THE USE OF AUTHENTIC HIGH-SCHOOL STUDENT SOLUTIONS AS A CHANGE AGENT WITH PRE-SERVICE TEACHERS

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This paper presents the results of a study designed to facilitate teacher candidates' shift from students of mathematics to teachers of mathematics. We develop a series of tasks each of which encourages candidates to reconsider rich tasks from increasingly teacher-centric points of view. The results show that using authentic high school student solutions, in particular Livescribe™ videos, provides a powerful experience in seeing solution paths they had not considered and in helping teacher candidates think about mathematics from someone else's point of view.

Keywords: Teacher Education-Preservice, Mathematical Knowledge for Teaching, Teacher Beliefs

Introduction

Successful teacher preparation programs provide learning experiences that scaffold a trajectory for teacher candidates in making the shift from "student" to "teacher" i.e. ushering them along a continuum of professional development. This transition is a crucial goal: to be a successful teacher a candidate must learn to see the tasks of school mathematics from other people's perspectives. However, this is not an easy transition, particularly for future school mathematics teachers, complicated, as it often is, by the fact that candidates' interest in mathematics and self-efficacy regarding mathematics arises from their personal success in solving complex problems rather than in their ability to understand others' solutions. In the course of attempting to scaffold this transition for teacher candidates and reflecting on what we have found to be helpful, a five-step approach has evolved whereby we feel we can effectively facilitate this transition for teacher candidates in the context of examining rich content tasks. The approach, which we refer to as the Mathematics as Teacher Heuristic (MATH), is designed to provide a set of experiences, in a two-week period at the beginning of a prospective secondary mathematics teachers' Methods class, that gradually require candidates to shift their mathematical view from a "learner/doer of mathematics" orientation to one embracing teacher-oriented perspectives. The MATH process requires candidates to engage with rich mathematics tasks but from points of view rarely considered in content-oriented coursework that, by their nature, are concerned with candidates' attempts to do their own mathematics rather than consider another learners' mathematics. In a two-week project at the very beginning of a Methods class, candidates complete a five-step process that consists of:

(1) solving a rich task as a "doer";
(2) assessing student work samples associated with the same task;
(3) constructing a set of questions for students who are stuck in the problem;
(4) developing scaffolded instructional materials addressing student challenges, difficulties and misconceptions (gleaned from earlier analyses); and
(5) reflecting on the process.

Each step of the MATH process encourages candidates to consider the rich tasks from increasingly teacher-centric points of view. (This is not to be confused with a "teacher-centred" approach to pedagogy, rather in this case, teacher-centric means thinking about mathematics as a teacher of mathematics needs to.) Assessing authentic student work and developing scaffolded instructional materials are activities that require candidates to consider interpretation of a learner's work and guidance for a learner (teacher-oriented tasks) rather than solving the problem on their own terms and presenting it for consideration (a learner's perspective).

In an earlier study (Meagher, Ozgun-Koca, Edwards, 2010) using the MATH process, we showed the particular impact of Task 2 above i.e. the power for teacher candidates of engaging with authentic high school student solutions in facilitating the transition from "doer of mathematics" to "teacher of mathematics." In that study we showed that this is perhaps the most crucial juncture for teacher candidates in the five-step MATH process i.e. the step that caused the greatest dissonance and resulted in the greatest level of reflection on the transition from "doer" to "teacher". In order to amplify this juncture and to study it more closely in this iteration of the MATH process we included a set of Livescribe™ videos along with the static solutions. Video technology allows the teacher candidates to see each step of the student work as well as being able to hear the students' description of their own thinking. As we will see below having a combination of video and static solutions is effective as there is great value to the teacher candidates trying to work out what the high school students were thinking, particularly when they took solutions paths other than the abstract algebraic approaches the teacher candidates were expecting.

The research questions guiding the research and the analysis are:

(a) To what extent does the MATH process help teacher candidates' transition from doers to teachers of mathematics?

(b) What is the particular impact on teacher candidates of engaging with authentic high school student solutions?

Literature Review and Relationship to Research

Many models for understanding the negotiation of the transition along the continuum of professional development from "student/doer" to "teacher" focus on dissonance and motivation in school settings, factors which occur regularly for in-service teachers in their day-to-day practice (Clarke & Hollingsworth, 2002; Edwards, 1994). Clarke and Hollingsworth (2002) and Loughran (2002) stress the importance of self-reflection in the development and evolution of teacher knowledge, beliefs, and attitudes. We believe that candidate teachers need to experience similar dissonances in university coursework order to accept the need for change and reflect on their change process. Providing such opportunities for change and reflection is, perhaps, more difficult in the case of candidate teachers at the very beginning of their course of study since they have fewer authentic teaching experiences to draw upon. Studies show that two main sources for dissonance in initiating candidate growth are in methods courses and student teaching.

Ball (1989) studied the role of a methods course for elementary mathematics teachers in helping candidates learn to teach. Ball highlights the importance of content knowledge and the experiences of candidate teachers as learners of mathematics. "Unless mathematics teacher educators are satisfied with what prospective teachers have learned from their experiences as students in math classrooms (and most are not), this highlights a need to interrupt, to break in, what is otherwise a smooth continuity from student to teacher" (p. 4). In this sense, we see Ball's call to break with experience, the "interruption," as an opportunity to create a dissonance.

Of particular relevance to the kind of shift we are asking teachers to make is the notion of "unpacking" discussed in the work of Adler & Davis (2006). In one of their examples of unpacking they present five different student responses to a standard question requiring finding solution(s) of a quadratic and note that, after seeing at a first level of analysis that all the students have found a correct answer, "The teacher will need to unpack the relationship between a mathematical result or answer and the process of its production" (p. 274). Adler & Davis note that the teacher is also faced with the challenge of interpreting the specific strategies used by each student and consider how those strategies, some of which are incomplete or problematic, will be orchestrated in a classroom setting to consolidate the learning of all students. Engaging in such understandings which are part of the

Mathematical Knowledge for Teaching (MKT) (Bass, 2005; Ball, Hill and Bass, 2005; and Hill, Rowan & Ball, 2005) construct involves the creation of a dissonance whereby teacher candidates are challenged to think about mathematics in ways that are not their own and displace them from the role of “doer” of mathematics.

Loughran (2002) explored the development of knowledge through effective reflective experience from teacher candidate to experienced teacher, comparing candidates’ views of teaching and learning with those of practitioners. He noted that candidates typically equated learning “with gaining right answers" (p. 41). Loughran's study illustrates the importance of giving candidates opportunities to face their views, reflect and reconsider them. Swafford et al. (1999) echo these findings for inservice teachers. Swafford et al. recommend the creation of environments for teachers that improve their content and pedagogical knowledge through reflection and collaboration.

Finally, the efficacy of using authentic student work in teacher development has been established (e.g. Kazimi & Franke, 2004), albeit often for inservice teachers in lesson study-like settings. Hiebert et al. (2007) have demonstrated how the use of student work can be effective in the preservice setting.

Methods and Methodologies

Participants

Participants in this study (n=38) were candidates enrolled in a methods course designed for prospective secondary mathematics teachers. At the very beginning of the methods class the teacher candidates worked in pairs for a two-week period on the five-step MATH process.

Candidate Tasks

Candidates were given the following set of tasks building from the problem in Figure 1 below:

> **The Bridgewater Problem.** At 1:00 p.m., two hikers began walking, the first from Amityville to Bridgewater, the second from Bridgewater to Amityville along the same path. Each walked at a constant speed. They met at 4:00 p.m. The first hiker arrived at Bridgewater 2.5 hours before the second hiker arrived at Amityville. When did the second hiker get to Amityville?

Figure 1: The Original Rich Problem

Task 1: the teacher candidates solve the Bridgewater problem using any method. This puts the teacher candidates firmly in the position of doer of mathematics and produces examples of their own mathematics that the students can then compare to other people’s mathematics.

Task 2: the teacher candidates review forty-five authentic high school students’ attempts at the Bridgewater problem and review five Livescribe™ videos of high school students (unsuccessful) attempts at the problem. The purpose of this task is for the teacher candidates to be confronted with many examples of how the problem can be solved including perspectives totally different from their own. In undertaking this task the students can begin the shift to thinking about the problem from another person’s perspective.

Task 3: using the Livescribe™ videos the teacher candidates construct questions they would ask those students to understand their thinking and help them make progress in solving the problem. This task requires teacher candidates to consider the incomplete work of a student and understand how that student was thinking about the problem. The purpose of this task is to require student candidates to continue their shift from doer to teacher by thinking about how they would work with students engaged in the task.

Task 4: the teacher candidates revise the task to construct a more scaffolded experience for students solving the task. The purpose of this task is to require candidates to engage in task design in such a way that they have to think about how someone else would do the problem.

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Task 5: the teacher candidates are asked to reflect as in Loughran (2002) on the process they have undergone and to articulate their thoughts on the shift they are making from doer to teacher.

Each step of the process encourages candidates to reconsider rich tasks from increasingly teacher-centric points of view i.e. reconsider the tasks in terms how a teacher has to understand learners' thinking and how to engage learners in a task.

The data collected consists of all teacher candidate work on each of the five tasks above. In our analysis here, we focus on Tasks 1, 2, 3, and 5 of the MATH process assignment. Due to the qualitative nature of data, our analysis concentrated on looking for patterns, categories, and themes. Building on the previous study's codes and literature review, we looked, in particular, for instances of dissonance, unpacking, and rich reflections.

Results

Working through the rich problem on their own terms and then working with the combination of video and static solutions resulted in three major elements which moved teacher candidates to think about their transition from "doer of mathematics" to "teacher of mathematics": (i) understanding the multiple solution paths students will bring to a problem, (ii) understanding the thinking process of students as they work through the problem, and (iii) changing their expectations of the level of work of which high school students are capable. The first of these was particularly powerful in creating the kind of "dissonance" (Edwards, 1994) and "interruption" (Ball, 1989) that can prompt teacher candidates to reflect. The videos were particularly powerful in the second of these as the teacher candidates undertook the task of "unpacking" (Adler & Davis, 2002) the student solutions as they sought to understand exactly how a high school student worked through a problem and exactly how that student got stuck.

Multiple Solutions

Our previous study (Meagher, Ozgun-Koca, Edwards, 2010) showed that in their own solutions to the problem the teacher candidates strongly privileged abstract algebraic methods. This finding was replicated in this study whereby all 17 of the 17 pairs of students who presented a solution used exclusively algebraic methods, i.e. they set up an algebraic expression for each walker and solved the quadratic arising from setting the expressions equal to one another.

The privileging of abstract algebraic methods was further underscored by the fact that while many used a graphical representation such as that in Fig. 2 below, 0 of 17 pairs used a Cartesian graph and 17 of 17 solved the quadratic equation by factoring rather than graphical methods.

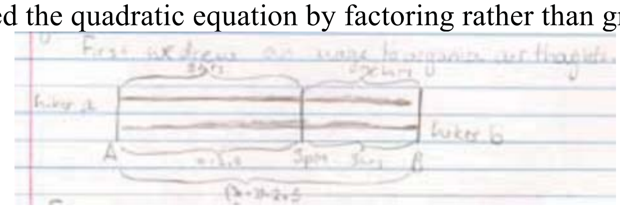

Figure 2: A Teacher Candidate's Graphical Representation of the Problem

The dissonance and/or surprise caused by the multiple solution methods used by the students was explicitly mentioned by approximately half of the pairs in their reflections: "The ability of students to solve high-demand problems in a number of ways was truly a surprise to us" (Student pair B8). Another pair commented: "In grading the student answers, we were particularly amazed at the variety of approaches. Some students performed a series of "guess, check, and revise," others created functions, some seemed to rely solely on number lines or graphs. Despite the variety, many of these attempts resulted in successful solutions" (Student pair A2). Note here the extra surprise that the teacher candidates felt that non-algebraic methods resulted in successful solutions. It was further interesting to observe how deeply the privileging of algebraic solutions (Cooney, 1999) is held for some students: "I knew that students could use guess and check, or solve things graphically, but I never knew it had been formalized" (Student pair A5); "In fact, many of the students were able to arrive at an answer with much less computation and mathematical rigor" (Student pair A7); and "Since we have had a lot more experience in problem solving than many of the high school students and have learned much higher mathematical concepts, using a method such as guess, check, and revise or reasoning through the problem with pictures was not our initial instincts for coming up with a solution." (Student pair A8). We see here that after some exposure to authentic multiple solutions produced by high school students, some of the teacher candidates are attached to the notion of abstract algebraic solutions as the best and most mathematical.

In our pilot study (Meagher, Ozgun-Koca, Edwards, 2010) we saw that the high school students often took a more embodied approach to the problem thinking of the hikers in real terms as actual people moving, whereas the teacher candidates tended to immediately think of the problem in terms of setting up equations to be solved. This approach is exemplified by the following comment: "At our level of math, we generally start with equations with a goal of solving them, whereas the students use less abstract math applications and more straightforward thinking and assigning values for items" (Student pair B6).

The teacher candidates acknowledged that, as they transition to being teachers, they need to move away from their own predilections for how a problem should be solved "One big idea … was that there are multiple ways to solve the same problem. It is important that we as teachers are open to letting students explore different methods and find their own strategy that works best for them" (Student pair B6), and "The big idea that we can take away from math project one is that every student's thought process is unique. In order to respond to this, it is important for us as educators to acknowledge the various ways in which students think to help them arrive at a solution" (Student pair B3). We see here that the examination of multiple solutions has facilitated the teacher candidates in moving from a "doer" perspective to a "teacher" perspective.

Livescribe™ Videos

As part of the assignment the teacher candidates watched five Livescribe™ (http://www.Livescribe.com/en-us/) videos of high school students attempting to solve the Bridgewater problem. Livescribe™ is a digital platform whereby a user, by employing a smartpen with an optical and audio recorder, can record a video of each of their written steps in solving a problem and a simultaneous synchronized audio of their thought process. The video can be played back with a "shadow" of what the user wrote permanently visible so the viewer can see everything the user ended up writing while they view the student writing in real time.

In order to provide extra insight into student thinking the teacher candidates watched videos of a number of students who struggled with the problem. The teacher candidates were then asked to provide questions they would ask the high school students to help them past their impasse. This task provided a useful challenge for the teacher candidates as they were forced to think about the problem from another person's point of view and to formulate questions that explicitly used what the student had already shown they knew to move then forward. The teacher candidates had many interesting responses as they struggled to negotiate the line between helping the high school student reflect but not telling them too much or telegraphing responses.

Many of the questions were unsuccessful: (a) too leading "What if you set up 2 equations? It might be helpful to use x and y, showing the distances before and after the hikers meet" (Student pair B4); "Do you remember the d=rt formula?" (Student pair B5); (b) Too vague: "What different approaches are there to solving the problem?" (Student pair A5); "What can you assume about the time that the hikers meet? What can you not assume?" (Student pair A10); or (c) not really questions "Can your picture be improved?" (Student pair A2).

Others were more successful in finding questions that might prompt students to reflect on their work thus far: (a) general questions "Have you used or represented all of the information given in the question?" (Student pair A1); "Why do you think we are told they maintain a constant speed?"

(Student pair B9); (b) and more specific questions related to the student's work up to getting stuck "What do your variables stand for in the context of the problem?" (Student pair B6); "What units would you give each of your variables?" (Student pair B8); and "Are the rates of the hikers the same?" (Student pair B5)

Thinking of good questions is challenging and, as we saw above, many of the student teachers found it difficult. Nonetheless watching the videos and thinking of questions to pose to the students was a powerful exercise. In the free-response reflection 11 out of 19 of the pairs explicitly mentioned the power of the smartpen videos in helping them think about student thinking and student approaches to the problem rather than thinking as a "doer."

The key value of the videos was to help teacher candidates in the transition from thinking about the problem for themselves to thinking about it from a student's point of view. An important part of this is for them to hear and understand a student's thinking process, i.e. to "unpack" (Adler & Davis, 2002) the student's thinking and to use that student thinking as a basis for helping the student move forward in their thinking. Listening carefully to students and understanding their thinking is a crucial element in teacher development. The value of this part of the experience was evident in the teacher candidate reflections. "Being able to put ourselves in the mindsets of our students is a key skill in helping our students figure out solutions on their own. In connection to this, brainstorming scaffolding questions for the students using the smartpen videos was one of the most useful tasks of this project. It helped us to see how students approached the problem and what things they struggled with." (Student pair A8). It is interesting to see here the teacher candidate focus on the student approach and struggles. This idea of how the student got to their impasse was noted by others, e.g. "Viewing the Livescribe™ files and watching the students work out the problem was a whole new experience. It allowed us to see and hear what the child was thinking while they were working out the question. It allowed us to see what went wrong and why it happened." (Student pair B5).

We can see that the use of video not only facilitates the transition to thinking from a student's point of view but also pushes teacher candidates in the direction of another crucial step: what questions can they ask which will move the student forward. The teacher candidates recognised this opportunity as evidenced by comments such as "some of those students had the right idea and with a little push … they would have reached a solution." (Student pair A2) and "it was nice to think about what questions we could ask to help them out. We didn't want to be too leading in our questions, so at first it was difficult to find questions." (Student pair B9). This transition along the continuum of professional learning begins to activate the MKT of teacher candidates, most particularly the combination of subject content knowledge and knowledge of how students think about and engage with content knowledge called "knowledge of content and students (KCS)" (Hill, Ball and Schilling, 2008, p. 373).

We should be careful to say that the smartpen videos could certainly have been replaced by "traditional" videos of students solving the problems, but it is clear from the evidence above that a video has a different effect on the candidates than static solutions or even solutions accompanied by a transcript. Furthermore Smartpens arguably make video data easier to collect.

Future Expectations for their Students

A significant advantage of having the teacher candidates work through the rich problem themselves is that they had considerable experience with the problem and had developed specific expectations both for what they expected to see in the student work and of how they expected high school students to work with the problem. As well as the "dissonance" (Edwards, 1994) caused by the variety of solutions offered by the high school students it was clear that the teacher candidates were impressed by the quality and ability shown by the high school students. This led many to comment explicitly that they have learned to expect more of high school students.

Many of the teacher candidates struggled with the Bridgewater problem and expected that ' students would find the problem extremely challenging. They were surprised to see that the high school students were able to solve the problem using less abstract methods than they themselves had used but also that many were capable of using abstract algebraic solutions. "One 'big idea' that we can take away from this project is that we should have higher expectations of our high school students." (Student pair A3); "I really had never thought that high school students would ever write anything so clear. They expressed their thought processes clearly, and thoroughly" (Student pair A5) and "The ability of students to solve high-demand problems in a number of ways was truly a surprise to us" (Student pair B8).

The expectations the teacher candidates had for their prospective students was changed by the quality of the work they saw: "Another aspect of the students' work that was interesting was how much the honors algebra students wrote for each problem, with the exception of one or two students" (Student pair A10). Perhaps more importantly, the exposure to multiple solutions helped the teacher candidates in the transition they must make to differentiating tasks and to thinking about how the same task can be framed differently for different populations. This idea, common among the responses is typified by the following reflection: "We've learned about how problems can be accessible and understandable by students at earlier ages with the help of scaffolding and technology. For instance, the Bridgewater problem would typically be a Algebra 2 level problem. However, there are geometric (visual) solutions as well as algebraic and numerical solutions. Students in Level 1 were able to set up the problem and some students even had a thought process that could have lead them to the right answer, thus even 2 mathematical levels below the suggested level for this problem students can find ways to achieve a solution" (Student pair A6). The experience prompted teacher candidates to reflect on how they would frame tasks "A 'big idea' that I can use in my future teaching endeavors is that when I create a lesson, I need to think not only about how I would solve the problem but the multitude of different ways that my students could solve the problem" (Student pair B2). They also learned that they can expect a lot of their students if they themselves understand their role as a teacher "One 'big idea' that we can take away from this project is that no problem is too hard for students to complete with the right guidance and scaffolding … When we were assigned this project, we thought that no high school student would be able to complete this problem because it involves deep thinking, but we were surprised by the results in the student work … Thus, we think a big idea we can take from this project is to never be too afraid to give our students a challenging problem because they may completely take us by surprise with the level of thinking they are capable of." (Student pair A8).

Conclusion

As teacher candidates negotiate the continuum of professional development it is crucial for them to transition from being "doers of mathematics" to being "teachers of mathematics." The study presented above shows that the MATH process, in particular the engagement by teacher candidates with authentic high school student work provides the kind of "dissonance" (Edwards, 1994) and "interruption" (Ball, 1989) that can facilitate that transition. Moreover, the use of Livescribe ™ video can engage teacher candidates in the kind of "unpacking" (Adler & Davis, 2002) that aids them in understanding mathematics from someone else's point of view and how to move that student's thinking forward. Of course the candidates' work analysed above takes place in a two-week period at the beginning of a methods class and represents a first intervention to shift the teacher candidates' thinking. Further research is needed to examine any lasting and transferable effect of the transition we see beginning here. In particular, as our research continues we will examine what happens in subsequent semesters when these candidates go into the field for student teaching: Are they able to enact effective questioning techniques? Do they encourage multiple approaches to solving problems? Do they encourage multiple approaches to problems but still privilege algebraic solutions as "better" or "more mathematical"?

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