THE INFLUENCE OF A TEACHER'S DECENTERING MOVES ON STUDENTS ENGAGING IN REFLECTIVE THINKING

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In this paper, we discuss how a teacher's decentering moves influence the discourse in her classroom. Research has revealed decentering to be a valuable construct in characterizing the interactions in a professional learning community. Our research explores the role of decentering in a teacher's ability to support student learning in the classroom. Specifically we focus on how a teacher's decentering moves influences her students' propensity to reflect on their thinking and solutions. Our findings illustrate how a teacher's improved decentering ability supports student reflection and achieving a conceptually oriented classroom.

Introduction

Researchers have found the construct of decentering to be a useful construct in analyzing a facilitator's ability to support mathematical discussions in a professional learning community (PLC) of secondary teachers (Carlson, Moore, Bowling, & Ortiz, 2007). The authors observed that as a facilitator increased her or his propensity and ability to discern the thinking of her or his peers, the mathematical discourse in the PLC became more substantive and conceptual. In this paper we adapt decentering to analyze a Precalculus teacher's interactions with her students. We discuss how a teacher's decentering actions relate to her students' opportunities for reflecting on their responses and thinking. We also characterize transitions in students' tendency to engage in reflective thinking in relation to the teacher's decentering actions.

Theoretical Framework

Piaget (1955) described decentering as the act of adopting a perspective that is not one's own. Steffe and Thompson (2000) built on Piaget's work in describing how one individual adjusts his or her behaviors in order to influence another individual, or a group of individuals, in a particular manner. Specifically, Steffe and Thompson used decentering to characterize the interactions betweens teachers and students.

A teacher decentering assumes that the student's behavior has a rationality of its own and tries to discern the mental actions driving the student's behaviors. As the teacher decenters, they build a model of the student's thinking, and then base his or her actions on this model. To contrast, a teacher not decentering interacts with students assuming that her or his thinking and the student's thinking are identical. Over the course of a conversation, a non-decentering teacher may notice that the student's thinking is different, but the teacher does not construct a model of the student's thinking, therefore acting in a non-decentered way.

Researchers (Carlson, et al., 2007) found decentering to be a key tool for facilitators to achieve successful discourse in secondary mathematics and science PLCs. When characterizing the PLC facilitators' decentering actions, the authors determined various Facilitator Decentering Moves (FDMs) in order to classify the PLC interactions. The decentering moves ranged from the facilitator acting entirely in a non-decentered way (FDM1), representing the lowest end of the spectrum, to the facilitator building a model of PLC members' thinking and how the members were interpreting her or his actions while adjusting her or his actions based on these models

(FDM5), representing the highest end of the spectrum.

Teuscher, Moore, and Carlson (2011) made further connections between a teacher's level of decentering and the student discourse in the classroom. In studying one teacher's use of research based curriculum, the authors noted that the teacher's question types and purposes changed as the teacher began to focus on her students' thinking and understanding of the concepts independently of her way of thinking about the tasks (decentering). We believe that student knowledge is unique to each individual and we conjecture that a decentering teacher will support her or his students' learning by enabling the students to build and reflect on their thinking and actions. We predict that classroom discourse will also shift toward more opportunities for student reflection as a teacher is decentering. When teachers promote student reflection and thinking, the students are placed in situations that Hiebert and Grouws (2007) call *opportunities to learn*.

Hiebert and Grouws (2007) suggested that the type of curriculum influences opportunities for students to learn and we conjecture that the curriculum may influence teachers' pedagogical decisions. A curriculum emphasizing a conceptual orientation may influence a teacher's pedagogical goals to promote an environment of conceptual discourse. On the other hand, a curriculum emphasizing a procedural orientation places value in students gaining procedural fluency. In characterizing these two goals of learning, procedural and conceptual, Hiebert and Grouws (2007) note two types of teaching patterns: teaching for skill efficiency and teaching for conceptual understanding. Teaching for skill efficiency means providing students with opportunities to practice and master mathematical procedures quickly and accurately. Teaching for conceptual understanding means providing students with rich mathematical tasks that support them making mental connections between mathematical ideas, procedures, and facts. Teaching orientations impact the nature of classroom discourse, promoting either a calculational or conceptual orientation (Thompson, Philipp, Thompson, & Boyd, 1994).

Stigler and Hiebert (1999) noted that even though an element in the system (in this case the teacher) may change, the system itself resists sudden change and will "rush to repair itself" due to cultural scripts. The system they refer to includes teachers, students, administrators, and parents. Cultural scripts are unwritten "rules" or behaviors that are shared by members of a society; in the context of teaching and learning, it is the expected role of teachers and students. Due to these constraints in achieving classroom shifts, we hypothesize that a teacher's decentering ability and corresponding changes in her or his students' activity will occur in gradual shifts.

For the present study, we investigate the following research questions:

1. How does students' discourse shift toward more opportunities for reflection as a teacher's decentering levels increase?
2. How does the nature of student reflection change when the teacher promotes and the curriculum supports a conceptual orientation in mathematics?

Methods

Claudia is a secondary mathematics teacher at Rover High School. Previous to the study, Claudia had a total of 20 years teaching experience including elementary, junior high and high school. At the time of this study, Claudia was assigned to teach Precalculus for the second time in her career. All mathematics and science teachers at Rover High School participated in a National Science Foundation Math and Science Partnership (MSP) project (No. EHR-0412537) designed to support secondary mathematics and science teachers in improving their instruction and their students' learning. The project leaders designed and implemented school-based interventions including graduate courses, workshops, and leadership for school-based PLCs at

Rover High School to support teachers in using inquiry-based instructional methods and a conceptually oriented curriculum. To support Claudia's efforts, the project leaders gave Claudia the opportunity to use a newly developed research-based conceptually oriented Precalculus curriculum (Carlson & Oehrtman, 2010), which we will refer to as the Pathways curriculum. In this curriculum, covariational reasoning, rate of change, proportionality, and problem solving are emphasized.

Claudia's two Precalculus classes used the Pathways curriculum and were videotaped daily to observe the interactions and discourse between Claudia and her students. Videos were digitized and viewed by the research team, with select videos analyzed to examine shifts in Claudia's interactions with her students. Three exemplary videos were selected for coding: one from the first week of class (August), one from five weeks into the school year (September), and one from five months into the school year (December). The research team coded the teacher-student interactions for students' mathematical reflections and the degree to which Claudia was decentering while interacting with her students.

To code student opportunities for reflection, the last two levels of descriptor C.9 "The teacher encouraged students to reflect on the reasonableness of their responses" from the Middle Tool Observational Tool was used (Reys, 2004; Tarr, et al., 2008). We first coded each teacher-student interaction that provided students' the opportunity to reflect on the reasonableness of their or someone else's responses. The second level of coding was within each interaction, the teacher moves depending on the whether the teacher encouraged or discouraged conceptual understanding (see Table 1) by either promoting a calculational or conceptual orientation (Thompson, et al., 1994).

Student Opportunity	Teacher moves
Student reflection on answers	1. The teacher asked students if they checked whether their answers were reasonable but did not promote discussion that emphasized conceptual understanding, or 2. The teacher encouraged students to reflect on the reasonableness of their answers, and the discussion involved emphasis on conceptual understanding.

Table 1. Student Opportunities (adapted from Reys, 2004)

To characterize Claudia's decentering, we used the construct of the facilitator decentering moves (Carlson, et al., 2007) in the context of interactions between Claudia and her students. Table 2 displays the construct of facilitator decentering moves (FDM); in the context of our study we referred to the teacher-student interactions as teacher decentering moves (TDM).

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Decentering Codes	Description
TDM1	The teacher shows no interest in understanding the thinking or perspective of a student with which he/she is interacting.
TDM2	The teacher appears to build a partial model of a student's thinking, but does not use that model in communication with the student. The teacher appears to listen and/or ask questions that suggest interest in the student's thinking; however, the teacher does not use this knowledge in communication.
TDM3	The teacher builds a model of a student's thinking and recognizes that it is different from her/his own. The teacher then acts in ways to move the student to her/his way of thinking.
TDM4	The teacher builds a model of a student's thinking and acts in ways that respect and build on the rationality of this student's thinking for the purpose of advancing the student's thinking and/or understanding.
TDM5	The teacher builds a model of a student's thinking and respects that it has a rationality of its own. Through interaction the teacher also builds a model of how he/she is being interpreted by the student. He/she then adjusts her/his actions (questions, drawings, statements) to take into account both the student's thinking and how the teacher might be interpreted by that student.

Table 2. Characterization of Teacher Decentering Moves (adaption from Carlson, et al., 2007)

Results

In this section we discuss the quantitative and qualitative results from the analysis of Claudia's three classroom videos. Each classroom session lasted 50 – 54 minutes. Table 3 displays the number of minutes of class time that was coded as a teacher-student interaction in which students reflected on their responses. The percentage of student contribution to these reflective interactions is listed after the minutes, measured as the time Claudia was not speaking during the interaction. Each instance of student reflection was coded according to whether or not the discourse promoted conceptual understanding (e.g., making connections between mathematical ideas).

Reflection type	August (minutes)	September (minutes)	December (minutes)
Did not empathize conceptual understanding	4.43 (28.91%)	0.00 (0.00%)	0.00 (0.00%)
Emphasized conceptual understanding	1.87 (28.91%)	5.39 (46.62%)	12.69 (52.10%)

Table 3. Teacher-student interactions categorized by reflection type

The percentage of total class time in which interactions included student opportunities to reflect on answers was 11.50% in August, 9.90% in September, and increased to 25.3% in December. The percent increase of time spent in these interactions, determined as the difference in minutes between August and December relative to the total class time in August, was 11.93%. The increase in student reflection on their mathematical responses is evident, but the table only conveys the percentage of class time for which this occurred. In August, the 11.50% spent in reflection was based on interactions that consisted of Claudia questioning her students during a whole class discussion. When students worked in small groups during the same lesson, the research team did not observe opportunities for student reflection. In both the September and December videos all student reflections occurred during work in small groups as the teacher circulated and visited with groups.

Although the time in which opportunities for student reflection increased from August to December, this data alone does not illustrate the quality or nature of these questions. In an attempt to provide a deeper characterization of Claudia's questioning, we analyzed her classroom videos to determine the role of decentering in her questioning, as well as transitions in the nature of her questioning and student thinking elicited by her questioning. The following three excerpts give examples of student reflection in the context of Claudia's interactions with her students.

In Excerpt 1a (August) Claudia read student responses to a recent quiz on average rate of change and asked students to determine what was missing or incorrect in the responses. In the transcript, '…' is used to indicate one speaker interrupting another speaker.

Excerpt 1a

C: Listen to this one and I'm going to tell (you?) the comment I wrote. Average rate of change is the overall distance, divided by the amount of time that tells you the average amount of miles per minute from p1 to p2. Now what's wrong with this answer? What is it actually telling me?
S1: How to find it.
C: So it's telling me the what? How to find it, but what about how to find it?
S2: The math.
C: The math, the formula, okay? So when it says what's the meaning of the average rate of change? It doesn't mean tell me in words what the formula is, it means: tell me what it means.

In Excerpt 1b (August) Claudia discussed with the class the meaning of the average rate of change of a diver's height from the water with respect to time using a table of values. Claudia and her students changed the time interval to half a second on the calculator and discussed the meaning of a rate of change of zero between two successive values in the table.

Excerpt 1b

C: Okay, so that would mean that what would happen, the person jumps off the board?
S1: Floats there for a second.
C: Okay, so is the diver suspended in mid-air for a half a second?
(Some students in class say no.)
S2: He wasn't suspended but…
S3: …went back in.
(Some students in class are making gestures of a concave down parabola.)
C: Oh, so he went up and back down, okay, but we're not seeing that right here in this table. But the way you could find out is change your interval again and what could we change it to this time?
S4: One-quarter.

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C: Point two five, or you could do smaller, you could do point two, whatever.

In Excerpts 1a and 1b, we characterize Claudia's actions at the TDM1 level because she was focused on students arriving at a specific answer with no indication that she was interested in the thinking or perspective behind the students' responses (Excerpt 1a: lines 6, 8-10; Excerpt 1b: lines 9-11, 13). Although she initiated the opportunity for student reflection (Excerpt 1a: lines 1-4; Excerpt 1b: lines 1-2, 4), meaningful discussion did not follow (Excerpt 1a: lines 5-7, Excerpt 1b: lines 5-8) and Claudia proceeded without pushing the students for further explanation (Excerpt 1a: lines 8-10, Excerpt 1b: lines 10-11). The opportunity for student reflection in Excerpt 1a did not promote conceptual understanding (lines 6-8). Although Claudia made an attempt to promote conceptual understanding in Excerpt 1b (lines 1-2, 4), she did not try to create a model of student thinking. Claudia instead questioned her students in ways that suggested she was seeking an answer, which did not promote deep or meaningful reflection.

In Excerpt 2, which occurred during a September class, students were in groups of three to four students. Claudia gave her students a task stating: "A local hotel currently rents an average of 28 rooms per night. The hotel management estimates that for every $5,550 spent on hotel renovations they will be able to rent an additional 6 rooms each night. Sketch a graph that represents the relationship between the number of rooms that can be rented in terms of the amount of money spent on renovations." (Carlson & Oehrtman, 2010, Module 2) This question was part of a unit on linearly and proportionality. Claudia circulated the room monitoring the students' activity and asked questions of students in different groups as they engaged in the task. Claudia's conversation with two students is presented in Excerpt 2.

Excerpt 2

C: So if you can pay for one room at a time that would mean, as soon as you pay the next 925 dollars you would get an additional room. Right?
S1: Correct.
C: Okay, what kind of relationship is that?
S1: (*thinking*)
S2: Step function.
C: (*Laughing*) No.
S2: It is not proportional.
S1: It is not proportional and it is not linear because…
C: Okay, why is it not proportional?
S1: Because you start at 28, and you can't spend negative money on the next room.
C: Are the changes proportional though?
S1: No.
C: So for each additional room built are they always going to pay nine hundred?
S1: I don't know, we are trying to think about how to describe this.
S2: Well you presume, because they're not only going to build half a room.
S1: If for every apple you need one orange and you have half an apple then you need half an orange, but this is not the case here because you spend half the money here you are still going to get one, or you can't spend half the money here.
C: Okay, are you assuming that you have to spend the 5,550 dollars in your graph, is that what you are assuming?

In Excerpt 2, we characterize Claudia's interactions with her students as TDM3 level because she made moves to guide Student 1 to her own way of thinking (lines 1-2, 4, 10, 12, 14).

Specifically, she was prompting the students to describe that there was a proportional relationship between the two quantities. Although her questioning (lines 1-2, 4, 10, 12), initiated opportunities for her students to reflect on their solution it did not support their reflections. In providing these opportunities, however, the students' utterances indicated that their thinking was different than Claudia (lines 6, 8, 9, 11, 13), and they persisted because they were convinced of the reasonable of their response. As the interaction progressed, Claudia's question in line 14 gave the students another opportunity to more fully verbalize their thinking. They justified why the situation with the hotel renovation was not proportional based on their understanding of proportionality (lines 16-19). By the end of the excerpt, Claudia began to reflect on the rationality of the student's thinking (lines 20-21). Immediately after Excerpt 2, Claudia allowed her students to describe their way of thinking, and this led her to accept their solution as a viable alternative to be respected (TDM4). She then asked Student 1 to present her solution to the class as one of the acceptable approaches to solving the problem.

In Excerpt 3 (December), students were in groups of three to four students discussing a problem about the relationship among the arc length, radius and angle. The context of the problem is a bug traveling on a fan blade with the unit of measure in radians and students are to explore the relationship between arc length, radius, and angle. The concept emphasized in the previous lesson was that one radian of angle measure corresponded to the arc measure of one radius length. Claudia circulated the classroom monitoring students' discussions as they engaged in the task. In Excerpt 3 Claudia was conversing with students in one group regarding the question: How does the angle measure change if the radius of the fan is changed, but the distance the bug travels (0.765 radians) is not changed?

Excerpt 3

C:	Okay, in radians is not changed though. You are thinking of the arc length, you are thinking of the distance the arc length.
S2:	That's not a distance the bug traveled that's the angle of the distance.
C:	But can we take an arc length and change it to a radian measure?
S1:	Yeah.
C:	Okay, so?
S2:	But that isn't the distance that the bug traveled.
C:	But if the arc length of an angle is given, can we find the distance the bug traveled?
S2:	If it is given in radians.
C:	Yes, if you have a radius and an angle you can, so you are just saying that the way the question reads, how does the angle measure change if the radius of the fan is changed, but the distance the bug travels in radians is not changed? Is because of how it is worded, he is thinking the arc length. Okay so if the…
S2:	But the bug travels would be, they say the units that they give you to use and the quantity they ask you to measure do not match.
C:	True.
S2:	That would be like, what is the gas mileage in liters, and it's like (*inaudible*).
C:	(*laughs*) Okay yeah, okay so let me look at this for a minute – how does the angle measure change if the radius of the fan is changed, but the distance the bug travels, 0.765 radians, is not changed? Well can you say that a linear measurement is 0.765 radians? Can I say that?
S1:	No, but that is what it says here.

C:	What is 0.765 radians, what does that represent for any circle?
S1:	The openness of the angle.
C:	But what does the actual 0.765 radians measure, what does that mean?
S1:	How– it's proportional to the radius so it is that much of the radius around the circle.
C:	Okay, so we could say that this is 76.5% of one radian in length? Could we say that?
S1:	Yeah.
C:	S2 what do you think? So if the measurement is 0.765 radians then the length of the arc is 76.5% the length of the radius.

In Excerpt 3, we characterize Claudia at the TDM4 level. She acknowledged understanding how Student 2 interpreted the question and made moves to help both Student 1 and Student 2 reach a common understanding of the question, based on her model of the students' thinking about the question (lines 1-2, 11-14). Claudia's questioning (lines 4, 6, 8-9, 21-22, 24, 26, 29-30, 32-33) initiated opportunities for student reflection (lines 3, 5, 7, 10, 15-16, 18, 23, 25, 27-28, 31). Specifically, she prompted the students to reflect on a misconception that the radius length could be only used to measure the angle subtended by an arc, as revealed through the comments of Student 2 (lines 15-16, 18). Claudia made moves to resolve this misconception by linking mathematics concepts from the prior lesson in regards to the correspondence between radian measure of the circle and its radius. The questions Claudia asked (lines 21-22, 24, 26, 29-30, 32-33) required the students to continue to reflect on reasonableness of their thinking until both students reached a common understanding (which was reached after this excerpt).

Conclusions

Hiebert and Grouws (2007) identified two critical features of classroom teaching that foster conceptual understanding : (1) explicit attention must be made to the connections between facts, procedures, and ideas of mathematics; and (2) students must "struggle" with mathematics, not in the literal sense, but in the sense that they need to put an effort in making sense of the mathematics. Excerpts 2 and 3 display Claudia supporting her students' thinking by questioning them in ways that led to their reflecting on the mathematics (Excerpt 2: lines 6, 8, 9, 11, 16-19, Excerpt 3: lines 3, 5, 7, 10, 15-16, 18, 23, 25, 27-28, 31). In these excerpts Claudia modeled a higher level of decentering (TDM3, TDM4) when compared to Excerpts 1a and 1b (TDM1), in which no decentering was present. Claudia's higher level decentering actions enabled mathematically rich conversations emphasizing conceptual understanding while generating more opportunities for student reflection. Also, classroom discourse shifted from a calculational orientation towards a conceptual orientation as students were asked to explain their thinking. Stigler and Hiebert (1999) had noted a system resists sudden changes due to cultural scripts. We had hypothesized that the changes in classroom norms would be gradual. Just as Claudia's progression in decentering was gradual as the semester progressed, so too was the students' propensity to reflect on their thinking.

Future studies should continue to investigate teachers' decentering abilities and how these abilities support student learning. Such studies should also include a focus on the attributes (e.g., content knowledge, attitudes, and beliefs) of a teacher that support shifts in her or his decentering abilities. Although one of the observational scale descriptors was adapted from the Middle School Observational Tool (Reys, 2004), the other descriptors should offer further insight in the decentering actions of a teacher and the learning and actions of students.

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