zKnowledge Foundations for Teaching Algebra for Equity

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This paper presents our initial work to develop trajectories of preservice teachers' knowledge of teaching algebra for equity. We have developed two overlapping hypothetical learning trajectories (HLTs) for teachers' learning to engage and motivate diverse students to solve algebra problems. These HLTs are being used to guide the development of assessments, interviews, and learning activities for a pilot group of preservice teachers. The HLTs also serve as learning materials to help the preservice teachers understand how their future middle grade students learn to solve algebra word problems.

This paper describes work in the first phase of an NSF funded project whose goal is to enhance preservice teachers' knowledge for teaching algebra to diverse students. Specifically, the project focus is to design, develop, and test technology-enriched teacher preparation strategies to address equity in algebra learning for all students. The foundation for this work is a set of Hypothetical Learning Trajectories (HLTs) for teaching algebra for equity, which will be described in the paper.

Moses and Cobb (2002) stated that "the most urgent social issue affecting poor people and people of color is economic access. In today's world, economic access and full citizenship depend crucially on math and science literacy" (p. 5). Ladson-Billings (2009), citing Haberman (1996), notes that "a serious effort toward preparing teachers to teach in a culturally relevant manner requires a rethinking of the teacher preparation process" (p. 143). We agree with Moses and Ladson-Billings and we concur with researchers who conclude that traditional teaching methods are one source of difficulties that students of color have in school-based learning. Weiner (2005) found that success in the classroom only came after a shift in teachers' attitudes about teaching, learning, and culture. Research (Irvine, 1990; Lewis, 2009) has found that White teachers are sometimes not sensitive to the cultural needs of African-American students in the classroom. Both verbal and nonverbal interactions between the student and teacher are often misinterpreted and can lead to negative consequences for the African-American student (Lewis, 2009).

The gap in mathematics achievement between White, Hispanic, and African-American students has been documented since 1973. Data from the past 30 years show that White students have consistently outperformed students of color for each of the testing points (Cooper & Schleser, 2006). Although there has been recent improvement among Hispanic and African-American students, the achievement gap is still an issue (The Nation's Report Card, 2007). Furthermore, several "national and international comparisons of student achievement indicate that it is between fourth and eighth grade when U.S. students in general, and minority students in particular, fall rapidly behind desired levels of achievement" (Balfanz & Byrnes, 2006, p. 144). There are several probable causes for the continuation of the achievement gap in mathematics,

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including, “weak and unfocused curricula, shortage of skilled, trained, and knowledgeable mathematics teachers, unequal opportunities to learn challenging mathematics, and unmotivated students” (p. 144).

The importance of teacher quality has been identified as a key element to closing the mathematics achievement gap [now known as the opportunity gap] and increasing achievement of all students (Ladson-Billings & Tate, 2006, The Education Alliance, 2006). Teachers who are highly qualified and certified to teach mathematics have stronger pedagogical and mathematics knowledge, and therefore are more likely to have a better understanding of how students best learn mathematics. Highly qualified teachers will have better instructional practices than less qualified teachers (Darling-Hammond & Sykes, 2003). Teachers who are highly qualified and who have the ability and knowledge to effectively teach mathematics can “produce as much as six times the learning gains produced by less-effective teachers” (Singham, 2003, p. 589). Preservice education is the starting point for developing high quality teachers for all students. In a review of research on preparing teachers for diversity, Sleeter (2001) noted that while many White preservice students expect to work with children of another cultural background, most have little knowledge of or experience in cross-cultural settings. Preservice teachers acknowledge family support as a factor in their own previous success in mathematics education, but also hold the beliefs that mathematics ability is inherited from the parents, and that socio-cultural factors can restrict or promote learning (de Freitas, 2008). Sleeter stated that "the great bulk of the research has examined how to help young White preservice students (mainly women) develop the awareness, insights, and skills for effective teaching in multicultural contexts. Reading the research, one gains a sense of the immense struggle that involves" (p. 101). She concludes that many of the programs and courses aimed at general multicultural awareness and knowledge have had limited success, partly due to the limited opportunities for direct and relevant experience that preservice teachers have in confronting real issues of diversity in the classroom.

Teachers in Ladson-Billings’ (2009) Dreamkeepers project offered these suggestions for teacher preparation programs: recruit teacher candidates who have expressed an interest and a desire to work with African-American students; provide educational experiences that help teachers understand the central role of culture; provide teacher candidates with opportunities to critique the system in ways that will help them choose a role as either an agent of change or defender of the status quo; systematically require teacher candidates to have prolonged immersion in African-American culture; provide opportunities for observation of culturally relevant teaching; and conduct student teaching over a longer period of time and in a more controlled environment (Ladson-Billings, 2009).

In designing the HLTs, we reviewed the research literature to examine effective strategies for equity both for preservice teachers and for middle grades learners. Cognitive, affective, and cultural factors were included as primary criteria for identifying strategies that might be effective. An important underlying goal is to explore and develop activities and strategies that are effective both for preservice teachers and the students they will eventually teach in the classroom.

Teaching and Learning Algebra

Specific learning models for algebraic thinking are being identified and used to guide the development and design of HLTs. We began by identifying research-based learning trajectories for specific emergent algebraic thinking at the middle grades (Chazan & Yerushalmy, 2002).

Although there is considerable research on teaching algebra, the nature of preservice teachers' learning trajectories for developing deep algebraic thinking is less well known. Our research focuses on identifying, describing, and documenting these trajectories so that knowledge of algebra, often found to be deficient in preservice teachers (You & Kulm, 2008), can be addressed during preservice work. Learning activities will be constructed around the strategies for providing access and encouraging student engagement in challenging mathematical problems.

There are many studies on teachers' knowledge for teaching function concept. For example, Norman (1992) found that the secondary teachers tended to have inflexible images of the concept of function that restricted their abilities to identify functions in unusual contexts and to shift among representations of functions. The sampled teachers were able to give formal definitions of function, were able to distinguish functions from relations, and were able to correctly identify whether or not a given situation was functional but did not show strong connections between their informal notions of function and formal definitions. Consistent with Norman's findings, Chinnappan and Thomas (2001) found that all four pre-service secondary teachers in their study had a preference for thinking about function graphically, weak understanding of representational connections, and limited ability to describe applications of functions. Difficulty in constructing functions was also observed by Hitt (1994), who found that teachers had difficulty in constructing functions that were not continuous or were defined by different algebraic rules on different parts of the domain. Even (1993) found that many prospective secondary teachers did not hold a modern conception of a function as a univalent correspondence between two sets. These teachers tended to believe that functions are always represented by equations and that their graphs are well-behaved. None of the teachers had a reasonable explanation of the need for functions to be univalent and over-emphasized the procedure of the "vertical line test" without concern for understanding. On the other hand, Chazan, Yerushalmy, and Leikin (2008) found that teachers who were provided an opportunity to discuss and use alternative approaches were able to rethink their ideas about functions and equations.

Description and Specifications of Learning Trajectories.

The first phase of the project focuses on two interrelated research questions: What are the trajectories of middle grade students' knowledge and skill in algebra? and What are the trajectories of preservice teachers' knowledge of teaching algebra for equity? We believe that the basis of preservice teachers' knowledge for teaching algebra for equity rests on their knowledge of students' learning and motivation. We have developed two separate but overlapping hypothetical learning trajectories (HLTs) for solving algebra problems. One HLT characterizes the learning development of middle grades students. The second HLT characterizes teachers' use of strategies to engage and motivate diverse students in learning algebra. The trajectories follow the model of Simon and Tzur (2004), who provided the following set of assumptions about the characteristics and use of a hypothetical learning trajectory (HLT):

1. Generation of an HLT is based on understanding of the current knowledge of the students involved.
2. An HLT is a vehicle for planning learning of particular mathematical concepts.
3. Mathematical tasks provide tools for promoting learning of particular mathematical tasks and are, therefore, a key part of the instructional process.
4. Because of the hypothetical and inherently uncertain nature of this process, the teacher is regularly involved in modifying every aspect of the HLT (p. 93).

The HLTs facilitate building on existing knowledge and developing deeper knowledge of the topics. In order to describe how this knowledge is built, we began with the model of Lamberg and Middleton (2009) in constructing HLTs. This model contains (a) descriptions of the conceptual scheme at each level of learning, (b) summaries of the cause/effect mechanisms that characterize students' current knowledge, (c) cognitive interpretations of current knowledge, including possible misconceptions, and (d) intermediary understandings that are necessary for bridging to the next level of the learning trajectory (p. 237). Figure 1 provides a first version of the HLT for solving algebra problems. The trajectory, based on findings from research and best practice, begins with a direct translation scheme and provides rationales and cognitive interpretations of reasons for students using this approach. The last column provides possible steps to bridge to a more sophisticated scheme for solving problems. The intermediary schemes represent increasingly sophisticated problem solving strategies. The highest level scheme in this hypothetical model is a heuristic approach based on Polya's (1945) work and subsequent research that has applied that scheme.

Conceptual Schemes	**Cause/Effect**	**Cognitive Interpretation**	**Bridging Steps**
Direct Translation Scheme: Translate directly from words into expressions or equations.	Students have been taught that certain words stand for operations or equivalence. (Koedinger & Nathan, 2004; National Council of Teachers of Mathematics, 2000). Textbook word problems often to support this strategy.	Students who can read and comprehend text may be successful using this strategy for some word problems. (Chapman, 2006) However, errors in translation are common, and many problems do not lend themselves to the approach. (Pape, 2004)	Practice using heuristics to understand problems, such as draw picture or diagram, make a table, identify the data and unknowns (Johanning, 2004, 2007). Delay writing equations, then write the equations in words.
Textbook Four-Step Scheme: Read, identify data, identify unknowns, write the equation.	In an attempt to reflect Polya's four step approach Polya (1945), yet keep a simple procedure, textbooks often use this strategy with simple word problems that provide reasonable success in applying this approach.	Students can use the strategy with simple word problems, as an aid in analyzing the data and assigning variables to the unknown(s) (Johanning, 2004, 2007). The approach does little to support finding relationships needed to write an equation (Greer 1993, Johanning, 2004).	Practice using heuristics to understand problems, such as draw picture or diagram, think of a similar problem, use numbers to find examples.
Generalized Pattern Scheme: Generate numbers and generalize the pattern.	In early grades, students learn to "plug in" numbers to verify statements. Since numbers are concrete and familiar, they can be used to generate data that satisfy the conditions in a word problem.	Students are familiar with numbers and can use them to satisfy problem conditions. They can use inductive reasoning to generalize familiar patterns or arithmetic sequences. (Zazkis & Liljedahl, 2002)	Practice using heuristics to understand problems, such as draw picture or diagram. Use differences to identify the type of pattern (linear, etc.)
Heuristic Scheme: Apply heuristics to understand, plan, carry out, look back (Polya)	An heuristic approach addresses the need to focus on understanding the relationships and conditions in a problem and the need to plan how to solve it before attempting to write equations (Johanning, 2007, Polya, 1945).	Students can begin to use heuristics for identifying the data and unknown, drawing diagrams, and tables. (Bednarz & Janvier, 1996; Kieran, Boileau, & Garancon, 1996). Then can use subproblems, special cases and develop strategies such as remembering similar problems (Kieran, 1996).	Reflect on the solution to find another one, or a better or generalized solution. Think about other similar problems previously solved.

Figure 1. HLT for writing symbolic representations for verbal or non-routine problems.

The HLT presented in Figure 2 addresses the issue of equity through the use of conceptual schemes that have potential to engage diverse students in learning algebra and problem solving. A central focus of our current research is to collect and analyze data in order to further understand and revise the proposed schemes.

Conceptual Schemes	**Cause/Effect**	**Cognitive Interpretation**	**Bridging Steps**
Situated learning scheme: Provide an instructional context that allows students to have concrete and hands-on experiences with math knowledge and skills. Build math learning on realistic, open-ended, culturally relevant problems that students solve using a variety of skills, concepts, and tools.	Traditional math instruction focuses on symbolic representations exclusively, or moves too quickly from concrete to abstract lessons. This kind of transition results in poor skill development and limited conceptual understanding. (Hollar & Norwood, 1999; Karsenty, 2002; O'Callaghan, 1998)	Students develop math understanding by constructing their own culturally relevant knowledge (Ladson-Billings, 1994), building from more concrete to abstract ideas. Activities that apply math in contexts support effective learning (Pellegrino, Chudowsky, & Glaser, 2001). Student engagement in planning and carrying out the activity builds "ownership" and understanding (Silva & Moses, 1990).	Use concrete materials such as balances, algebra tiles, and everyday objects to provide concrete (Their, 2001) and hands-on experiences, before introducing formal symbols, definitions and rules. Problem solving activities can be used that gradually are more open-ended and provide opportunities for students to devise solution strategies.
Culturally relevant context scheme: Use contexts for activities that are based in and relevant to students' cultures and lives.	Many students do not see the relevance of math and have low self-expectations for learning math. Providing interesting and relevant contexts can be motivational. (Ladson-Billings, 1997)	Learning does not take place unless students are engaged in the lesson (Their, 2001). A "zone of proximal development" is necessary in which students can learn which enhances motivation for learning math.	Adapt math activities and problems that have relevant contexts, individualized to the interests of a particular class, group, or individual student (Ladson-Billings, 1995).
Critical pedagogy scheme: Provide learning activities in which students investigate the sources of mathematical knowledge, identify social problems and plausible solutions, and react to social injustices.	Many students do not see the social relevance of math. Providing interesting and relevant social contexts can be motivational. (Ladson-Billings, 1995)	Problem-based learning engages students in using math to address and solve problems that are drawn directly from possible social or community issues. The context can motivate and engage students (Boaler, 2000).	Adapt math activities and problems that have social contexts, individualized to the interests of a particular ethnic or interest group, or individual student (McLaughlin, Shepard, & O'Day, 1995; Stinson, 2004).

Figure 2. HLT for engagement in meaningful, interesting, and culturally relevant algebra problems.

These HLTs are being used by the project in two ways. They serve as frameworks to guide the development of assessments, interviews, and learning activities for a pilot group of preservice teachers. These data will inform the revision and further development of the HLTs and the learning activities aimed at developing preservice teacher knowledge of teaching algebra for equity (KATE). The HLTs also serve as learning materials, along with problem solving activities, to help the preservice teachers understand how their future middle grades students learn to solve algebra word problems. The teachers develop activities and lessons that address the bridging steps necessary to progress in the trajectories. Finally, they present lessons in Second Life to a simulated class of diverse middle grades students.

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