TEACHER FOLLOW-UP: COMMUNICATING HIGH EXPECTATIONS TO WRESTLE WITH THE MATHEMATICS

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In-depth analyses of six elementary mathematics classrooms (with varying student achievement) captured student and teacher interactions and documented the critical role of teacher follow-up in shaping opportunities for low-income students of color to engage in algebraic reasoning. This paper details the efforts of teachers as they, contrary to existing school practices, found ways to support student engagement around explanations. Examining the variation in teacher follow-up that existed across classrooms illuminated how one high-achieving classroom teacher's use of follow-up communicated a set of high expectations around mathematical work for students and supported them to wrestle productively with the mathematics.

Across a number of studies, Cognitively Guided Instruction (CGI) research documents the relationship between teachers' knowledge of student thinking and student mathematical achievement. We have found that teachers draw upon their knowledge of students' mathematical thinking as they pose problems, encourage the use of a range of strategies, and support students to share their ideas. We also have evidence of variability in the ways teachers engage in these practices, and we are interested in how that variability relates to student outcomes. We believe that knowing more about the details of teachers' classroom practice within the context of student outcomes can help both to elaborate Hiebert and Grouws' (2007) argument for allowing students to "wrestle" with the mathematics and to address our concern of the kinds of opportunities provided for marginalized students.

Efforts to detail the aspects of teachers' mathematical classroom practice that support the development of mathematical proficiency are not new (see Franke, Kazemi & Battey, 2007; Lampert, 2001; Stein, Engle, Smith & Hughes, 2008; Wood, 1998). For example, researchers have studied how posing rich mathematical tasks, allowing students to solve problems in different ways, and encouraging students to share their strategies may foster learning (Franke, Kazemi & Battey, 2007; Hiebert & Grouws, 2007) or how teachers might orchestrate sharing of student explanations and use students' explanations to highlight a mathematical goal (Lampert, 2001; Forman & Ansell, 2002; Forman, Larreamendy-Joerns, Stein & Brown, 1998). And while evidence shows these practices are not likely in many classrooms, they are even less likely in classrooms of low-income students of color (Anyon, 1981, Ladson-Billings, 1997; Lubienski, 2002; Means & Knapp, 1991). Our project builds on these studies and focuses particularly on tying the classroom practices of teachers working to make use of students' mathematical thinking to student outcomes.

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Teacher Practice and Student Learning

This study was a follow-up to a larger experimental study that found teachers who had participated in one year of CGI algebraic thinking professional development knew more about students' algebraic thinking and the students in their classroom performed better on algebraic reasoning assessment items than a comparison group (Jacobs, et al., 2007). Yet, even within the CGI professional development group, variability existed; thus we wanted to understand what teachers were doing in classrooms with higher student achievement to support student learning that may not have been occurring in the classrooms of teachers in the same professional development but with lower achievement outcomes. The teachers of this project were second or third grade teachers with student achievement on our algebraic thinking measure that was consistently high, low, or on average fell to the middle range. We then observed in these classrooms while the teachers were engaged in working on relational thinking, doing so in a way that captured the teacher's interactions with students in whole class settings, as well as all of the talk of twelve randomly selected target students (see Webb et al., 2008, 2009 for details).

The schools participating in this study served predominately low-income students of color and are considered some of the lowest performing in the state (99% Latino and African-American students combined in each school, over 98% students receiving free or reduced lunch, 49-64% English Language Learners, and scored 1-3 out of 10 on California's Academic Performance Index). The district was expected to: use curricula that focused on developing skills over conceptual understanding, "drill" rather than engage in mathematical conversation, and target the students "on the bubble" of proficiency to raise test scores. However, this characterization of the schools only reflects one aspect of the district. At the time of this work, the local district leadership was working with the schools to meet a new vision for developing mathematical understanding, showing what students were capable of, and creating school learning communities. The schools involved in the project had been working hard to meet the needs of students in a variety of ways. The elementary school principals all wanted to participate in our work, seeing algebraic thinking as critical to their students' success and believing that it was appropriate content to take on.

We engaged schools in site-based professional development focused on relational thinking[1], providing professional development first for one group of schools and the following year for a comparison group. Our experimental study showed that the students in both the algebraic thinking group and the comparison group thought relationally and were able to solve a range of algebraic thinking problems. While not all students in the schools understood mathematics in ways we would hope, this study showed that students across the district were capable of engaging in algebraic thinking and the schools were willing to support students in doing so.

Understanding that working on algebraic thinking in the ways we suggested through professional development was contrary to many existing practices, we conducted this follow-up study of six classrooms to understand in detail exactly how teachers were finding space to create opportunities for students to engage in algebraic reasoning.

Classroom Practice

Our observations of these six classrooms found that on the surface they looked much like what we would expect in a CGI classroom. The teachers posed problems that often encouraged more than one strategy, asked students (98% of the time) how they solved the problem, and asked for elaboration of student thinking (76% of the time). These practices are not what we would expect more typically in elementary classrooms, and we would argue that these practices

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are important for creating opportunities for students to engage in wrestling with the mathematics. These practices alone, however, did not lead to productive student outcomes for all students in these classrooms (See Table 1), suggesting that closer examination of the ways in which teachers engaged students and content was needed to understand opportunities for student learning.

Table 1. Mean student achievement[a] across classrooms

	Group 3 (low-achieving classrooms)		Group 2 (medium-achieving classroom)	Group 1 (high-achieving classrooms)		
	Guo	Collins	Gomez	Lane	Lee	Theissen
Mean	18	27	50	60	75	88
(SD)	(30)	(35)	(46)	(46)	(35)	(25)

[a]Percent of algebraic reasoning posttest problems correct.
Note: Differences across classrooms: $F(5,68) = 5.91$, $p<.001$.

Problems Posed

Looking more closely at the problems teachers posed – taking into account that they were all working on common CGI algebraic reasoning content of relational thinking – we did find that there were differences in the problems posed. If we focus on one teacher from each of the achievement groups—Ms. Guo, Ms. Gomez, and Ms. Lee—we can examine closely how the problems posed were the same and different. Ms. Guo posed the most mathematically powerful sequence of problems that built toward a relational understanding of the equal sign. The majority of Ms. Lee's problems were drawn from ones the students had written. She chose problems that provided opportunities to use relational thinking, but without a particular sequence except when she added a problem to build from student responses. Ms. Gomez's problems often did not lend themselves to using relational thinking, as they focused mainly on multi-digit computation. An examination of the problems teachers posed across classroom observations is one approach to understanding opportunities for student mathematical learning. Yet, the teacher with the "best" problem sequence had the lowest achieving students, suggesting that while the tasks differed in the mathematical opportunities provided for students, they alone did not determine student outcomes.

Student explanations. As we looked further at what was occurring for students in these classrooms, we focused on the explanations students provided to the problems posed. We tracked student answers and explanations, given both to other students and to the teacher, within the whole-class setting and in small-group or pair-share settings. Students were participating in giving explanations in each classroom and for a high percentage of problems. (see Table 2).

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Table 2. Overall level of student participation

	Group 3		Group 2	Group 1		
	Collins	Guo	Gomez	Lee	Lane	Theissen
Percent of target students who ever gave an explanation or answer	85	100	100	100	87	83
Percent of problems in which students gave explanations	64	80	88	100	86	70

It was not just the giving of explanations that differentiated classrooms, but rather the kinds of explanations that were given. The degree to which the explanations were complete and accurate varied across classrooms (see Table 3).

Table 3. Work contributed by students during the whole class: Percent of classwork problems with student contributions of each type

	Group 3		Group 2	Group 1		
	Collins	Guo	Gomez	Lee	Lane	Theissen
Correct/complete explanations	55	40	88	100	86	70
Incorrect/incomplete explanations only	9	40	0	0	0	0
Correct answer but no explanation	36	20	13	0	14	30
Incorrect answer but no correct answer or explanation	0	0	0	0	0	0

Note: Numbers in a column do not always sum to 100, due to rounding.

We found that students who gave complete, correct explanations scored better on the student achievement measures designed to measure algebraic thinking (see Table 4). We have examined this finding in a number of ways and each time we find the same pattern: it is not the mere act of explaining that is related to positive student achievement, but rather the giving of correct and complete explanations.

Table 4. Correlations between Student Participation and Achievement Scores

Highest level of student participation on a problem[a]	Achievement Score[b]
Gives explanation	.52***
Correct and complete	.59***
Ambiguous, incomplete, or incorrect	-.01
Gives no explanation	-.41***

[a]Percent of problems in which a student displayed this behavior. Problems discussed during pair-share and whole-class interaction are included. [b]Percent of problems correct. (n=74)

There are two critical points to be made here. First, "correct and complete" is not simply about making an explanation that gets to an end. Correct and complete explanations are about students articulating the steps in their solution in a way that details the mathematical relationships. In explaining a solution to 24 + 19 = 25 + ___, rather than saying the answer has to be 18 "because of the 24 and 25", the student would say (or point to) "24 is one less than 25 so whatever you add to 25 has to be one less than 19". The latter articulation makes explicit the particular relationship between the 24 and 25 and thus the 19 and the unknown. In complete explanations students are articulating in a way that as one listens and watches they can tell exactly what the student did to solve the problem. Second, as this is correlational data one might argue that students in Ms. Lee's class simply knew the math better and that is why their explanations are complete and correct. While we have no data that supports that these students came in knowing more, it is a possibility. What makes it less likely as an argument is that as we examined what teachers did to support students to make correct and complete explanations we found clear differences within and across classrooms. As the data we share will show, when teachers followed up in particular ways, it was more likely that students would produce correct and complete explanations.

Engagement around Explanations

Teacher Follow-Up

As we continued our examination of classroom practice across classrooms with a range of student achievement, we looked closely at the interactions among students and teachers as students gave explanations. We wanted to know how teachers were supporting students to give complete and correct explanations. We focused specifically on teacher follow-up that resulted in the articulation of more student thinking, whether their initial explanations were complete, incomplete, or ambiguous. The teachers with the highest achieving students followed up in ways that led to more explanation being given by the student (see Table 5).

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Table 5. Experiences of target students when engaging with the teacher around their contribution during the whole class: Percent of target students who experienced each type of student participation

	Group 3		Group 2	Group 1		
	Collins	Guo	Gomez	Min	Lane	Theissen
More student thinking was provided beyond the initial explanation[a], engagement ended with a correct/complete explanation	8	18	17	92	60	83
More student thinking was provided beyond the initial explanation[a], but the engagement never led to a correct/complete explanation	0	45	17	0	0	0
No more student thinking was provided beyond initially correct/complete explanation	31	55	25	8	33	8
No more student thinking was provided beyond initially incorrect/incomplete/ ambiguous explanation	15	64	0	8	0	0
No more student thinking was provided beyond the student's correct answer	69	27	9	23	60	17
No more student thinking was provided beyond the student's incorrect answer	46	36	0	8	7	0

[a]In most cases, the target students themselves provided the additional thinking; in a few cases, another student in the class provided the additional thinking.

Looking closely at the follow-up, we expected to see that the teachers supporting more complete and correct explanations would be using particular approaches to do so – more probing questions, revoicing, providing tools, and so on. We did not find this to be true. All of the teachers drew upon a repertoire of follow-up moves. It was how they used these moves in relation to the students and the mathematics that seems to support students to provide more explanation, take an incorrect explanation to correct, or an explanation from incomplete to complete.

Returning to our one focus teacher per group, Ms. Guo, Ms. Gomez and Ms. Lee, we found that not only did they draw upon a repertoire of follow-up moves, but that the substance of their follow up was quite different. We examined these interactions in detail to gain insight into the different experiences students had as they solved problems and gave explanations (Franke et al, 2009).

Ms. Guo, the teacher with the lowest achieving students, posed productive problems, asked students how they solved the problems, and while she followed up on student thinking less often than the other teachers, she did follow up. For 45 percent of her students, however, her follow up did not lead to more explanation. Often this was due to her not pressing the student idea and posing a new problem or to confusion about what she was asking.

Problem: a= a; True or False?

Marcus: It's true, true true.
Lindsay: It's a letter. It's an alphabet letter. It's true because A is the same as A.
Marcus: It's true for everything.
Ms. Guo: You guys think that this one is true?
Marcus: Yes.
Lindsay: Yes. Because A and A is the same letters.
Ms. Guo: So they are the same letters, so that makes it true?
Lindsay and Marcus: Yeah.
Ms. Guo: We'll check it out.
Marcus: Everything is true, huh?

Here Ms. Guo followed up on Marcus and Lindsay's thinking and even asked a question that was specific to what students had said. However, she did not press either of the students to articulate what they mean by "A is the same letter as A" and then what that means for the problem they were solving. We do not learn more about either Lindsay or Marcus' thinking as they engaged with Ms. Guo.

In addition, Ms. Guo often followed up but rather than press the student, she revoiced or posed a new problem for the students to consider. In the next example Ms. Guo posed 100 + ☐ = 100 + 50 as a follow-up to a previous unresolved discussion. The class had begun to refer to this idea as "partners", that is, 50 is "partnered" with the other 50. Here Fernando used his notion of needing the same numbers on both sides to argue that 50 goes in the box.

Problem: 100+ ___ = 100 + 50

Ms. Guo:	Ok. One hundred plus box over here is the same as…one hundred plus fifty. What would go inside that box now and why? Fernando?
Fernando:	One hundred…no, the fifty will go right there because it has to be the same number.
Ms. Guo:	What has to be the same number?
Fernando:	One hundred fifty and the other side has to be one hundred fifty too *(unclear whether Fernando is saying 150 or 100 and 50)*.
Ms. Guo:	Ok, so you are adding these together. You said this side has to be *(bell rings)* one hundred fifty?
Fernando:	No.
Ms. Guo:	Oh, what were you saying?
Fernando:	That they have to be those because… 'cause it has to have the same numbers.
Ms. Guo:	It has to have the same numbers. Ok.

Fernando:	Fifty, fifty, and one hundred.
Ms. Guo:	Ok, this side has a fifty and this side has a fifty. Ok. I see that relation going across. But what if I did this though? What if I did this? If I called…I did one hundred and I called box, B for box. And if I did one hundred plus (*writes: 100 + B = 100 + 75*)…what would B have to be?

It was never clear if Fernando was considering numbers on each side as equivalent quantities across the equal sign or matching to find the same number in the number sentence. Ms. Guo followed up by posing another problem, but not one that would necessarily clarify the mathematical issue of "partners". This issue came up a number of times as the lesson progressed, causing difficulty as students began to match numbers regardless of the position in the number sentence (such as a + a = b + b being always true because "the a's have partners"). The issue is never resolved.

Ms. Gomez, whose problems were often focused on a computational solution and not related to the algebraic thinking, followed up about half of the time and half of that follow-up led to more explanation. Ms. Gomez asked follow-up questions directed at the mathematics, but she did so to help the student through the solution that they could not complete. So the follow-up, while framed as a question, was structured to help the student see the next step rather than to elicit more student thinking. Here Ms. Gomez stopped in while Miguel and Maricela were working on the problem.

Problem: 14 ÷ 2 = 3 x __ + 1

Miguel:	Three times…
Ms. Gomez:	You are working on number one right?
Miguel:	Three times one…
Ms. Gomez:	Ok, but you are working on number one… So remember, we always ask ourselves, which of the two sides is complete. The left side or the right side?
Miguel:	Left side. *(pointing on paper with pencil)*
Ms. Gomez:	Ok, so can you solve the one that's complete? *(Miguel nods)* So, do you think that's a good idea if you solve that first? *(Miguel nods)* Ok. *(Miguel looks up at teacher)* So what does that tell you?
Miguel:	That fourteen divided by two equals, is the same as seven?
Ms. Gomez:	Ok, very well read. Now so, if this side equals seven, what does this side… Oh my, you've already solved it.

Ms. Gomez wanted to help Miguel with an approach that starts with solving one side of the equation and then using that to help you figure out the other side. This is an example of a problem Ms. Gomez posed that does not readily lend itself to relational thinking, so her approach of solving one side may be best. She did not, however, ask Miguel how he was thinking about the problem nor did she ask him to think about the whole problem, what it is asking and what he knows in relation to what it is asking. If Ms. Gomez's approach made sense to you as a student, it could be very helpful; if not you could be lost. Ms. Gomez did not leave incorrect strategies unaddressed, yet she often did not ask any additional questions when a correct and complete explanation was given.

Ms. Lee, who chose productive relational thinking problems from those her students had written, regularly followed up on students' explanations and regularly supported students to provide additional explanation. She followed up on the mathematical ideas embedded in the student explanations and could always find something productive in what the student had offered, using it to ask a follow up question. Her questions took into account both the mathematics and the student's thinking. Ms. Lee had posed the problem, 11 + 2 = 10 + __. During the whole-class share she followed up on Andrew's explanation by asking him a clarification question and then specific to his explanation as well as the mathematics.

Problem: 11 + 2 = 10 + __

Andrew:	I put eleven here, put a two right here, then I plussed it, and it was thirteen. I put take away… take away (*holds fingers up*). I wrote thirteen right here. I put right here a caret and put thirteen. I put here a thirteen. Three, thirteen take away, take away ten and then I minus, I minus (*counting on fingers*)… ten. And then *(inaudible).*
Ms. Lee:	Wow. Okay. This is really interesting. Okay, let's look at this. Does everybody understand how you got eleven plus two equals thirteen?
Students:	Yeah.
Ms. Lee:	Okay. Why did you minus ten? And where did you get that ten from?
Andrew:	Cause on the three I added first… I went three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen (*inaudible*).
Ms. Lee:	Okay, is there anywhere else in the problem that has a ten?
Andrew:	Yes here (*points to the 10 to the right of the equal sign*).
Ms. Lee:	There. Okay, does anyone see that connection?

As she followed up with Andrew, she asked a question particular to his explanation and to the mathematics within the problem that has the potential of encouraging him to think relationally. She also worked to connect their interaction to the whole class.

Ms. Lee also, more than any other teacher, asked extension questions about the mathematical properties. In sharing his strategy D'ante stated that he knew when he took zero from 30 it would still be 30. Ms. Lee followed up on this particular idea.

D'ante:	Because thirty take away zero is still thirty.
Ms. Lee:	Okay, is that true? Is that true that thirty take away zero is still thirty?
Students:	Yes. *(a small number of students)*
Ms. Lee:	How do you know that's true?
Student:	When you have thirty and you don't take away nothing, so it's still thirty.
Ms. Lee:	Can you say it louder so everyone can hear you?
Student:	Because you have thirty fingers and you don't take away nothing so it's still thirty.
Jennifer:	*(loudly, to herself)* 'Cause you don't take away nothing. You have nothing to take away.
Ms. Lee:	Is that always true?

This conversation continued for over five minutes and then Ms. Lee took it back to the problem at hand, connecting to why it would help you to use this idea. This type of follow-up was typical in her classroom. Following up on the students' ideas, while following up on a relevant math idea, and pushing beyond the solution given occurred in the highest achieving classes.

Another way to consider the follow up that occurred in Ms. Lee's class is to think about the way that her follow-up sets up expectations for students and the effect this has on opportunities for mathematics learning. In their studies of teachers of African American students, Clark and his colleagues (2009) found that teacher expectations stood out as a major factor in shaping the mathematics learning of students. Battey and Stark (2009) showed how deficit beliefs created impoverished mathematical practices for Latino and African American students. Ms. Lee's actions reflect her belief that students are capable of high-level mathematical work and thus expected to participate as such. Ms. Lee demonstrated follow-up (in ways similar to what Kazemi and Stipek (2001) term "press") to each students' ideas in a way that communicated these high expectations. Ms. Lee did not shy from addressing the mathematical properties or working with variables with her second and third graders. She had the students write the problems they solve together. She asked follow-up questions to everyone (each student shared an explanation at some point across two class sessions), questions that pushed them to articulate their mathematical ideas but in a way that was connected to their own ideas. Ms. Lee found something positive –mathematically—in what each student shared and used it to move the group's mathematical work forward. A growing literature suggests that this focus on what students can do, positioning students competently, matters and matters often for the lowest achieving students (Battey (under review); Boaler & Greeno, 2000; Boaler, 2003; Empson, 2003). In addition, she expected and supported students to do this with each other. This is not only about holding high expectations; it is a set of actions that communicate those expectations.

Student Follow-Up

We are finding that students do the same types of follow-up that their teachers do when they are in small groups or pairs together. In Ms. Guo's class the students in pair share frequently discussed ideas for solving the problem, but they did not complete the solutions nor did they work to make sense of each other's ideas. The goal seemed to be to get ideas out on the table, much like Ms. Guo did, with little to no follow-up to further detail these ideas or move them toward completion. Ms. Gomez's follow-up in both pair-share and whole-class discussions communicated an expectation of completing the problem, often with a particular procedure that the class had been working on. Rarely were there interchanges among students that moved beyond working through a procedure to involve probing of explanations. Students in Ms. Lee's class exhibited further probing of each others' ideas as they shared explanations and worked toward finding more than one way to approach the problem. The ways in which students engaged with each other in pair share reflected the kinds of opportunities created as Ms. Guo and Ms. Gomez engaged students.

As we further examined the interactions in Ms. Lee's class, however, we recognized that there were other potentially productive outcomes occurring through these interactions beyond the sharing of explanations. Students were learning a way of participating in mathematics that included: sharing their ideas to a greater degree of detail, listening and engaging with someone else's ideas, asking mathematical questions based on another's strategy, understanding someone else's ideas in relation to their own, and so on. The following interchange demonstrates both how students follow up like their teacher (Ms. Lee) and how they are learning a range of things beyond how to correctly solve a mathematics problem.

In this example Ana and Michael were working during pair share on a problem posed to their class: $8 + 8 = 15 + \square$. Michael thought the answer to the problem was 15 (likely due to a similar "partner" misunderstanding as in Ms. Guo's class). Ana, who knew the answer was one, worked to get Michael to see that he was not thinking about it correctly.

Problem: $8 + 8 = 15 + \square$

Ana: It's not... it's not... it's not fifteen, because eight plus... I'll explain it back to you so you can understand that it's one. Because eight plus eight sixteen…

Michael: I already tried two times.

Ana: Huh?

Michael: I already tried two times.

Ana: It's not... it's not fifteen, because eight plus eight is sixteen. So this one has to... has to equal sixteen—"the same".

Michael: *(pause)* The equal sign means "the same" and there's eight (*trails off*)...

Ana: But it, the equal sign means that "the same answer"… It has to be the same answer. Like if... if I put... (*pause)* I could switch it: fifteen plus one is the same as eight plus eight.

Michael: OH! I know what you're talking about.

Ana: It's just gonna be sixteen.

Michael: So it's worth sixteen right here (*gestures to one side of paper)* and sixteen right here *(gestures to the other side)*. Ohhh!

Ana's first move to challenge Michael's incorrect idea was to explain that the expressions on both sides had to equal sixteen, but that explanation did not make sense to him. She emphasized the language "the same." Michael agreed with the idea of "the same", but this did not resolve his incorrect answer. Ana persisted. She added that it was not only "the same" but it was "the same answer." As that explanation did not convince Michael, she made yet another attempt to get her point across: she flipped the number sentence around to say "fifteen plus one is the same as eight plus eight." It was this specific move that helped Michael understand the quantity sixteen on both sides of the equal sign, moving him away from his incorrect answer of fifteen.

Ana helps Michael with a way to think about the solution that made sense to him. You can see him gesture in a way that shows you can move the 15 + 1 together and the 8 + 8 together and they are the same because they are both just 16. This interchange exemplifies the impressive caliber of mathematical discourse that occurred in Ms. Lee's classroom. Ana explains; Michael listens and works to make sense. Ana listens to Michael and tries a different approach she thinks might be more successful. Ana finally makes a mathematically powerful move that helps Michael make a connection about the quantities on both sides of the equal sign. There is a sense in watching this interchange that Ana is confident that she can communicate with Michael, and she has the skills and tools to do so. The types of interactions in this exchange occurred in other pairshare episodes in Ms. Lee's class. It is though an interchange that contrasts significantly those in Ms. Gomez's and Ms. Guo's classrooms.

Summary

Freund (2011) found in examining the aspects of mathematical proficiency developed within theses classrooms that developing the types of classroom norms seen in the higher achieving classrooms was not a matter of simply telling kids to work with each other or explain their thinking (all of the teachers did this). It was a combination of norms statements as well as communication of expectations through interaction that determined what constituted engagement around a mathematical explanation (Cobb et al, 2001). We can see through the work shared here and Freund's study that teachers and students together are developing a set of norms that shape participation in ways that allow students to make complete and correct explanations and create a culture of what it means to do mathematics together.

Ms. Guo, Ms. Gomez, and Ms. Lee serve as exemplars of what occurred across the six classrooms with differing student outcomes. Each of these teachers engaged their students in mathematics that countered the prevailing district culture. Each of these teachers was working to reach each of their students and engage them in developing an understanding of algebraic ideas often not addressed in second and third grade and certainly not addressed in the lowest performing schools at these grade levels. Each of these teachers carried out practices that research would contend are productive for developing mathematical understanding. What these three cases do is help us understand the nuance of Ms. Lee's interaction with her students and how that is related to student outcomes. Ms. Lee through her follow-up interactions connected with students' mathematical ideas and pressed students to detail their ideas for themselves and the class. She expected her students to participate in high-level mathematics and worked with them so they would know how.

Conclusions

In understanding the classroom practices that support student achievement in mathematics the data from this project suggest that posing productive mathematics problems while necessary, is not sufficient. Asking students the initial "how did you solve that" is necessary, but not sufficient. Following up on the students' explanation is necessary, but not sufficient. The way one follows up matters. It matters that the follow up presses students to provide more explanation and the explanation gets to something that is complete and correct. It matters that the follow-up is connected to what the student has articulated or shown as well as to the mathematical ideas being addressed in the lesson. It matters that, as the teachers are following up, they are supporting students to learn a way of engaging with the mathematics and creating a culture of doing mathematics that signals that wrestling with the mathematics means asking each other questions, considering others' ideas, and connecting mathematical ideas. It matters that teacher and student interactions create a set of expectations and a way of positioning students that make explicit that each child brings mathematical knowledge to the interactions and can engage in challenging mathematical work. It matters that the interactions highlight persistence, detailing thinking, listening, questioning, as well as the process of getting stuck and unstuck. We would argue that this type of work constitutes creating opportunities to "work and wrestle" with the mathematics in ways that set expectations through action for each of her students.

Endnote

1. Relational Thinking involves children's use of fundamental properties of operations and equality to analyze a problem in the context of a goal structure and then to simplify progress towards this goal. (Carpenter, Franke, & Levi, 2003; Empson, Levi, & Carpenter, 2011)

References

Anyon, J. (1980). Social class and the hidden curriculum of work. *Journal of Education, 163*(1), 67-92.

Battey, D. (under review). “Good” mathematics teaching for minority students and the poor: The mediation of instruction through relational interactions. Submitted to *Educational Studies in Mathematics*.

Battey, D. & Stark, M. (2009). Equitable classroom practices: Avoiding ability attributions from misconceptions in mathematics. In C. Malloy (Ed.), *Mathematics for All: Instructional Strategies for Diverse Classrooms* (pp. 167-177). Reston, VA: National Council of Teachers of Mathematics.

Boaler, J. (2002). *Experiencing school mathematics*. New Jersey: Lawrence Erlbaum Associates.

Boaler, J. (2003). Learning from teaching: Exploring the relationship between reform curriculum and equity. *Journal for Research in Mathematics Education, 33*(4), 239 - 258.

Boaler, J. & Greeno, J.G. (2000). Identity, agency and knowing in mathematical worlds. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning.* Westport, CT: Ablex.

Carpenter, T.P., Franke, M.L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school*. Portsmouth, NH: Heinemann.

Clark, L., Johnson, W., & Chazan, D. (2009). Researching African American mathematics teachers of African American students: Conceptual and methodological considerations. In D. Martin (Ed.) *Mathematics Teaching, Learning, and Liberation in the Lives of Black Children*, (pp. 75 - 102). New York: Routledge.

Cobb, P., Wood, T., & Yackel, E. (1993). Discourse, mathematical thinking, and classroom practice. In E. A. Forman, N. Minick & C. A. Stone (Eds.), *Contexts for learning: Sociocultural dynamics in children's development* (pp. 91-119). New York: Oxford University Press.

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. P. E. (2001). Participating in Classroom Mathematical Practices. *Journal of the Learning Sciences, 10*(1&2), 113-163.

Empson, S. B., Levi, L., & Carpenter, T. P. (2011). The Algebraic Nature of Fractions: Developing Relational Thinking in Elementary School. To appear in J. Cai & E. Knuth (Eds.), *Early Algebraization: A Global Dialogue from Multiple Perspectives*, 409-426. New York: Springer.

Empson, S. B. (2003) Low-Performing Students and Teaching Fractions for Understanding: An Interactional Analysis. *Journal for Research in Mathematics Education, 34,* 305-343.

Forman, E. A., & Ansell, E. (2002). Orchestrating the multiple voices and inscriptions of a mathematics classroom. *The Journal of the Learning Sciences, 11*, 251-274.

Forman, E. A., Larreamendy-Joerns, J., Stein, M. K. & Brown, C.A. (1998). You’re going to want to find out which and prove it: Collective argumentation in a mathematics classroom. *Learning and Instruction* 8(6), 527-548.

Franke, M.L., Kazemi, E., Battey, D. (2007). Understanding teaching and classroom practice in mathematics. In F. Lester (Ed.) *Second handbook of research on mathematics teaching and learning* (225-256). Charlotte, NC: Information Age Publishing.

Franke, M. L., Webb, N. M., Chan, A. G., Ing, M., Freund, D., & Batter, D. (2009). Teacher questioning to elicit students’ mathematical thinking in elementary school classrooms. *Journal of Teacher Education. 60,* 380-392.

Freund, D. (2011). *Opportunities to Develop Mathematical Proficiency: How Teachers Structure Participation in the Elementary Mathematics Classroom* Unpublished Doctoral Dissertation, University of California, Los Angeles. CA: Los Angeles.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371-404). Charlotte, NC: Information Age Publishing.
Jacobs, V., Franke, M.., Carpenter, T., Levi, L. & Battey, D. (2007). Exploring the impact of large scale professional development focused on children's algebraic reasoning. *Journal for Research in Mathematics Education 38* (3), pp. 258-288.
Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *Elementary School Journal, 102*, 59-80.
Lampert, M. (2001). *Teaching problems and the problems of teaching*. New Haven & London: Yale University Press
Lampert, M. (1989). Choosing and using mathematical tools in classroom discourse. *Advances in Research on Teaching, 1*, 223-264.
Lubienski, S. T. (2002). Research, reform, and equity in U.S. mathematics education. *Mathematical Thinking and Learning, 4*(2&3), 103-125.
Means, B., & Knapp, M. S. (1991). Cognitive approaches to teaching advanced skills to educationally disadvantaged students. *Phi Delta Kappan, 73*(4), 282-289.
Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation*, *2*, 50-80.
Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10*(4).
Webb, N. M., Franke, M. L., Ing, M., Chan, A., De, T., Freund, D., & Battey, D. (2008). The role of teacher instructional practices in student collaboration. *Contemporary Educational Psychology, 33,* 360-381.
Webb, N. M., Franke, M. L., De, T., Chan, A. G., Freund, D., Shein, P., & Melkonian, D. K. (2009). Teachers' instructional practices and small-group dialogue. *Cambridge Journal of Education, 39:1, 49-70.*
Webb, N. M. (in press). The teacher's role in promoting collaborative dialogue in the classroom (annual review article). *British Journal of Educational Psychology.*
Wood, T. (1998). Funneling or focusing? Alternative patterns of communication in mathematics class. In H. Steinbring, M. G. Bartolini-Bussi, A. Sierpinska (Eds.), *Language and communication in the mathematics classroom* (pp. 167-178). Reston, VA: National Council of Teachers of Mathematics.