DEVELOPING ELEMENTARY TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE OF RATE-OF-CHANGE IN ENGINEERING THERMODYNAMICS: A DESIGN STUDY

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The developing pedagogical content knowledge of elementary-certified teachers was supported in a course on concepts in algebra and calculus. Engineering thermodynamics was used as a domain through which participants could model rate-of-change for a variety of functions. The design of the professional development course is examined, and teachers' struggles and triumphs with course content described. Results show teachers' backgrounds in elementary algebra proved a barrier to their ability to treat relationships among variables as mathematical objects. Once these barriers were overcome rate-of-change served as a powerful support to model complex thermal applications that utilized rate-of-change as an index of efficiency.

This paper describes a design-research project that focuses on developing the mathematical knowledge for teaching of elementary-certified teachers who are interested in becoming STEM specialists in the middle grades. The case described is of the design of a graduate-level course focusing on the mathematics of change and how it relates to engineering thermodynamics. We first describe our approach to teacher knowledge, and why pedagogical content knowledge is the key lever for practical change in the classroom. Then we address design-theory and why it is an appropriate paradigm for studying teachers' mathematical learning at the same time as we attempt to improve it. Next, we describe the course design, dwelling on modeling as the pedagogical approach to the course, the mathematical content and its sequencing, and engineering thermodynamics as an ideal context for the development of essential concepts of calculus. Lastly, we describe the ways in which teachers interacted with this content, and particularly how they grappled with their "fuzzy" prior knowledge of algebra, attempting to repair holes in their understanding, even while continuing to move forward on more advanced content.

Pedagogical Content Knowledge

Teachers' knowledge has long been promoted as a lynchpin variable, connecting standards and curricular innovation with classroom practice and student learning. Ostensibly, teachers' understanding of content, pedagogy, students, and curriculum enables them to better design instructional environments, tailor activities to the needs of their students, and to assess and improve their practice for the purpose of increasing student learning outcomes.

Lee Shulman and colleagues are generally credited with the initial promotion of teacher knowledge in this lynchpin role (e.g., Grossman et al., 1989; Shulman, 1986; Wilson, Shulman, & Richert, 1987). They generated a number of descriptions of teacher knowledge as an attempt to show teachers' knowledge as multi-faceted, drawing from a number of experiences in their early lives as well as university coursework, teacher education, and practice (Shulman, 1986, p. 9). In general, research has focused on three categories of teacher knowledge: (a) subject matter knowledge, (b) pedagogical content knowledge, and (c) curricular knowledge. Content knowledge refers to both subject matter knowledge and its organization or lack thereof, while pedagogical content knowledge involves content knowledge as it is directly related to the

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teaching of specific subject matter. Finally, curricular knowledge attunes to knowledge of the programs of study and instructional materials and technology used by teachers and students.

There is some evidence from certification studies as well as large-scale surveys and quasi-experimental studies that indicates that teachers' knowledge has a significant impact on student gains in achievement over time (Darling-Hammond, 2000; Hill, Rowan, & Ball, 2005). This information is mixed, showing mathematical content knowledge as only moderately correlated with student achievement gains. Pedagogical Content Knowledge also shows a significant moderate correlation with student gains separable from mathematical content knowledge. This separability supports Shulman and colleagues' initial assumptions that several funds of knowledge interact to impact instruction and subsequent learning.

More recently Hill, Ball, & Schilling (2008) argue that the knowledge required for teaching mathematics, Mathematical Knowledge for Teaching (MKT), is multifaceted, incorporating aspects of: 1) common content knowledge held by all mathematically sophisticated occupations, 2) content knowledge specialized to the teaching of mathematics, 3) knowledge of student learning of mathematical content, 4) knowledge of the practices of teaching mathematics, and 5) knowledge of mathematics-related curriculum. Each of these knowledge components has some effect on teachers' ability to develop, select, and deliver tasks at an appropriate developmental level at the appropriate time in mathematical sequence and student learning. Moreover, each of these components supports and updates the other components as teachers' learning of mathematics and mathematics-relevant pedagogy grows. It is likely that, unless each of these aspects of MKT is addressed, and their interdependence emphasized, that professional development for mathematics improvement will result in only modest student gains over only a short term (Ball & Bass, 2000).

Of these facets of MKT, pedagogical content knowledge is of primary concern because it lies at the confluence of the content and the curriculum; it is the knowledge base of mathematics education: PCK comprises the knowledge and skills required for one to transform the other facets of MKT (like mathematical content, knowledge of curriculum, knowledge of students) into a set of experiences, activities, and environments that optimize the likelihood of students learning. In our program, we attempted to improve elementary teachers' content knowledge related to fundamental concepts in algebra and calculus utilizing Modeling Instruction as the pedagogical theory for both teacher learning and for student learning. It was by integrating Modeling Instruction with teachers' own content learning experiences that we hoped to impact their ability to understand and apply this content in an appropriate pedagogical manner with their own classrooms—hypothetically connecting new content with new knowledge of curriculum and pedagogy—addressing three of the five aspects of MKT through PCK.

Modeling as an Integrative Construct

One of the key goals for professional development is for teachers to develop new models of content, teaching, and curriculum. Models are conceptual structures—mental (and oftimes physical or inscriptional) representations of real things, real phenomena. *Modeling* as it refers to content knowledge, involves building, testing and applying conceptual models of natural phenomena and is a practice that is central to learning and doing science and mathematics (Hestenes, 1992). In fact, it can be accurately stated that Mathematics, Science, and Engineering are all fundamentally modeling enterprises. Educationally, modeling has been touted as a unifying theme across science and mathematics education as recommended by both the National Science Education Standards (1996) and the Common Core Standards for Mathematics (2010).

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In sum, we think with models, we think through models, and we think about models. Our approach to instruction attempted to embody this fact and exploit this facet of human cognition.

Modeling Instruction as an instructional theory aims to correct many weaknesses of the traditional lecture/demonstration method in science and mathematics education, including the fragmentation of knowledge, student passivity, and the persistence of naive beliefs about the physical world. Modeling Instruction objectives begin with the goal to engage students in understanding the physical world by ***constructing and using scientific models*** to describe, to explain, to predict and to control physical phenomena. To do this, students must gain proficiency with ***basic conceptual tools*** for modeling real objects and processes, especially mathematical, graphical and diagrammatic representations. This is the underlying curricular model we employ at all levels in the reported program: Build and use models of important scientific content, utilizing important mathematical and representational tools.

It has been shown in a variety of studies of student learning that context is critical for coming to understand mathematical concepts and skills. Moreover, the capacity to create models of scientific phenomena, and to test those models is dependent on the development of mathematical ways of thinking about the phenomena, including the ability to make sense of patterns in data. For this reason, modeling truly is an integrative construct, connecting mathematics and scientific content through meaningful activity.

Unfortunately, teachers are not exposed to mathematical modeling of scientific phenomena during their undergraduate careers to any great extent. This is especially true for middle school teachers. With typically one course in mathematics at the College level, and only one non-calculus-based science course, middle school teachers trained as generalist elementary education majors are just not equipped to handle the traditional mathematics curriculum let alone Algebra I and Geometry which are currently common offerings in US schools. The same can be said for middle school science, where teachers may teach life science, physical science and earth and space science in any given year. It is therefore imperative that the curriculum and instruction that takes place for teachers closely resembles that which they are expected to provide for their students—yet not be so prescriptive as to squelch personal style, initiative and inspiration. We structured our classroom tasks and pedagogy based on the past 20 years of research on modeling (citations from Lesh, Hestenes here).

Design Research and Curriculum Development

Lamberg & Middleton (2009) describe how design theory can provide a rigorous paradigm for disciplined inquiry in learning and instruction, and a pragmatic framework for defensible change in curriculum and instruction. Briefly, this theory outlines seven "phases" through which a research program progresses, beginning with more grounded methods, where the researcher attempts to understand a little-researched area of inquiry, consolidating evidence about learning and practice, until a local theory of change can be generated that seems to explain and predict positive movement along a trajectory of change. This *theory*, then is used to design curricular tasks, which serve as operational definitions of that theory. Through iterative testing and revision, tasks are made more effective, while at the same time, the theory is refined, refuted, and made more useful and explanatory (Author, 2008). Design research, thus has two primary outcomes: A theory of change that describes how learning progresses across a defined set of topics, and a mechanism for at least partly driving that change—curriculum materials or other supports that have been refined over time to be optimally (or at least *reasonably*) effective.

Design theory is a kind of modeling theory that focuses on the development of systematic

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knowledge about teaching and learning. As such, it is consistent with the theory of learning and instruction we employed in the reported research: Modeling Instruction. The study reported here is situated in Phases 2 and 3: Development of the Artifact (in this case, the course curriculum and activities), and Feasibility Study (initial trialing and iterative improvement of the course).

Description of Modeling Course

Participants

The focus teachers in this study were 11 elementary certified teachers. Two additional students (masters' degree graduate students in mathematics education with no prior teacher training) participated as well. Of the teachers, two had a calculus course early in their collegiate experiences, but the rest had no mathematics beyond basic College Mathematics and Mathematics Methods for Elementary Teachers. All were female, and all were teaching elementary grades subjects. On a pre-Calculus assessment, none scored higher than 50%, and the majority scored around the 20% or chance level.

Course Foci

The modeling course for teachers focused on developing key mathematical content related to modeling scientific and engineering concepts. We emphasized discussing the most common linear, polynomial, exponential, and inverse square functions and their representations, focusing especially on the analysis of change in the context of applied problems. We focused on these concepts in the context of modeling situations of thermodynamics, which is the study of energy transformation. We chose thermodynamics because we hypothesized that most of the teachers had everyday experience with heat and temperature, energy, pressure and work, whether formally in a classroom or informally solving problems in their daily lives. We emphasized convection and conductance as key ideas in thermodynamics because they were the most accessible concepts in thermodynamics while also providing convenient ways to talk deeply about rate of change: Newton's law of cooling for convection and thermal exchange in conductance both are examples of exponential functions, while radiation follows the classic inverse-square law. Our goal was to formalize the in-service teachers' knowledge while exploring applications of thermodynamics in engineering contexts, environmental science, and biology such as passive solar heating and cooling, properties of insulators, and heat capacity of materials.

Thermodynamics is an area of science where several key variables, such as entropy, cannot be measured directly; one can only measure them by examining related variables and doing the algebra. So in this case the mathematics is absolutely critical for understanding the science—thermodynamics cannot be understood at even a basic level without it. In general, we wanted to emphasize the deep connections between mathematics and science, and help the teachers understand how one can better understand science by knowing the mathematics.

Structure of Course

We attempted to structure the course so there was a common substrate of activities in each homework assignment and weekly meeting of the course. Each week, teachers were engaged in the following tasks:

Book club write-ups. Students read 1 to 3 chapters from *Calculus Made Easy* (Thompson, 1914) each week. To get them to reflect deeply on the content of that book, students were required to write a review of the chapter(s). In their review, we asked them to focus on what they

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know, what they wanted to know, and what they had learned: 1) What you know: A description of the reading, as if you were describing the concepts to an 8th grade algebra student; 2) What you want to know: Analysis and insights you have about the study of change, and especially those things you feel confused about; and 3) What you have learned: Students listed those ideas and ways of thinking they had that they didn't have prior to reading the text.

Weekly Modeling Problem. In each class session, we presented the students with a new thermodynamics problem to model. Following class, they were required to do a formal lab write-up of their model. Their analyses addressed the following points: 1) A description of the relationship they attempted to model; 2) A description of the methods they used to model the relationship; 3) The problem solution presented in a general form (e.g., a rule, procedure, or symbolization, which was often a combination of formulae, graphs, and data tables); and 4) An analysis of why the problem solution was correct.

Problems emphasized convection rates, conductance/insulative properties of materials and radiation rates, which mathematics paralleled the content being developed in the Book Review assignments, but required application in messy, data-rich situations.

Weekly Mathematics Problems. Finally, based on the modeling problem and ideas introduced in the book club, we presented students with a set of algebra and calculus problems to attempt. Their response to each problem included all their work, including mistakes, and their answer, if they arrived at one; and a reflective statement describing the core mathematical idea the problem was addressing.

For the reported course, we organized the mathematics content into 7 modules:

- Proportional Reasoning: Direct, Inverse, and Joint Variation
- Average Rate of Change: $\Delta y/\Delta x$, Analyzing Functions Qualitatively, Rational Expresssions
- Families of Functions: Linear, Polynomial, Exponential/Logarithmic/Power
- Comparing Functions within Families: Coefficients, Exponents, Bases, Local Maxima/Minima, Inflection Points
- Derivatives as local average rate of change
- Derivatives at a point; Derivatives as Functions; Limits
- Accumulations; Area under a curve

Structure of Daily Sessions

Each 4.5-hour class session was broken into three 1.5 hour activities. One activity focused on discussion of the week's readings and calculus concepts. The second activity focused on discussing any difficulties participants had with understanding and completing homework activities. The third activity engaged teachers in modeling thermodynamics problems and applying the readings and problems.

Each modeling session was organized into ***modeling cycles*** which move students through all phases of model development, evaluation and application in concrete situations — attempting to promote an integrated understanding of modeling processes and acquisition of coordinated modeling skills (Lesh & Yoon, 2007). Typically, the instructor set the stage for classroom activities, ordinarily with a demonstration and class discussion to establish common understanding of a question to be asked of nature. Then, in small groups, students ***collaborated*** in planning and conducting experiments, collecting and analyzing data to answer or clarify the question. Teachers were required to present and justify their conclusions in oral and/or written

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form, including a ***formulation*** of their own models for the phenomena in question and an ***evaluation*** of the set of models by comparison with data. Technical terms and concepts were introduced by the instructor only as they were needed to sharpen models, facilitate modeling activities and improve the quality of discourse.

Additionally, in 3 of the 8 sessions, we carved out an hour to examine the Common Core Standards for Science and Mathematics for middle grades, and map the key content they were learning in the course to the key mathematical ideas emphasized by the Standards. Teachers developed maps of the content to show how it fit together in an integrated fashion in science and mathematics classes.

Analysis of Sessions and Course Revision

The research team consisted of three mathematics educators with engineering backgrounds, and an expert in teacher professional development. The research/teaching team met each week following the daily sessions. In the manner of Cobb et al (1997), we reviewed student work, planned subsequent sessions, and revised tasks and assignments based on the discourse engendered in the daily sessions. In particular, we reviewed the difficulties students had in their conceptual understanding of the algebra, and their facility with its manipulative skills, and developed new tasks to help them bridge this knowledge, while still forging ahead thinking about derivatives and rate-of-change as a function.

Sources of data included their homework, book club reflections, modeling problems, and their posts to the class discussion group, which we developed to be able to handle ad-hoc questions just-in-time.

Results and Discussion

We set out to focus on a mathematically rich course focused on thermodynamics, and eventually we were able to talk about difficult mathematical concepts to explain and model scientific phenomena. However, the level of resistance and frustration the teachers exhibited in the first three weeks of the course was palpable. Their homework reflections and midterm course evaluations suggested that they were not thinking about modeling phenomena. Instead, they were highly frustrated or confused about concepts like algebraic field properties and representation of proportional quantities using fractions. We used their class discussions, homework assignments, and book reviews to gain insight into what sense they were making of modeling thermodynamics ideas using mathematics.

We found that the majority of students' ways of thinking about the mathematics was not at a level where discussions about rate of change could be interesting or productive to the teachers. Given our data and feedback from the teachers, we concluded there were two major obstacles to their understanding. First, a chief barriers we faced was the confluence of factors associated with teachers' knowledge and the nature of disciplinary knowledge and how these factors could be made compatible and even complementary. Second, there was a real disconnect between the community of mathematics and that of the sciences regarding what was being modeled and what its conceptual substrate is. With motion and mechanics the connections between the particular functions being modeled is pretty close, but when there is no macroscopic motion to serve as a grounding metaphor (i.e., in thermodynamics), students must rely on more abstract understanding of variable quantity, function, rate, direct and indirect proportions, and the combinations of these ideas as they are manifest in the typical power functions and exponential functions we see in science applications not derived from motion examples.

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Teachers' Content Knowledge

Most of our students had not taken a course in mathematics, or had significant content professional development since their 1st or second year of college. Most teachers, because they are elementary certified, took required courses up through college mathematics, and therefore have little experience with algebraic methods, functions and functional reasoning, the mathematics of change, or geometry and its relationship to algebraic relations. As a result, teachers' knowledge of and understanding of variable, function, field properties and their tie to the manipulative skills of algebra, basic concepts of exponents, factors and multiples, and proportional reasoning was generally poor and uneven across the cohort.

Nearly all teachers possessed poor mathematical self-efficacy. Reasons for this include poor prior performance resulting from poor prior learning experiences, knowledge that their coursework was not sufficient for a developed conceptual understanding and procedural skill, and the fact that much time has passed in which they have not continued mathematical learning nor practiced mathematical skills previously learned. As a result, teachers' confidence in their conjectures and insight appeared to be tentative; their ability to judge their learning and understanding (meta-cognition) was under-developed.

Given the breath and depth of these problems, we took steps taken to deal with these issues. We immediately stepped back from a pre-Calculus level of instruction to a high school algebra II level, to an 8th-grade pre Algebra/Algebra I level with some high school ideas integrated. We focused on scaling back on the quantity of homework to allow for students to focus on ideas discussed in class, and developed homework activities based on class discussion the day following, so that teachers can do relevant work the following week. We used 3 of 8 to focus on the most basic ideas of quantities, algebraic manipulative skills and their conceptual basis in the field properties, recognizing the form of familiar functions (linear, polynomial, exponential). Once we helped the teachers construct a foundation of basic mathematical concepts, we used rate of change as an integrative concept to help students look at different regions within the same curve, and the same regions across different curves, to examine the behavior of these functions, and their similarities and differences.

Teachers' Disciplinary Knowledge

In science, the mathematical models developed can be derived from specific circumstances to assist the student to be able to describe their behavior. In mathematics, mathematical models must have a generality across applicable situations—this implies that, if an idea is developed within a context, there must be other application contexts where the student is expected to transfer their knowledge, and some opportunity to make the mathematical abstractions explicit and their transferable structure overt.

Much of school science is done without any mathematics or any significant mathematics. This make students not lazy per se, but the teachers were disinclined to mathematize or to frame scientific questions or conjectures in mathematical terms other than just basic correspondence relations. Functional reasoning was not a habit of mind. Overreliance on solving for a number detracted the teachers from the examination and description of relationships. We believed that the general practice of making science "conceptual" typically eliminates the critical mathematical structure of the concepts in favor of more qualitative or general depictions of phenomena

The use of software for scientific data analysis was tremendously helpful to both the teachers

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and us. However, the learning curve for students to learn the software takes a lot of time and focus away from the mathematical content itself. Graphing Calculators are excellent tools, but a distinct effort had to be made to develop some proficiency with these tools. Excel was also excellent, but the teachers had little to no experience with spreadsheets or other data systems, so this also took much of their time and energy. The important aspects of these programs are to allow students to see the form of relationships, to create an explicit algebraic relationship by manipulating cell formulae (excel) and lists (graphing calculators). As the semester progressed, the teachers moved from viewing technology as a barrier to understanding to using it as a tool for thinking about modeling complex thermodynamic situations.

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