GROWTH IN SECONDARY TEACHERS' CONTENT KNOWLEDGE AND PRACTICE IN DISCRETE MATHEMATICS

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Thirty-five teachers teaching advanced topics in high school mathematics were engaged in a year-long intensive professional development project intended to increase their Mathematical Knowledge for Teaching in the area of discrete mathematics. Project activities embodied a models and modeling perspective on pedagogy and learning. Results showed that participating teachers' content knowledge improved significantly in discrete mathematics, their classroom practice improved significantly. Additionally, participating teachers' students outperformed their counterparts in a matched-control group in achievement. Results provide evidence of the effectiveness of mathematical modeling for developing teachers' mathematical knowledge for teaching, and on improving secondary mathematics teachers' practice.

With the recent push to increase the general mathematical competence of US high school graduates, the State of Arizona has instituted new standards incorporating significant aspects of discrete mathematics, including graph theory, combinations and permutations applied to the solution of conflict problems, probability and statistics, and shortest path algorithms (Standards Committee, 2008). This subject matter is becoming increasingly important in scientific and economic modeling, and is now considered essential background for college- and work-readiness.

Two problematic issues arise when attempting to infuse discrete mathematics content into the current high school curriculum. The first concerns the ability of teachers to effectively deliver high quality instruction given that discrete mathematics has historically been given only cursory (if any) treatment in their college-level mathematics courses (Rosenstein, Franzblau, & Roberts (1991, Eds). The second issue, and one that is at least as important as teacher knowledge is the fact that there is no coherent set of curricula (i.e., texts, materials) for teachers to use to structure their implementation of the new standards.

To address these concerns, a partnership was formed between a large Southwestern University, and a large urban school district to substantially increase the pedagogical content knowledge of high school mathematics teachers in the areas of discrete mathematics and mathematical modeling, and to create a coherent course of study around discrete mathematics appropriate for a 4th year of high school mathematics. The project worked toward four measurable goals and used formative evaluation as a guide for on-going program development and refinement to meet the goals.

Goals

Wiest, L. R., & Lamberg, T. (Eds.). (2011). Proceedings of the 33rd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Reno, NV: University of Nevada, Reno.

1. Increase mathematical knowledge for teachers (MKT) for Junior-Senior level teachers (e.g., Hill, Ball, & Schilling, 2008);
2. Create a coherent, rigorous curriculum for fourth year courses that develop student knowledge and skills in Discrete Mathematics and modeling (e.g., Zawojeski, Lesh, & English, 2003)
3. Increase student achievement specifically focusing on fourth year courses that develop student knowledge in Discrete Mathematics and modeling
4. Develop student's content knowledge through the use of workplace technology

Goals were designed to ensure that this learning and growing expertise was sustained and shared with district instructors and colleagues.

Project Activities

The partnership identified 10 campus instructional leaders and an additional 25 mathematics teachers proportionally distributed by student population across high schools. The project developed an integrated system of PD, curriculum, and technology implementation in the area of Mathematical Modeling. Mathematical Modeling was chosen because it integrates all aspects of the College and Work-Readiness Standards in Arizona, because it draws easily upon students' prior knowledge of Algebra and Geometry, and because it provides important contexts for learning and applying discrete mathematics in realistic scientific and workplace scenarios Figure 1 (below) shows project activities arranged across the 14 months of project funding.

The meat of the program has been a monthly series of 8-hour professional development sessions designed to increase teachers' pedagogical content knowledge in modeling discrete mathematics topics. These sessions are bookended by week-long (40 hours each) Summer

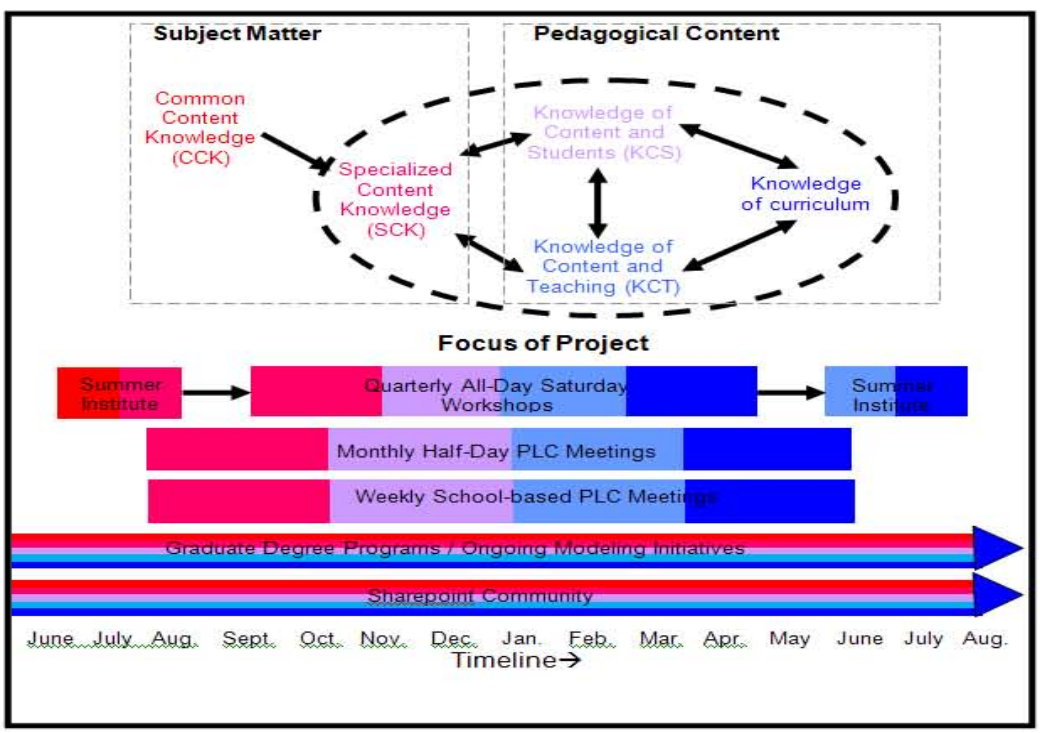

Figure 1. Professional development foci and timeline

Workshop. Teachers are organized across schools into curriculum development teams, tasked

with designing the 4th-year curriculum structure and tasks for subsequent implementation in the 2010/2011 academic year (see Figure 1).

Initially, forty two teachers were provided 135 contact hours of professional development utilizing two 40-hour intensive summer workshops that introduced participating teachers to the content of discrete mathematics, including graph theory, combinatorics, probability and statistics. Institute faculty emphasized a variety of researched best practices, focusing on modeling high cognitive demand tasks in problem-based-learning scenarios. The use of readily-available software and mathematical technology was emphasized throughout these sessions. Technology included simulation tools, probeware and data analysis tools, and graphical/visualization tools. Throughout the school year, additional content instruction was provided to teachers in each of 9 additional 8-hour Saturday sessions.

Also throughout the school year teachers implemented these best practices by developing and piloting curriculum for a new 4th-year course designed to embody the College Ready and Work-Ready Standard for Arizona Mathematics. This course is targeted towards non-calculus-intending students, and is pedagogically based on mathematical modeling. Teachers engaged in iterative design cycles (Lamberg & Middleton, 2009) to develop and test lessons and whole units in their classrooms, videotaping their practice, and collecting student work samples for use in subsequent professional development. These data records were used to improve the designed courses and develop teachers' guides to assist other teachers in implementing the College and Work Ready Standards in their classrooms, focusing on discrete mathematics, probability and statistics. All work products were uploaded to the project Sharepoint® website to enable ready access for teacher teams to updated materials.

Evaluation

Participants

The project was evaluated using a pre-post, quasi-experimental design with a matched comparison group. Each school was asked to provide a comparison teacher for any participating teacher. Matched comparison was based on number of years teaching, grade level taught, and math courses taught. Twenty-seven project teachers completed all of the 135 hours of professional development, and had complete data. Forty teachers, who received no professional development in discrete mathematics or mathematical modeling comprised the comparison sample.

Measures

To measure teachers' practical change, the Reformed Teaching Observation Protocol (RTOP) was employed. The RTOP was developed as an observation instrument to provide a standardized means for detecting the degree to which K-20 classroom instruction in mathematics or science is "reformed". Pre-RTOP data have been used to assess the quality of instruction against an external metric of excellence, and fidelity of implementation. In addition, for teacher content knowledge, a cognitive growth instrument, The Discrete Math Content Measure (DMCM), was developed (the pre-test was administered in 2009 and the post-test administered in 2010). The overall reliability (Chronbach's Alpha) of the exam as a whole was 0.70. Student achievement was measured with AIMS scores and CRT (course) scores yearly.

Observational Protocol

There were five observers that completed the RTOP evaluations for the participant group. The observers completed RTOP training through center for Research and Innovation in Mathematics and Science education (RIMSE), with Arizona State University. Each participant

and contrast group teacher was observed in a typical class where they taught discrete mathematics topics: Once in the Fall semester, 2009, once in the Spring Semester, 2010. There were no major differences in the timing or method of the administration of the RTOP evaluations in the pre or post RTOP.

Results

Teacher Knowledge

Results from the DMCM analysis shows a significant difference between the 2 groups (*p*-value = .00) as post-test scores for the Participant (*M* = 18.29) were higher than the comparison group (*M* = 11.95) See Table 1 Results from the DMCM analysis for the 27 program Participants show that there were gains from pre (*M* = 10.47, *SD* = 3.41) to posttest (*M* = 18.29, *SD* = 3.29), and the gains were significant (*p*-value = .00). Results from the analysis for the comparison group revealed that there was little change between pre-test mean scores (*M* = 11.42, *SD* = 2.72) and post test mean scores (*M* = 11.95, *SD* = 3.17) and the difference was not significant (*p* = .24).

Discrete Mathematics Content Measure: Participant Group			
Group	**Pre-score Mean (SD)**	**Post-score Mean (SD)**	**Significance p < .05**
Participant	**10.47** (3.41)	**18.29** (3.30)	**.00**
Control	11.42 (2.72)	11.95 (3.17)	.24

Table 1. DMCM comparison of participant and comparison groups

Teacher Change in Practice

Differences in pre- and post-classroom observation (RTOP) average scores indicate a positive gain in those items relating to reformed and inquiry-based classroom from mean of 59.78 at the beginning of the MSP to a mean of 68.56 following the MSP professional development intervention. The gains were significant (*p* = .028). The Post RTOP score (*M* = 68.56, *SD* = 17.29) revealed a nine-point gain from the Pre RTOP score (*M* = 59.78, *SD* = 17.59), and the gain was significant (*p* = .028). PUHSD had a 15% change in pre-post RTOP scores.

With regards to the relationship between teachers' RTOP scores and their student achievement, we found that, for our data, the Spearman rho between the RTOP & post-DMCM was to be .50, a moderate-to-high, statistically significant correlation (p = .007). This finding indicates that project teachers who learned more mathematics, also taught mathematics in a more reformed manner on average, and that this reformed practices was related to higher student achievement.

Student Achievement

The project measured student growth using the State Arizona Instrument to Measure Standards, a Criterion-Referenced test bench-marked on the State Mathematics Standard. Unfortunately, in the middle of the project, the State elected to change the test, and so pre- and post administrations are not measured on the same scale. Overall in the State, scores on the 2010 AIMS are lower than on the 2009 administration due to this change. For the 2010 administration of the Arizona Instrument to Measure Standards (AIMS), there were 610 students in participating

classrooms, primarily 10th graders, taking the test for the first time. Of those students, 338 students (55%) scored proficient or above and 272 students (45%) scored basic or below. The comparison group teachers tested 899 students that were 10th graders. Of those students, 405 students (45%) scored proficient or above and 494 students (55%) scored basic or below. This provides some evidence to suggest that the teaching practices in the MSP program contributed to student learning.

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Course	Group	Mean	Std. Deviation	N
Algebra 1	Control	36.59	11.041	191
	Particip	42.26	11.502	111
	Total	38.68	11.525	302
Geometry	Control	51.80	13.510	574
	Particip	52.96	15.038	353
	Total	52.24	14.115	927
Algebra II	Control	66.83	10.567	81
	Particip	73.40	7.066	70
	Total	69.87	9.661	151
Pre-Calc/ Calculus	Control	73.67	5.508	3
	Total	73.67	5.508	3
Academic Decathlon	Particip	70.80	9.834	5
	Total	70.80	9.834	5
AIMS prep Math	Control	44.34	12.992	50
	Particip	50.37	10.622	49
	Total	47.32	12.200	99
Enrichment	Control	47.64	11.582	22
	Total	47.64	11.582	22
Total	Control	49.57	15.201	899
	Particip	53.10	15.731	610
	Total	51.00	15.510	1509

Table 2. Descriptive Statistics on the AIMS-Mathematics test for Students of Participating and Control Group Teachers

A univariate post-test, experimental/control group model was applied to the analysis of student achievement data. The dependent measures included students' AIMS mathematics raw score, the ACT composite mathematics score, ACT Algebra/Coordinate Geometry score, and ACT Plane Geometry/Trigonometry score. It was hypothesized that students of teachers participating in the Modeling MSP would show greater student achievement on these outcome measures than teachers who had not participated in the program. In addition, because typical high school courses (e.g., Algebra 1, Geometry, Algebra II, etc) would show different levels to which course content fit the content in the MSP, we analyzed differences in student achievement by course, and we accounted for the interaction effect between project participation and course by performing a factorial analysis. All family-wise alpha coefficients were set to .05 (1-tailed). Table 2 shows results for the analysis of the AIMS-mathematics test by course and by participation.

Results indicate that students of participating teachers outperformed their peers in classes taught by control group teachers. When analyzing the main effect for course, as expected, students in higher level courses showed greater AIMS achievement. Most importantly, the main

effect for overall mathematics achievement as measured by the AIMS-mathematics test was found to be superior for students of participating teachers when compared to their matched control group (F(1,1497) = 8.58, p = .048). In analyzing the interaction of the two conditions, the data show that for every class where comparison was possible, participating teachers' students outscored control group students. Overall, these differences are statistically significant (F (3,1497) = 3.87, p = .009). Details of the analysis are presented in Table 3 below.

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Observed Power
Intercept	Hypothesis	153458.779	1	153458.779	329.403	.002	1.000
	Error	1000.119	2.147	465.870[b]			
Course	Hypothesis	99905.196	7	14272.171	36.117	.001	1.000
	Error	1966.882	4.977	395.161[c]			
Particip	Hypothesis	4396.818	1	4396.818	8.585	.048	.575
	Error	1867.314	3.646	512.157[d]			
Course * Particip	Hypothesis	1945.671	3	648.557	3.866	.009	.825
	Error	251110.353	1497	167.742[e]			

Table 3. Univariate ANOVA--Effects of participation and course on student achievement on the AIMS-mathematics test.

These findings are in alignment with the goals of the project. Since all project activities were designed explicitly to embody the Arizona Mathematics Standard, it was expected that on a test of this Standard, students of teachers who had more experience learning the mathematics related to the Standard would perform better. Additionally, it provides evidence that modeling as an environment for enhancing teachers' understanding and pedagogical skill in applying the Standard content is a viable strategy.

In addition to the AIMS data, the ACT was utilized as a nationally-validated outcome measure to augment the AIMS data. We analyzed student achievement using three of the ACT scale-scores: ACT Mathematics Composite, ACT Algebra/Coordinate Geometry subscale, and ACT Plane Geometry/Trigonometry subscale. Results are mixed, showing no overall effect of project participation on ACT Mathematics Composite scores, or on ACT Plane Geometry/Trigonometry scores. A main effect for Course was detected for each of these subscales, with students enrolled in higher courses performing better on both (See Table 4 below).

On the ACT Algebra/Coordinate Geometry subscale, a significant main effect was found for Course, but not for Participation. The interaction effect, however was found to be statistically significant. A Scheffe post hoc test revealed that in Algebra II, the largest represented course in the study, Participating teachers' students outperformed those of Control teachers ($p < .05$).

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Observed Power
Intercept	Hypothesis	3107.319	1	3107.319	798.648	.000	1.000
	Error	3988.149	1025.040	3.891[b]			
Course	Hypothesis	1401.903	7	200.272	15.917	.001	.999
	Error	84.387	6.707	12.582[c]			
Particip	Hypothesis	.023	1	.023	.003	.957	.050
	Error	284.312	35.854	7.930[d]			
Course * Particip	Hypothesis	75.155	5	15.031	2.425	.034	.771
	Error	8028.120	1295	6.199[e]			

Table 4. ANOVA--Effects of participation and course on student achievement on the ACT algebra/coordinate geometry subscale.

Discussion

Results show that teachers, given an intense, sustained set of professional development experiences organized around mathematical modeling, can learn significant new content, change their practice to reflect modeling instruction, and increase the mathematical achievement of their students in the process. Several problems, however, have been identified related to attracting and retaining teachers over the long haul. In particular, retention of participants was an ongoing challenge. The number of original participants was identified as 60; however 43 instructors participated. Participants dropped out for various reasons: one teacher passed away, one was taking care of a dying parent, one started their Master's degree and had a time conflict, three teachers were involved in remediation and wanted to discontinue, two were getting paid to do Saturday School so there was a conflict with their schedules, and others stopped coming without explanations.

Moreover, implementing a two-week summer workshop may not be the most effective way to provide the bulk of the learning. Teachers experience a high level of mental fatigue and express feelings of being overwhelmed using this format.

RTOP observations and teacher comments continue to demonstrate the need to model and explicitly identify reformed teaching practices and an understanding of how students learn mathematics content. During the final weeks of the project, teachers were expected to implement specific instructional strategies in their classroom. During the RTOP Evaluation Dissemination session, were able to share experience and ask follow-up questions regarding each RTOP category and item. We conducted the RTOP Evaluation Dissemination after the first RTOP. The team felt that the process of identifying each item, addressing strengths and weaknesses in the overall group and modeling exemplar teacher performance (with MSP teacher participants) would help the majority of teachers be more successful in terms of developing pedagogical content knowledge and providing more effective mathematics instruction for their students.

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