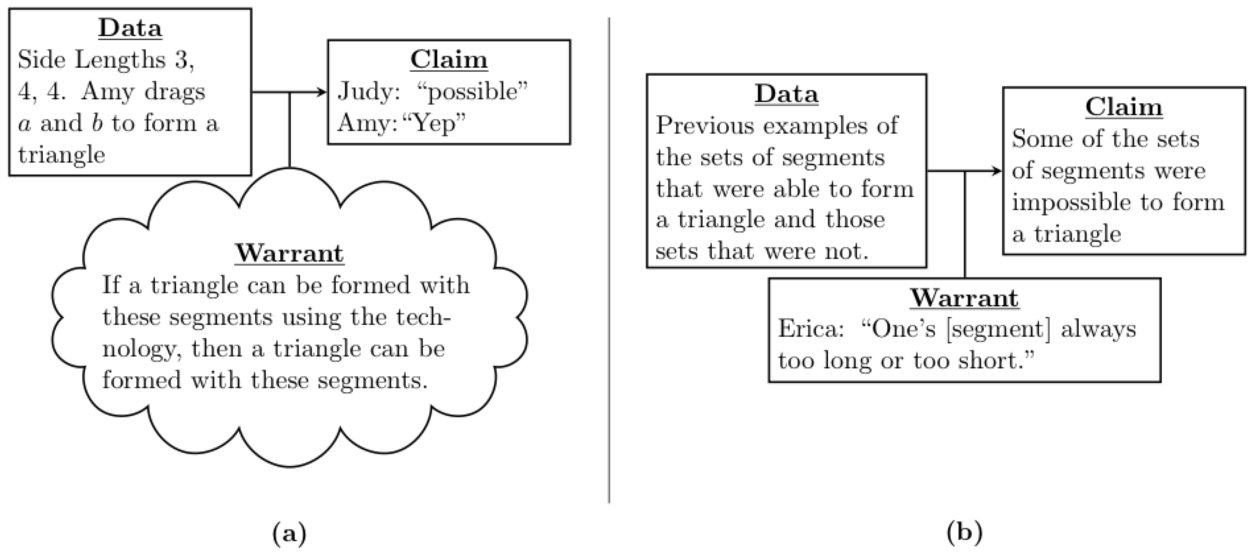

Figure 2. Arguments in which (a) students use technology and provide an implicit warrant and (b) students do not use technology and provide an explicit warrant

However, when the pairs of students were not actively using technology, the students were mainly working on generalization type tasks. For example, one of the arguments common to all pairs of students in structure was in response to the question on the task sheet, "Why was it impossible to construct a triangle with some of the given lengths?" One student, Erica, stated, "One's [segment] too long or too short." This argument is illustrated in Figure 2b. The question asked the students to generalize across the examples. The data used by the pairs of students to support their claims were their answers to the examples sets of segments on their task sheet. To gather this data, the students used technology. However, when responding to this question, the data had been previously collected and their reasoning was not based on their active use of technology, but on the product of their previous uses.

Arguments created by the pairs of students working in the non-technological environment. The arguments created by the pairs of students in the non-technology class were similar in content and structure with regards to the use of the non-technological tools and the explicitness of the warrant. Of the 19 arguments in which the pairs of students used the tools, they only provided an explicit warrant for 1 (5%) of these arguments. Of the 37 arguments in which tools were not actively employed, the pairs of students provided an explicit warrant for 30 (81%) of these arguments.

Similar to the pairs of students in the technology class, when the pairs of students were using the tools, they were attempting to determine whether a triangle could be formed. In these arguments, the data were the lengths of the segments; the arranging of the snap-cube segments; and, the appearance of the figures the students formed using the snap-cube segments. In one argumentation episode, Clair and Jim were determining whether a triangle could be formed with segments of length 3, 4, and 4. Clair arranged the snap-cube segments and was able to form a triangle. Jim said, "Possible." Clair agreed saying, "Yep." This argument is illustrated in Figure 3a.

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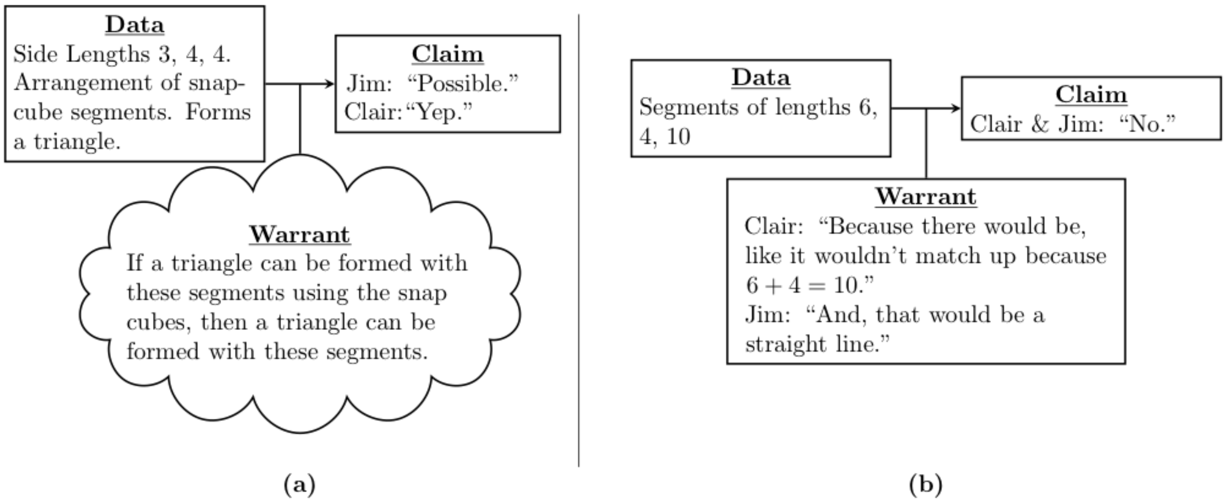

Figure 3. Arguments in which (a) students use non-technological tools and provide an implicit warrant and (b) students do not use tools and provide an explicit warrant

When not actively using the tools in the creation of their arguments, the pairs of students were generally working on generalization tasks and the majority of their warrants were explicit. However, the pairs also created arguments with an explicit warrant that was based on their previous uses of the non-technological tools. For example, Clair and Jim created an argument of this structure as they discuss their solutions to a homework problem. The problem asked the students to determine whether a triangle can be formed with segments of lengths 6, 4, and 10. Clair said, "The first one you said no and I agreed. I said no because there would be like it wouldn't match up because 6+4=10." Jim added, "And that would be a straight line." This argument is illustrated in Figure 3b. Although, in this example, the claim created by pair of students was not a generalization, they justified their claim by using a known theorem and stated what would occur if they attempted to form a triangle with segments of the given lengths.

Additional Data Collection

Many times the pairs of students in both classes collected additional data to verify or refute a previous claim. All three pairs of students in both classes created arguments of this structure. The students' decision to seek additional data may be due to a number of factors including an explicit challenge to a claim, the uncertainty of a claim, the uncertainty of a claim due to the lack of precision in the use of the technological tool, and the ways in which the students used the technology to collect the initial data. Even though the pairs of students collected additional data for a variety of reasons, the students in the technology class always used technology to collect this additional data, usually using the drag feature of the technology. For example, Amy and Judy were determining whether segments of lengths 2, 7, and 4 would form a triangle. Amy dragged the sliders accordingly and Judy claimed, "That looks totally impossible, that is huge." Amy immediately responded, "You don't know that though; that's the only thing." Judy agreed with the sentiment and stated, "I know, the weirdest looking stuff may be possible." Amy dragged the endpoints and is unable to form a triangle. She stated, "It's impossible." This argument is illustrated in Figure 4. In this argument, Amy challenged Judy's claim indicating appearances can be deceiving. Judy agreed and the pair of students sought additional data to verify or refute that claim.

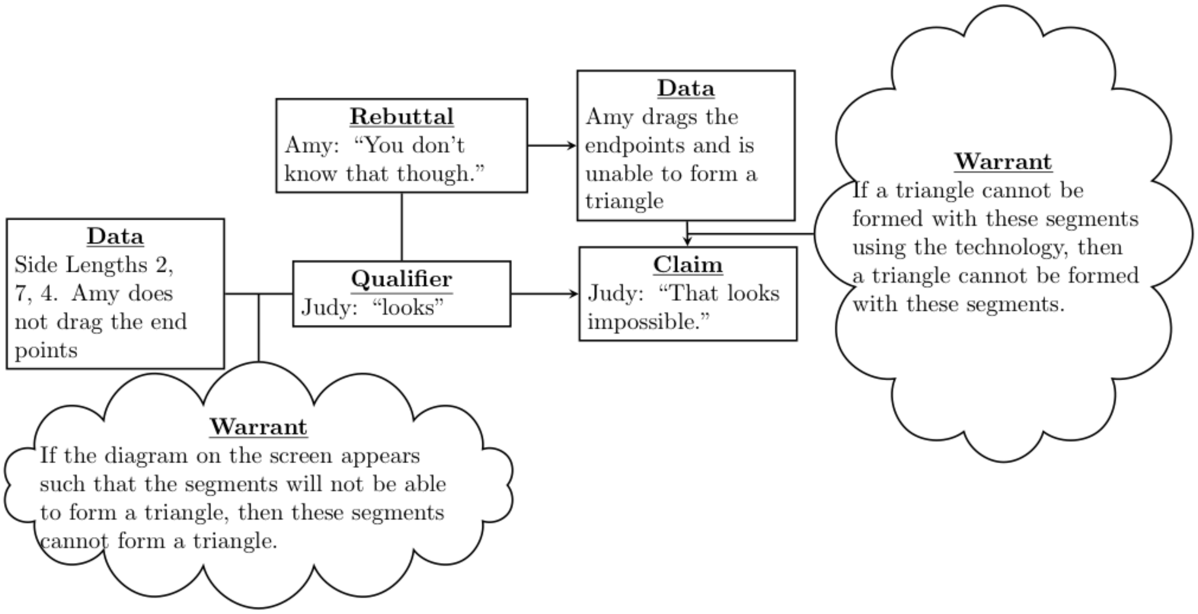

Figure 4. An argument in which additional data is collected using technology.

For the majority of the arguments in which the pairs of students in the non-technology class collected additional data, the students used tools by rearranging the snap-cube segments to determine whether a triangle could be formed, using a ruler to create a sketch to determine if a triangle could be formed, or focusing on different features of the tools. Although the pairs of students mainly used the tools to collect additional data, this was not always the case. At times, the students used known facts such as definitions and theorem as additional data to verify or refute a claim. For example, Ellen had written on her paper that a triangle could be formed with segments of lengths 10, 7, 24. The teacher asked her, "So, you think 24?" She responded, "As long as it's more than 17, it don't [sic] matter." Then, the teacher simply stated, "So 10, 7, 24…okay, okay." Ellen asked, "Hold on is it more or less?" The teacher pointed to the board and asked, "What does it say up there?" Written on the board was the triangle inequality theorem. Ellen began erasing her paper and changed her answer. Ellen collected additional data by reading the theorem on the board. Realizing that her initial claim was incorrect she made a new claim.

In the first example of this section, Amy challenged Judy's claim regarding the appearance of the diagram, which prompted the pair to collect additional data by dragging the figure to determine if the initial claim was correct. Perhaps, when these students' initial claims were challenged, the students felt the need to collect additional data using tools to verify their claims. In the second example, Ellen's uncertainty of her claim prompted her to question the basis for her reasoning and sought additional data to determine whether her reasoning was correct and, hence, whether her claim was valid. In other words, when a claim is challenged, the students used tools to collect additional data in order to resolve the challenge. When the students were uncertain about their claim, they collected additional data, either using tools or definitions and theorems, to determine whether they were using sound reasoning.

Discussion

One major finding of this study is when students actively used the technological or non-technological tools, they were more likely to create arguments with non-explicit warrants. In contrast, when the students did not actively use the tools, they were more likely to create

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arguments with an explicit warrant. Hollebrands, Conner, and Smith (2010) had similar findings in their study of the arguments college geometry students created when working with technology. When the college geometry students provided explicit warrants for their claims, the students were generally not using technology. The authors attributed this finding to the students' prior experiences in learning mathematics at the collegiate level where the students were expected to provide formal proofs, which require explicit warrants. This same attribution cannot be made to the middle school students in this study because it is unlikely they had been exposed to formal proofs. Rather, the lack of explicit warrants when the pairs of students were using the technological and non-technological tools may be attributed to the visual nature of the tools and the task on which the students were working.

The students' lack of explicit warrants in their arguments created while actively using the technological and non-technological tools may be related to the visual nature of the tools. Each student did not have his or her own personal set of tools. Rather, the pairs of students shared the tools and would alternate between using them. Thus, the students shared the same visual display presented by the students' uses of the tools (the screen for the students in the technology class and the arrangement of the snap-cube segments for the students in the non-technology class). Because the pairs of students had the same visual referent, they may not have been compelled to provide an explicit warrant. Lavy (2006) investigated the types of arguments middle school students created while using an interactive computerized environment. The author found that the students not only used images on the screen and commands as data, but also in their reasoning. Lavy concluded, "In a visualized environment, it is obvious that visual evidences can serve as reasoning in an argument, since all the work in this setting has the same character" (p. 168). Thus, the pairs of student may not have been compelled to provide an explicit warrant because they felt their reasoning was obvious to each other. In fact, when the students' challenged each other's claims the students generally used the tools to collect additional data rather than make their reasoning explicit (see Amy and Judy's argument illustrated in Figure 4).

In general, when the students were using the technological or non-technological tools, they were determining whether given sets of segment lengths would form a triangle. However, when the pairs of students were not actively using the tools, the students were mainly working on generalization type tasks. Even though the students in the current study did not actively use the tools when working on generalization type tasks, other researchers (e.g. Healy & Hoyles, 2001) found that students will create generalizations while using technology. Some pairs of students in Healy and Hoyle's (2001) study used a DGE to investigate relationships among the angle bisectors of a quadrilateral. The pairs of students did not create the same constructions and did not arrive at the same conclusions. However, those students that were successful were able to construct and measure aspects of their diagrams and developed generalizations while using the DGE. The students in the current study did not have the option of creating their own diagram. Instead, the students in the current study used a teacher-generated pre-constructed sketch that limited the students in how they could modify and/or measure aspects of the diagram. Perhaps, the students in the current study would have been more likely to use the technology while working on generalization type tasks if they had been given the opportunity to create their own diagrams.

Endnotes

1. All names are pseudonyms

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