MAINTAINING COGNITIVE DEMANDS OF TASKS THROUGH SMALL GROUP DISCUSSIONS IN PRESERVICE ELEMENTARY MATHEMATICS CLASSROOMS

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Enacting tasks at high levels of cognitive demand helps preservice teachers make sense of mathematical ideas and serves as a model for instruction. Small group discussions can be useful pedagogical tools for maintaining a task's cognitive demands. In this article, we contrast two small group discussions within a preservice elementary classroom to illustrate characteristics of productive small group discussions.

Introduction

Current research on mathematical tasks examines the levels of cognitive demands at which tasks are written, set up, and implemented in elementary and middle school classrooms in the hopes of shedding light on effective classroom instruction (Stein, Smith, Henningsen, & Silver, 2000). Tasks that require high levels of cognitive demands prompt students to explain their reasoning and make connections between different representations or procedures. Since reasoning and sense making are critical components of conceptual understanding, finding ways to maintain high levels of cognitive demand during task implementation is important. Research shows, however, that teachers often inadvertently reduce the cognitive demands of a task during its implementation by making problems easier to solve, often by providing students with too many hints or by removing prompts for student-constructed justifications (Stein, Grover, & Henningsen, 1996).

One way to maintain high levels of cognitive demands may be through the use of mathematical discourse. There is evidence demonstrating the benefits of using discourse in the mathematics classroom (Walshaw & Anthony, 2009). When students are pushed to articulate their ideas and listen to the thinking of their peers, they are able to make better sense of their own thinking. Serving as a basis for current reform efforts, the National Council of Teachers of Mathematics (2000) identifies communication as an essential standard of mathematics education, stating that students must learn to communicate their mathematical thinking to others and evaluate the mathematical thinking of their peers. Nevertheless, many classrooms continue to implement traditional teacher-centered instruction (Hiebert et al., 2005).

In this research report we will present an analysis of preservice elementary teachers' small group discussions regarding a mathematical task written at a high level of cognitive demand. Through careful examination of videotaped discussions, we will identify characteristics of small group interactions that maintained the level of cognitive demand inherent in the task. We will contrast them with other small group interactions that lowered the task's cognitive demand level. Focusing on small group discussion yielded new information as small group interactions provide opportunities for learning that whole class formats do not (Yackel, Cobb, & Wood, 1991). For example, preservice teachers may feel more comfortable sharing their ideas in a small group setting. Although our original intent was on examining the role of whole-class discussion in fostering preservice teachers' conceptual understanding, we began to quickly notice that small group interactions played a key role in helping preservice teachers make sense of important mathematical ideas.

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Theoretical Framework

Research on mathematical tasks and their cognitive demands has been on-going for over twenty years. Mathematical tasks play a large role in determining the mathematics that students will see in the classroom (Doyle, 1988). Tasks that emphasize computation and memorization result in students acquiring procedural skills without understanding why they work; tasks that focus on solving rich, contextual problems help students attend to the concepts underlying the problems (Stein & Lane, 1996).

Research suggests that preservice teachers should become accustomed to solving cognitively demanding problems, since the expectation is they will be posing cognitively demanding problems to their own students (Stein et al., 2009). The cognitive demands of a task refer to the kinds of thinking processes entailed in the task (e.g. memorization, use of procedures, use of complex thinking and reasoning strategies). Stein and colleagues suggest that the cognitive demands of a task can change as the task moves through three phases of development – writing, set up, and implementation (Stein et al., 1996; Stein et al., 2000; Boston & Smith, 2009). For example, a mathematical task may be written at a high level of cognitive demand, but presented and/or implemented at a lower level of cognitive demand.

Rubrics have been developed to assess the cognitive demands of tasks as written and as implemented (Boston & Smith, 2009). The *Instructional Quality Assessment – Academic Rigor* (IQA-AR) rubric includes two forms, the *Potential of the Task* and *Implementation of the Task.* Both forms scores range from 0 to 4. A score of 0 indicates that the lesson tasks were nonmathematical in nature. Score levels 1 and 2 represent low-level cognitive demands. Score levels 3 and 4 represent high-level cognitive demands in which the connections to meaning and understanding are implicitly (score level 3) or explicitly (score level 4) required by the task.

When comparing tasks' levels of cognitive demands at the written and implementation phases, Suzuka and colleagues (2009) found that teacher educators have difficulty maintaining a task's cognitive demands when implementing the task in a preservice teacher classroom. Two primary reasons could explain their difficulty: sometimes teacher educators focus too heavily on the mathematics of the task, thereby disconnecting the task from the work of teaching; or, work focuses too heavily on pedagogical concerns without engaging preservice teachers in understanding mathematical content. In order to keep tasks focused on MKT development, the authors suggest asking preservice teachers to attend to their classmates' thinking, pressing preservice teachers to explain their own thinking, and being explicit about the connections between mathematical tasks and teaching.

While reform efforts during the past twenty years call for the use of classroom discourse (Senk & Thompson, 2003), research has also recently begun examining classroom communities where discourse is used to promote mathematical learning. Hufferd-Ackles, Fuson, and Sherin (2004) created the *Levels of Math – Talk Learning Community* framework, which identified four components of a math-talk learning community: questioning, explaining mathematical thinking, source of mathematical ideas, and responsibility for learning. Each component is rated from level 0 to 3 to reflect changes in how teachers and students interact to make sense of mathematical ideas. Classrooms at a level 0 are teacher-centered, where the teacher is the only questioner, students provide short answers and never elaborate on their thinking, the teacher is the only source of mathematical ideas, and students are passive listeners to what the teacher is saying. Classrooms at a level 3, however, are primarily student-centered, where students and teacher question one another, students provide more elaborate justifications of their mathematical ideas, and students take responsibility for helping each other learn mathematics.

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Though research on mathematical discourse and small group interactions is abundant at the K-12 level, little is known about discourse in preservice teacher classrooms. Dixon, Andreasen, and Stephan (2009) conducted a classroom teaching experiment in a preservice elementary teacher mathematics content course in order to understand how classroom norms are established. They observed that making sense of others' reasoning was one of the most difficult, yet important, norms to establish. Research on discourse in preservice teacher classrooms also showed that math-talk learning communities (as defined by Hufferd-Ackles et al., 2004) can be established by prompting preservice teachers to question, explain, and clarify each others' thinking (Rathouz & Rubenstein, 2009). These studies, however, did not focus on small group interactions. As such, we do not yet have a clear picture of the characteristics of student-to-student talk in preservice teacher classrooms.

Methods

The study took place in the spring of 2010, during a semester-long preservice elementary mathematics content course at a large private university in the northeastern United States. It is the second course in a series of two required mathematics content courses for undergraduates seeking state certification as elementary or special education teachers. The course emphasizes preservice teachers' conceptual understanding of content and seeks to develop productive mathematical talk as a classroom norm. Each section consisted of approximately sixteen participants who were randomly assigned to four small groups. Two classroom sections of this course were videotaped and audio taped during eighty minute class sessions over a two-week period.

The curriculum used in the study was a four-part unit on geometric measurement created by the researchers. Our analysis focuses on a two-day lesson about surface areas of prisms and cylinders. Day one required participants to work through mathematical tasks regarding the lateral and total surface areas of prisms. Day two required participants to work through mathematical tasks focused on lateral and total surface areas of cylinders, as well as the relationship between the surface areas of prisms and cylinders. The tasks were written to be cognitively demanding, with repeated prompts for participants to justify their reasoning to others and make generalizations across topics.

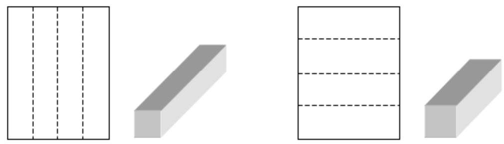
We used two sets of rubrics to collect data about each task and its implementation. We used the IQA-AR rubrics for the potential of the task and for the implementation of the task in order to assess the cognitive demands of the tasks before and during implementation, respectively (Boston & Smith, 2009). In order to assess the nature of student-to-student talk within small group discussions, we used the *Levels of Math Talk* framework (Hufferd-Ackles et al., 2004).

Results

During day one of the surface area task, small groups in both sections worked on a mathematical task consisting of four questions (shown below). Each group had to fold one piece of paper lengthwise and a second, identical, piece of paper widthwise, find each prism's dimensions, and calculate their surface areas. Participants then needed to identify similarities and differences between the surface area calculations of both prisms. Each group had approximately fifteen minutes to complete the task.

3. Take two pieces of $8\frac{1}{2}$ in. by 11 in. paper and fold them into two different rectangular prisms with square bases. Label one, Prism X, and the other, Prism Y. The large rectangle that is used to form each prism is called the "lateral surfaces rectangle." Note that the prisms are

missing their bases. On the drawing below, label the dimensions of both prisms.



Prism X Prism Y

4. a) What are the dimensions of the lateral surfaces rectangle for Prism X? How can we use the dimensions of the prism to find these measures?
   b) Calculate the area of the lateral surfaces rectangle of Prism X, the area of the bases of Prism X and the total surface area of Prism X.
5. a) What are the dimensions of the lateral surfaces rectangle for Prism Y? How can we use the dimensions of the prism to find these measures?
   b) Calculate the area of the lateral surface rectangle of Prism Y, the area of the bases of Prism Y and the total surface area of Prism Y.
6. Explain any similarities and differences between the surface area calculations in questions 2 and 3.

Cognitive Demand Analysis for Potential of Task

We rated this task at a level 4 cognitive demand using the IQA-AR rubric for potential of the task. The task engages participants in complex thinking because they are asked to compare and contrast two different orientations of seemingly identical prisms. They are asked to follow a procedure (creating two different prisms from the same sized sheet of paper) to illustrate an important mathematical concept (prisms can have the same lateral surface area but different total surface areas). In other words, the numerous computations of total and lateral surface areas required by the task serve a greater purpose – conceptual understanding of the differences between total and lateral surface areas of prisms. In the last question of the task, participants are prompted to provide explicit evidence of their understanding by comparing and contrasting the calculations that were performed on the two prisms. In doing so, they are making connections between two representations of a rectangular prism.

Instructor F's Small Group Discussion

Participants create the two paper rectangular prisms using sheets of paper with dimensions 8 ½ inches by 11 inches, following the instructions written in question 3. In the following small group discussion, participants are finishing their calculations of prism Y's surface area. After having found that the length of a side of prism Y's base is 2.75, they are helping each other with the rest of the calculations that need to be done in order to answer question 5. Once this is complete, they, move on to a discussion of question 6.

S1: We're saying that the lateral surface area would be 93.5 because it's the same piece of paper but the bases are going to change.
S4: So the base is?
S1: It would be 2.75 squared.
S4: 7.5625.
S2: Plus 2 times 7.56.

S1:	Put another one [prism Y base area] because you need 2 of those. 108.625. So it's not the same.
S3:	Well yeah because the squares [points to paper rectangular prism] are different sizes.
S3:	The lateral surface areas are the same, but…
S2:	Yeah, the lateral surface areas are the same.
S3:	But the bases…
S2:	The bases are different.
S3:	…are different sizes, so the larger base is going to have a larger surface area?
S2:	Yeah.
S3:	Um, what number are we supposed to go up to?
S1:	I think up to 6.
S3:	Oh, okay…are you sure?
S1:	Yup, that's why we had to do different squares on the…
S4:	So why is the surface area different?
S1:	Because this [points to paper rectangular prism]…they have the same lateral surface area [points to top of paper rectangular prism], but the boxes [points to base of paper rectangular prism] of the squares of the bases are different. Because these [points to bigger paper prism] squares are bigger so it makes it more.

In this discussion, the small group interactions result in a shared understanding of why the total surface areas of the two prisms differ. Initially, S2 and S1 notice that S4 has only added one base area to prism Y's lateral surface area, explaining two base areas are needed to accurately calculate total surface area. S3 revoices S1's remark that the total surface areas of the two prisms are not the same since their base areas differ. S3 and S2 again revoice the idea. Although S3 is satisfied with this explanation, S4 is not and asks the group to explain why the prisms' surface areas differ. S1 restates the explanation in his own words using folded paper prisms to illustrate his reasoning.

We rated this small group discussion at a level 4 in cognitive demand using the IQA-AR rubric for implementation of the task. There is clear evidence of participants' reasoning, as group members articulated an explanation for why the surface areas of the two prisms are different. When S3 responds to S1's initial comment that the surface areas are not the same, she is making sense of the mathematics. She could have simply stated that the surface areas differ because the two prisms are different, but this would have shown vague understanding. Instead, she states that the difference is a result of different sized bases. This shows that she understands that it is the base areas of the two prisms that distinguish one prism's total surface area from the others' – the lateral surface areas are the same. Also, S1 is making an explicit connection between representations when he lays one paper prism next to the other and points to their different sized bases.

We rated the small group discussion at a level 3 for questioning, explaining mathematical thinking, source of mathematical ideas, and responsibility for learning using the *Levels of Math Talk* framework. These high levels of math talk are due to the comfort with which group members express their ideas and help one another understand them. Participants initiate their own student-to-student talk without prompting from the instructor, and they do not hesitate to ask questions of each other to clarify ideas. Participants freely interject their own ideas throughout discussion, either correcting each others' mistakes as evidenced by S2's and S1's noting that S4 omitted one base to calculate the total surface area. Participants repeatedly justify their ideas and with little prompting from each other, as seen when S3 spontaneously began to articulate her explanation for why the surface areas differ. Participants in this group take responsibility for their own learning, as seen when S4 asks for clarification after the other group members were finished.

Instructor C's Small Group Discussion

Participants in this small group also worked on the same mathematical task as the group in Instructor F's class. In the following discussion, the group members are finishing their computations of the total surface areas of both prisms and need to explain any similarities and differences between the surface areas of the prisms.

S2: So we know the lateral surface area….
S1: Is 8.5, oh wait sorry what did you get for the lateral surface area?
S2: Is 93.5
S3: What's 93.5?
S2: Is the lateral surface area….one piece of paper.
S1: Yeah same as the last one.
S2: Right. And then to that we have to add the bases [points to bases of prism] which are now different.
S3: The bases are….2.75?
S2: Yeah. So 7.5…now you got to multiple that by 2…15.
S3: What was the area of one of them?
S2: One of them is 7.56. So the total is 108.425, which is bigger.
S1: Okay, so similarities and differences between the surface area calculations…the lateral surfaces were the same, but the bases were different and therefore the total surface area…
S1: Okay, are we supposed to keep going?
S3: Well he said till 8.

In this discussion, the group focuses on making correct calculations of prism Y's total surface area. S1 asks S2 for the lateral surface area of prism Y (93.5), after which S3 asks S2 what 93.5 represents. As S2 begins to articulate the next step in the procedure for calculating total surface area, S3 asks her for the area of the base of prism Y. The only discussion surrounding mathematical concepts occurs at the end, when S1 states that the difference in surface areas is due to the difference in base areas. None of the other participants follows up on this claim at all.

We rated this small group discussion at a level 2 in cognitive demand using the IQA-AR rubric for implementation of the task. The purpose of this group's talk is to make sure that all group members arrive at the same numerical results for the calculations of total surface areas of prism X and prism Y. Since the group as a whole does not engage in any discussion around S1's statement, it is not apparent that the other group members agree with this statement or understand why it is true. Throughout their discussion, participants follow a well-known procedure for calculating surface areas; they are not engaged in any complex thinking or meaning making.

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We rated this small group discussion at a level 1 using the *Levels of Math Talk* framework. Participants ask each other questions, but they do not listen attentively since questions are repeated (S3 asks for the lateral surface area immediately after S1 asks the same question). S1 is the only participant who describes her non-computational thinking, but her claim is not explored any further by the other students. In fact, small group discussion focuses solely on procedures for finding total surface areas, rather than the concepts underlying the problem.

When comparing Instructor F's small group interactions to Instructor C's small group interactions, clear differences appear. In Instructor F's class, group members maintain the task's high level of cognitive demand during task implementation. They do this by engaging in thoughtful discussion about the key mathematical ideas of the task, making sure to provide reasoning when appropriate. In Instructor C's class, however, group members are focused on getting the correct answer to the problem and neglect the last part of the question that asks for an explanation of the similarities and differences between calculations.

Differences in both group's level of math talk are also evident. Instructor F's group is at the highest level of math talk because they make repeated efforts to explain their thinking. In fact, when analyzing classroom video of these interactions, one notices a sense of urgency, among the group members, to make sure everyone understands the concepts under investigation. They are not afraid to interject their own ideas into a conversation because they know their ideas will be valued. On the other hand, Instructor C's group members exhibited a low level of math talk. Since participants were focused on getting the correct answer instead of making sense of their answers, questions were short and only elicited superficial responses. Also, since S1's claim is not explored by other group members during the small group discussion, one might conclude that the group uses another source of mathematical ideas, such as the instructor or whole class discussion, instead of each other.

Discussion

In this article, we provided examples of how differences between two small groups of elementary preservice teachers working on the same task lead to maintaining or lowering the task's cognitive demands. In Instructor F's small group discussions, participants maintained the cognitive demands of the task by fully answering the questions posed and justifying their answers. In Instructor C's small group discussions, participant discussion resulted in lowered cognitive demands because this group only answered the computational portion of the questions posed and neglected to provide explanations of why they performed their computations or reached their conceptual conclusions.

We also evaluated the two small group discussions using a rubric for levels of math talk (Hufferd-Ackles et al., 2004). We found that Instructor F's small group discussions were rated at a higher level of math talk than Instructor C's small group discussions. Participants in Instructor F's class were actively involved in asking each other questions and defending their own ideas, while participants in Instructor C's class rarely asked each other to explain their thoughts and were hesitant to pose clarifying questions.

Based on our analysis, we believe that the focus of preservice teachers' attention during small group discussion can impact whether tasks' cognitive demands are maintained or not. Small groups who focus on providing explanations, justifying procedures, and making generalizations appear to maintain a task's high level of cognitive demand. We observed a shared desire among all group members in Instructor F's class to make sense of the task. Participants were consistently asking each other for explanations and were often dissatisfied with correct solutions that they did not understand clearly. In contrast, small groups who focus solely on finding correct answers, do not justify their procedures, and do not attempt to generalize their ideas beyond specific cases risk lowering the cognitive demands of a task. In Instructor C's small group, there did not appear to be a shared need to develop an understanding of the mathematics of the task. Participants seemed satisfied when they arrived at the same numerical answer, and chose not to push for meaning or clarification.

Although it is unclear why one group developed a shared need to make sense of the task while the other did not, it seems that a small group working together in a preservice Instructor Course can develop its own culture that helps to determine how the group functions. Future research should explore the source of small group cultures. Do small groups adopt the culture of their classroom? Does what the instructor of the course value – respectful discourse and focus on meaning – dictate how small groups operate? In our study, Instructor F focused whole-class discussion on sense making while Instructor C focused his discussions on answer checking.

It is also possible that maintaining high levels of cognitive demand is associated with high levels of math talk during small group discussions. A future study could investigate the interplay between a task's cognitive demands and the levels of math talk used to solve the task: does a small group's success in solving cognitively challenging problems relate to the degree to which they use productive math talk during discussions? Our study suggests that math talk can help maintain high levels of cognitive demand, and that using contrasting cases might be a useful method for investigating this further.

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