WRITING PROBLEMS TO BUILD ON CHILDREN'S THINKING: TASKS THAT SUPPORT PRE-SERVICE TEACHER TRANSITIONS

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A critical practice in teaching elementary mathematics is posing problems that build on children's mathematical thinking. As such, teacher educators must provide pre-service teachers (PSTs) with a set of learning experiences to support PSTs in this practice. In this study, we present our analyses of PSTs' responses to a sequence of three methods course activities that engaged them in increasingly complex tasks requiring the PSTs to write problems in response to authentic student work.

Keywords: Children's Thinking; Teacher Education–Preservice; Mathematical Knowledge for Teaching

Introduction

Research suggests that a critical practice in teaching elementary mathematics is posing problems that build on children's mathematical thinking (Carpenter et al., 1999). An implication of this research is that teacher educators must provide pre-service teachers (PSTs) with a set of learning experiences to support PSTs in engaging in this critical practice. However, as a field, we know little about the design, enactment, or sequencing of these kinds of experiences. In this study, we present our analyses of PSTs' responses to a sequence of activities that engaged them in increasingly complex tasks requiring the PSTs to write problems in response to student work.

Theoretical Frame

Shulman (1986) suggested three types of knowledge are important for teaching - subject matter knowledge, pedagogical content knowledge, and curricular knowledge. Since that article, Ball and colleagues (Ball, Hill, et al., 2005; Hill, Sleep, et al., 2007) have built on Shulman's work and have provided the Mathematical Knowledge for Teaching (MKT) framework further defining subject matter knowledge (SMK) and pedagogical content knowledge (PCK) and identifying subsets of these knowledge bases. We are grounding this study in two subsets of PCK—knowledge of content and students, "knowledge that teachers possess about how students learn content" (Hill, Sleep, et al., 2007, p. 133); and knowledge of content and teaching, "mathematical knowledge of the design of instruction, includes how to choose examples and representations, and how to guide student discussions toward accurate mathematical ideas" (Hill, Sleep, et al., 2007, p. 133). These subsets of the PCK construct are useful as we are asking PSTs to think about how students solved particular problems and then use that knowledge of students to design subsequent instruction.

Also relevant to this study is the *professional noticing of children's mathematical thinking* construct (Jacobs, Lamb, & Philip, 2010). Three interrelated skills: *attending* to children's strategies, *interpreting* children's understandings, and deciding how to *respond* on the basis of children's understandings comprise the construct. Within our methods course, we ask PSTs in several instances to talk about what they notice in student work (via video clips and written) and discuss what they think students do or do not understand. Finally, we ask PSTs to use what they know about students to generate a next problem.

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Methods

Data were collected from thirty-three, first semester, senior level PSTs (32 female, 1 male) enrolled in an elementary mathematics methods course taught by the first author in fall 2011. The data include PST responses to three different activities, each of which are from a set of methods course materials written by the second and third authors (Drake, Land, et al., 2011). The activities were designed to scaffold and support PSTs as they developed the capacity to make sense of student strategies and to write appropriate subsequent tasks for students. Each of the activities is set in the context of actual classrooms. The three activities were posed over the first six weeks of the course and were sequenced in order to provide PSTs with various experiences analyzing and writing effective tasks based on student thinking. The first activity (Natalie's Class the Next Day) was designed to give PSTs the opportunity to notice and analyze how an experienced teacher used her students' current knowledge of division with fractional remainders to design a subsequent story problem and number choices. The second activity (Counting Sequences) required the PSTs to write an opening number routine (ONR) and problem, including number choices to address a class-wide addition misconception. The third activity (Fishbowl Problem) asked PSTs to analyze 14 students' multiplication strategies and write a subsequent problem with number choices to address the wide range of learners. We organized the activities to form a trajectory along several dimensions – moving from noticing an expert teacher's task design to having PSTs design tasks themselves, moving from designing a task to address a single misconception to writing a task that addressed a wide range of student understandings, and moving from PSTs noticing an expert teacher's number choice to selecting numbers for a pre-written task to writing an entirely new task.

Data Analysis and Results

Natalie's Class the Next Day

Prior to completing the Natalie's Class the Next Day task, PSTs watched a video with transcript of Natalie and her 2nd grade class as they solved two partitive division story problems:

> Problem #1 Trisha and Allie are sharing ______ chocolate chip cookies. If they are shared equally, how many will each of them get?
>
> 2 4 5 8 9 12 13
> 30 31 50 51 66 67 83
>
> Problem #2 Trisha, Allie, Lance, and Kathy are sharing brownies. If they are sharing _____ brownies equally, how many will each person get?
>
> 4 5 8 9 16 17 20
> 32 33 44 45 48 49 50

Multiple number choices are given to provide for differentiation. Students were to choose the row of number choices "just right" for them. In Problem #1, even numbers were posed followed by the next consecutive number (with the exception of 2 and 83). In Problem #2, multiples of four were posed followed by the next consecutive number. The use of next consecutive numbers was intended to provide a scaffold in that students could use what they knew about one number choice to help with the next. Included in the video are examples of student work, teacher/student interactions, and a sharing session where students explain their various strategies. After discussing the video, PSTs are asked to complete the following activity:

> The next day, Natalie posed the following problem. Solve the problem for a few of the number choices. Then, answer the questions below.
>
> There are ____ miniature candy bars. Dustin, Jose, Sam, and Joe are going to share the candy bars. If they split up the candy bars equally, how many will each of them get?
>
> 11 17 22 35 48
> 65 83 75 99 104

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1. Why do you think Natalie posed this particular problem next?
2. What do you notice about the number choices in this problem compared to the number choices given the day before?

Analysis: Natalie's class the next day. As we examined responses from the PSTs, we focused on their responses to question two. The next problem Natalie posed is also a partitive division problem and extends the second problem from the day prior in sharing a set of objects among four people. We analyzed PSTs' responses according to their noticing of three aspects of Natalie's number choices: (1) the numbers in both rows are both *larger* numbers than the day before; (2) the numbers are more *complex* in that students had to think not only about sharing remainders of zero and one, but also two and three as well; (3) the next consecutive number *scaffold* that had been used the day before has now been removed. Two authors independently coded the PSTs' responses for evidence of these three facets with 94.9% agreement (94/99).

Here is a sample response from one of the PSTs, Jaceylyn (all names are pseudonyms):

> The first thing that stood out to me about these number choices was that they were generally larger than the ones offered on the previous day. Next, when I actually started working with them, I found that these number choices granted me with quite different answers than the day before. On the previous day the answers had either been whole numbers, or sometimes involved a half as well, but today the answers came out with remainders of ¾ or ¼.

This response was coded as identifying larger numbers as well as more complex numbers.

Results: Natalie's class the next day. We examined the 33 responses to the Natalie's Class the Next Day activity in two ways: (1) how many of the facets of the number choices were identified by each PST, and (2) number of PSTs that identified each facet. The results are presented in Tables 1 and 2. We interpreted this data through the lens of MKT, specifically as indication of knowledge of content and teaching; knowledge of how to choose examples and design instruction. From the data one can see that ~48% of the PST identified either zero or one facet of the number choices, ~42% identified two of the three facets and only a small percentage (~9%) were able to identify all three. We posited that it might be more likely for PSTs to notice the larger numbers and the lack of scaffolds than recognize the complexity of the numbers, as the first two required less developed knowledge of content and teaching.

Table 1: Number of Facets Identified

# Correctly identified	# PSTs
0	7
1	9
2	14
3	3

Table 2: Percentage of Each Facet Identified

Facet	# PSTs	% PSTs
Larger numbers	13	39.4
More complex numbers	18	54.5
No scaffolds	15	45.5

Counting Sequences

The Counting Sequences activity begins with the PSTs watching a video with transcript of Jenny's first grade class. For her ONR, Jenny poses the following counting sequences to her students that focus on base-ten concepts: 30, 40, 50, ______, ______, ______. 44, 54, 64, ______, ______, ______. 57, 67, 77, ______, ______, ______. 157, 167, 177, ______, ______, ______. Jenny's students are able to solve the tasks by counting by 10. Students also notice the units place remains the same and the tens place number increases by one each time. They are able to solve the sequence that "crosses over" from a 2-digit number to a 3-digit number. The video ends with Jenny posing a story problem about a paleontologist:

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> A paleontologist had _____ dinosaur bones. He found some more. Now, the paleontologist has _____ dinosaur bones. How many bones did he find?
> (10, 70), (20, 84), (26, 126), (15, 65), (60, 150), (42, 53)

The activity provides PSTs with a description of student work from the Paleontologist Problem:

> Most of the children solved the paleontologist problem by using a hundreds chart, but many counted by ones when counting up to the second number instead of counting by tens. Some children did count by tens. For 20 and 84, the children who were counting by tens either counted by ones from 20 to 84, or counted by tens to 80, then counted 4 more. Nobody solved for 42 and 53 (Drake, Land et al., 2011).

Following this description, the story problem Jenny used the next day (without number choices) is given, "Today, the paleontologist is looking for fossils. He already had ____ fossils in his collection. He found some more. Now, the paleontologist has ____ fossils. How many fossils did he find?" The counting sequences activity was then posed for the PSTs to complete:

> Now that you have seen the Counting Sequences video (and its transcript), consider these questions related to students' solutions to the Paleontologist Problem.
>
> 1. What is the disconnect between how students counted in the opening routine and the counting strategies they used when solving the problem?
> 2. Why do you think the disconnect exists?
> 3. Considering this disconnect, generate two artifacts for the next day's lesson: an opening number routine and number choices for the Paleontologist problem given below. Briefly justify your choices.

In this activity, we were interested to see if: (1) the PSTs could recognize many children did not see their counting by tens strategy in the sequence activities as applicable in solving the join-change unknown story problem; (2) they could posit reason(s) for the disconnect; (3) they could design an ONR to address the reason(s) stated in 2; and (4) they could select appropriate number choices for the next day's problem. We believe this task was a natural progression from the previous task, as it required PSTs to interpret and respond to a general mathematical misconception within a class of children.

Analysis: Counting sequences. Prior to analyzing this data set, the authors collaboratively examined several responses to this activity from a previous course and through open and emergent coding (Strauss & Corbin, 1998) established a series of codes and operational definitions for: (1) identifying the disconnect explicitly and accurately (yes/no/no response); (2) number of reasons given for the disconnect (0, 1, 2 or more); (3) identifying the degree to which the ONR addressed the reason(s) given (low, medium, high); and (4) classification of the types of number choices we believed were appropriate for Jenny's students (count by 10s from a decade number as given in Jenny's original Paleontologist Problem, count by 10s from a non-decade number, count by 10s and 1s). We operationalize the degree to which the ONR addressed the reasons for the disconnect by examining the approaches the PSTs took in selecting the type of task, structure and/or number choices for their ONR. PSTs who used the same approach as Jenny, or used approaches that did not connect to their reason, were ranked low. PSTs who attempted at least one new type, structure or number choice related to their reason, and did so in a manner we *believed* could be effective, were rated medium. PSTs who made significant changes (more than one new approach) related to their reason, and did so in a way we were *confident* could be effective, were rated high. Reliability percentages for each of the four categories are as follows: Disconnect: 90.9%; Reasons: 75.8%; Degree: 78.8%; Number Choices: 87.1%. We discussed disagreements and reached consensus on the final codes.

Chelsea's response follows. The numbers correlate with the questions given above:

> 1) When students were counting in the counting sequences opening routine, they were counting by tens and realized that the second digit of the number was remaining the same. However, once they tried solving the problem, the students began counting by ones, and it threw them

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off to try counting larger numbers by ones.

2) When counting by tens, the second digit of the number remains the same. It creates a pattern and makes it easy to continue in an almost rhythmic-like pattern of repeating "10, 20, 30, 40, 50..." and so on. However, when counting by ones, the second number changes along with the first number and this can be very confusing for kids if they are counting "10, 11, 12, 13, 14, 15, 16, …"

3) Opening Number Routine – Fill in the blanks with the missing numbers.

   10, _____, _____, _____, 50, 60, _____, _____, 90, _____
   5, _____, 25, 35, _____, _____, 65, 75, _____, _____, 105
   100, _____, _____, 130, _____, 150, _____, _____, 180, _____, _____

   Problem for the next day
   Today, the paleontologist is looking for fossils. He already had ____ fossils in his collection. He found some more. Now, the paleontologist has ____ fossils. How many fossils did he find?
   [10, 30] [5, 25] [100, 175] [3, 43]

I chose these numbers because I started out with simpler numbers that they could easily apply their counting sequence strategy to a word problem (10, 20, 30). I then moved on to [5, 25] because starting at 5 and counting by tens is slightly more difficult. Next I did [100, 175] because starting at 100 is difficult, and they also have to count by 5's once they get to 70. Finally, I placed the hardest number choice last because the students have to count by tens, but they are starting at 3, which will throw them off to see a 3 as the last digit, and they will really need to understand the process of counting by tens to get from 3 to 43.

The above example was coded as (1) *yes* to identifying the disconnect; (2) *0* for not identifying a reason for the disconnect; (3) as *low* for the degree in which he/she addressed the disconnect as it is the same approach used by Jenny; and (4) as having *counting by tens from a decade number, counting by tens from a non-decade, and counting by tens and ones* in the number choices.

Results: Counting sequences. Of the 24 PSTs who attempted to identify the disconnect within Jenny's class (9 no response), 18 were able to accurately do so (75%). 18 of those 24 (75%) were able to posit at least one reason why the disconnect may have occurred. When it came time, however, to design an opening number routine that would address the disconnect, more than 50% of PSTs simply posed "more of the same" approaches Jenny used. Ten PSTs (30.3%) made an attempt to try something different, but only five PSTs (15.2%) were able to do so in a way we felt confident would afford the children multiple opportunities to make the connection between skip counting by 10s in patterns and using skip counting by 10s as a strategy for solving join-change unknown addition problems. The number choices data were more encouraging. There was a high percentage of PSTs (42.4%, 14/33) who included at least two of the three appropriate number choices or all three of the appropriate number choices (48.5%, 16/33) in the next day's problem. Three PSTs included only one of the appropriate number choice types. One emerging pattern from these data is our PSTs seem to be able to understand and identify student thinking, but often struggle using this information to effectively address it.

Table 3: Counting Sequences Results

Disconnect			Reasons			Degree			Number Choices		
Y	N	NR	0	1	2+	L	M	H	10sD	10sND	10s1s
18	6	9	15	9	9	18	10	5	25	29	24

Fishbowl Problem

The Fishbowl Problem is set in the context of Molly's 2nd/3rd mixed age classroom. This task was built around PSTs' examination of examples of student work from 14 children in Molly's class in response to the following multiplication problem:

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> Sam had ____ fish bowls. He had _____ fish in each bowl. How many fish did Sam have? Molly presented 4 pairs of number choices for her students to pick from: A: (2, 10), (5, 10); B: (4, 20), (8, 20); C: (3, 11), (6, 11); D: (4, 12), 8, 12).

The task for the PSTs was as follows:

1. First, consider Molly's learning goals—what are they?
2. Next, look at the student work on the following pages. What do you find interesting? What evidence can you identify that students are or are not making progress toward the learning goal(s)?
3. Write a problem for the next day along with a rationale. What do you think will be an appropriate problem that will meet the range of needs in Molly's classroom? Reference at least three students or group of students specifically in your rationale.

We believed this activity was an appropriate next task for the PSTs' development as it required them to analyze and make sense of several children's thinking, to write a story problem appropriate for the entire class and simultaneously attend to specific strategies and learning goals when writing number choices. This activity is very similar to the work of teaching and required PSTs to use many different knowledge bases to effectively complete the activity. Molly had different goals for different groups of children in her room. For some children she wanted to see if they were able to skip count by multiples of ten. For others, she wanted to see if they could notice and use the doubling relationship between the pairs of numbers she had chosen for them to solve. She included number choices like 11 and 12, to see if any children would solve using the distributive property and their knowledge of tens.

Analysis: Fishbowl problem. In our analysis of the children's work, we classified their approaches into one of four categories: (1) direct modeling: children in this group either could not solve any of the multiplication tasks, or did so by directly modeling the solution with drawings; (2) skip counting: children in this group skip-counted by 10s and/or multiples of 10; (3) repeated addition/break apart by place: children in this category solved tasks by writing the multiplication problems as repeated addition and then broke the 2-digit numbers like 11 and 12 apart by place value and added the 10s and 1s separately; and (4) doubling: the children in this group also used repeated addition to solve the first number choice in the pair, but were also able to recognize the relationship between doubling the number of groups and doubling the product.

Similar to our analysis of the Counting Sequences Activity, the authors first collaboratively examined several responses to this activity from a previous course and established a series of codes and operational definitions for writing an appropriate story problem (yes/no) and demonstrating understanding of children's strategies (yes/no). In our analysis of the PSTs' number choices, we coded their responses in terms of addressing current student understanding and in terms of addressing Molly's learning goals. As we coded the responses in terms of addressing students' current understanding, we first looked for evidence in the rationale that the PSTs were *attempting* to choose numbers for specific individual's (or groups of children's) strategy. If we found evidence, we then examined the number choices they selected in order to determine if they had *successfully* done so. We coded their number choices in terms of learning goals in a similar manner. If PSTs explicitly mentioned a learning goal in their rationale, we coded it as an *attempt*. If an attempt was made, we then determined if the number choices were appropriate. If so, we coded it as a *success*. Reliability percentages were calculated for each category and ranged from 73.5% – 93.9%. Consensus was reached on all disagreements.

Samantha's response follows as an example. For space purposes only her problem is shared:

> Olivia has ____ drawers. She has ____ pencils in each drawer. How many pencils does Olivia have?
>
> I want to address the same goals, but have structured them so some are easier than her first examples, some the same difficulty, and some harder.

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Group A (3, 10) (4,10) (6, 10) (11, 10) *Here I want to practice going over 100 to provide some extra challenge. I also wanted those struggling to recognize the relationship between the 3 and 10 and the 4 and 10.*

Group B (1, 20) (2, 20) (4, 20) (8, 20) *Here I want the students to start on the 20s to focus on the relationship between the first and second number in the problems, but also the first numbers over the sequence.*

Group C (4, 11) (5, 11) (7, 11) (5, 12) *Here I want students to apply their knowledge of counting by 10s and then adding 1s to solving the problem. Hopefully having the second number switch to 12 will have these extend that knowledge.*

Group D (2, 11) (2, 12) (2, 13) (2, 14) *I wanted the students who've really gotten a hang of this 10s and 1s concept to apply it and to see patterns by keeping the 2 consistent.*

In this case, the above problem was coded not attempting, and thus, not successful, in addressing specific student's strategies. However, it was coded as attempting and successful in choosing numbers for specific learning goals.

Results: Fishbowl problem. The data supports the preliminary result from the Counting Sequences activity. We can see by this stage in our sequence a vast majority of the PSTs (31/33, 93.9%) made sense of the student work provided and were able to write an appropriate story problem type (28/33, 84.8%). When it comes to writing number choices for the next story problem however, it becomes evident that: (1) PSTs have difficulty in addressing multiple groups of student thinking simultaneously; (2) when PSTs do attempt to write specific number choices to address or further student thinking, they are not often successful in doing so (9/17, 52.9%; 8/26, ~31%; 7/15, ~47%; 5/12, ~42%); and (3) PSTs have difficulty writing number choices that attend to both student thinking and learning goals.

Table 4: Fishbowl Problem Results

		Number Choices for Students								Number Choices Learning Goals					
		Low/Direct Model		Skip Count		Repeat+ /BABP		Doubling		Skip by 10s		Doubling		Distributive Property	
Story?	Understand?	A?	S?	A?	S?	A?	S?	A?	S?	A?	S?	A?	S?	A?	S?
28	31	17	9	26	8	15	7	12	5	22	11	17	12	14	10

Discussion

As we interpret the results from this sequence of activities through the work of Jacobs and her colleagues (2010), we conclude PSTs have become more adept at *attending* to and *interpreting* student thinking. The activities however, have not helped the PSTs to make similar progress in *responding* to student thinking. One possible reason for this result is that our sequence of tasks does not provide enough educative supports to develop PSTs' ability to respond appropriately to student thinking. We have not explicitly attended to the question, "What makes a number choice appropriate or inappropriate to support/extend a student's current way of thinking?" An activity that presents an example of student thinking and requires PSTs to select and justify an appropriate number choice from a list of possibilities might help to develop PSTs' ability to interpret, evaluate and write appropriate number choices. These conclusions can be explained in terms of the construct of MKT. Our sequence of activities appears to support the development of PSTs' knowledge of content and students. Through repeated exposure to authentic student work (both video and written), PSTs have improved in their ability to make sense of and evaluate students' thinking strategies in a variety of mathematical contexts. This knowledge base is paramount in *attending* to and *interpreting* student thinking. PSTs' knowledge of content and teaching however has not shown similar improvement. Though the PSTs have demonstrated an ability to interpret student thinking and "diagnose" mathematical inconsistencies, they have not yet developed the appropriate content knowledge base to respond effectively in "prescribing" the next treatment.

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