TALKING ABOUT PEDAGOGY, STUDENTS AND MATHEMATICS

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When student teachers and cooperating teachers have conversations during the student teaching experience, research indicates that they spend considerable time talking about classroom management. In an effort to focus student teaching on the practice of eliciting, understanding and using student mathematical thinking, we altered the structure of student teaching. In this altered student teaching experience, the frequency of conversations involving mathematics was significant (almost 60%) and the frequency of those involving classroom management was correspondingly quite small (about 4%). The nature of the mathematical conversations focused on how students were making sense of the mathematics and how teaching facilitated that process. The management conversations that did occur also focused on strategies that could facilitate students' mathematics learning.

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The conversations that take place between student teachers (STs) and their cooperating teachers (CTs) constitute a significant portion of the "enacted curriculum" of student teaching. As explicit occasions for reflection and collaboration, these conversations represent much of what STs have the opportunity to learn. Research has shown, however, that these ST/CT conversation sometimes focus more on classroom management than on mathematics and facilitating student mathematical learning (Peterson & Williams, 2008). As one CT put it,

> I still don't think that the challenge of teaching mathematics, at least at the junior high level, is mathematics. It's not having a better understanding of what we need to teach. I think its understanding how to control them, and I feel that is the key to helping the mathematics at this level in the junior high. (Peterson & Williams, 2008, p. 470)

The traditional structure of student teaching actually seems to support such a focus on classroom management (Leatham & Peterson, 2010a). CTs often decide much of the curriculum of student teaching, and they tend to view the purpose of student teaching as STs experiencing real classrooms with real student behavior (Leatham & Peterson, 2010b). This paper presents results from a study that investigated ST and CT discussions in a nontraditional student teaching structure—one that had been purposefully designed to change the focus of student teaching away from students' behavior and onto students' mathematics (Leatham & Peterson, 2010a). As part of this structure, STs and CTs observed each other teach and then participated in post-lesson reflection meetings. We analyzed these reflection meetings to see how much and in what ways these participants talked about mathematics, students, pedagogy and classroom management. The results of this analysis show that STs and CTs are indeed capable of talking a great deal less about classroom management and a great deal more about mathematics and their students' mathematics. We also show that the nature of these conversations better aligns with mathematics teacher educator expectations and desires with respect to STs' opportunities to learn.

Theoretical Framework

We view mathematics teaching as the purposeful facilitation of student mathematics learning. For us the essence of mathematics teaching lies in the interactions between teachers, students and mathematics (cf. Cohen & Ball, 2001). As teachers and students interact around mathematical ideas, students provide evidence of their current understandings and teachers seek to provide experiences that are likely to help students build on those understanding. We thus see as a foundational teaching practice the elicitation, understanding and use of students' mathematical thinking. As described previously, we designed the student teaching structure under examination with the express purpose of facilitating STs' understanding and development of this practice.

In defining *mathematics teaching* in this way we do not mean to discount the myriad other responsibilities mathematics teachers have as part of their job. As we have discussed elsewhere (Leatham & Peterson, 2010b), we have found the metaphor of a shoe store apprentice useful when distinguishing among these aspects of a teacher's work. There are two main arenas of knowledge an apprentice cobbler needs to learn: 1) how to make shoes and 2) how to run a shoe store. Similarly, an apprentice teacher needs to learn 1) how to facilitate student learning and 2) how to manage a classroom. Although in each case the apprentice needs to learn both aspects of the job, the former is far more important in general and, we would argue, should take precedence over the latter. What good is having a well-run shoe store if you cannot make quality shoes? What good is creating a well-run classroom, if you cannot facilitate student learning? For us, "teaching" is facilitating students' learning (making shoes), not running the classroom (running the shoe store). Thus our emphasis in moving the conversations during student teaching away from classroom management toward facilitating student mathematics learning is to focus teachers on what we believe to be the most important aspect of teaching.

We find *opportunity to learn* a useful construct for describing the enacted curriculum in a given classroom—the content students have the opportunity to learn as evidenced by the content with which they interact and the ways in which they do so (Schmidt, Cogan, & Houang, 2011). The content of student teaching for our study is "mathematics teaching" and STs interact with this content through observation, teaching, discussion and reflection. We view the content of these activities as the content STs had the opportunity to learn. In this paper we do not seek to provide evidence of what STs actually learned. Rather we seek to provide evidence of what they had the opportunity to learn. Our intent is thus to explore in what ways a particular structure of student teaching seems to provide the opportunity for STs to learn the practice of eliciting, understanding and using student mathematical thinking.

Methods

In order to provide an appropriate background for this study, we first described the setting—in this case the structure of the students teaching experience in which these STs and CTs were participating. We then discuss the how the data were collected and analyzed.

Structure of Student Teaching

As mentioned earlier, we defined the primary purpose of student teaching to be learning how to elicit, understand and use students' mathematics thinking. We then reorganized the student teaching experience so that the structure would support this purpose. See Leatham and Peterson (2010a) for greater detail on the structure and the rationale behind it. The primary facets of the structure are as follows: Two STs were paired with a single CT. Two or three pairs of STs at neighboring schools (consisting of at least one high school and at least one middle or junior high school) were joined to form clusters. During the second week of the 15-week experience each CT taught a lesson for all the STs in the cluster to observe. The university supervisor (US) also observed this lesson. After the lesson the STs, CT and US participated in a reflection meeting (similar to those held in a lesson study model). During each of the next three or four weeks each pair of STs prepared a single lesson. On the assigned day the pair of STs each taught their lesson to a different class while the US, CT and the other STs in the cluster observed. Again they all held a reflection meeting soon after the second lesson. During these first five weeks the STs also participated in other learning to teach activities that focused their attention on students' mathematics—conducting focused teaching observations, writing reflection papers, interviewing students, and prompted journaling. During weeks 6-13 the pairs of STs took primary responsibility for the CTs' classes, each taking on about half of the CT's regular teaching load. During week 14 the STs turned their classes back over to the CT, again prepared just a single lesson that was observed by all in the cluster, and held associated reflection meetings.

Each reflection meeting was structured in a similar way. In general, one observing ST facilitated the meeting by first asking the teaching STs to describe the mathematical goal of their lesson, how the lesson was designed to accomplish that goal, and how they felt it played out in their two classes. The STs then asked each other questions and shared comments about the lessons. After this conversation seemed to have run its course then the facilitator invited the CT to share their questions and comments, then the US was invited to do the same. Although the CT and US could participate in the earlier discussions, they were encouraged to wait until the end of the meeting to initiate conversation.

Participants and Data Collection

One cluster of six STs student taught under this structure during the Fall 2006 semester, with one pair at a high school and two pair at neighboring junior high schools. Eight STs participated during the Fall 2007 semester. They were divided into two clusters, each consisting of one pair at a high school and a second pair at a neighboring junior high or middle school. The participants in this study include these 14 STs, their 6 CTs (one of the teachers was a CT both years) and the three USs (one for each cluster). The 2006 cluster held 5 early reflection meetings and a sixth near the end of the semester. Each of the 2007 clusters held four early reflection meetings and a fifth near the end of the semester. We video recorded each of these 38 reflection meetings.

Data Analysis

We transcribed each of the 38 reflection meetings and then coded the transcripts according to the content of the conversations. Because the designed purpose of the student teaching experience was to learn how to elicit, understand and use students' mathematical thinking, and the structure was designed to facilitate that focus and lessen focus on classroom management, we coded for these fundamental aspects. That is, we looked for evidence of participants talking about pedagogy (P), mathematics (M), students (S), and behavior (B) (the latter being the way we tried to capture "classroom management"). Statements were coded as P if they referred in any way to the actions or words a teacher might or did say. Any reference to specific mathematical language, procedures or concepts was coded as M. Reference to students in general or by name were coded as S. Statements were coded as B if they referenced behavioral expectations or problems or referenced school-related issues outside of or tangential to the act of teaching. It is important to note that all B codes were applied as additional codes to statements that already had some combination of the PSM codes.

Our unit of analysis was an idea unit, which was typically comprised of one or two sentences. Thus, throughout the data set, we basically coded each sentence for whether it referenced in some way these four basic themes. Multiple researchers coded trial transcripts until intercoder reliability reached 80%, at which point each transcript was coded by a single researcher.

Results

Approximately 11,000 statements across the 38 reflection meetings received some combination of the PSM codes. Approximately 4% of these statements also received a B code. Thus, on average there were about 12 statements related to classroom management in a given reflection meeting, which on average contained almost 290 statements. These broad stroke findings suggest that the focus of these conversations was *not* classroom management. To further examine how discussions of classroom management did and did not occur in these discussions we show the distribution of the behavior codes across the PSM codes (see Table 1).

Table 1: Distribution of Behavior Codes across the PSM Codes

	% of all comments	% of these statements coded as Behavior	% of Behavior-coded statements coded here
Pedagogy	17.9	2.4	12.3
Students	6.9	18.1	36.0
Mathematics	6.5	0.0	0.0
PS	16.1	10.2	47.0
SM	15.9	0.3	1.3
PM	19.8	0.1	0.5
PSM	17.0	0.6	2.9

Behavior and mathematics are seldom combined as a focus. There are no instances of M-coded statements also coded as behavior and very few among the other M-related statements. Taken together, less than 5% of all B-coded statements also received an M code. When one considers the nature of mathematical conversation, perhaps this is not surprising. But the literature has repeatedly found that STs and CTs *do* talk about behavior and *do not* talk about mathematics. These results provide some explanation as to why—the two topics do not often go together. When STs, CTs and USs focus on students' mathematics they talk a lot about mathematics and not as much about classroom management.

Almost half of the B-coded statements are coded as PS, with the remaining half going primarily to S (about 1/3) and the rest to P (about 1/6). So, STs and CTs are more likely to talk about behavior when they are talking about both students and pedagogy and less likely when talking about simply pedagogy (although such statements exist). Also of interest—it is the B-coded statements that were coded as S and PS where there are significantly more B codes than the normal percentage. That is, although only 6% of all statements were S-coded, 36% of the B-coded statements were S-coded. Similarly, B-coded statements were much more likely to be PS-coded than were statements in general (47% compared to 14%). The middle column in Table 1 further illustrates this pattern. B-coded statements make up about 18% of the S-coded statements and about 10% of the PS-coded statements. B-coded statements make up a very small portion of any of the other categories. Discussions that focus solely on students or students and pedagogy are much more likely to focus on classroom management.

Almost 60% of all statements that received some combinations of the PSM codes were mathematical in nature (combining codes M, SM, PM and PSM). Thus the participants in these reflection meetings were directly talking about mathematics in a majority of their conversations.

We now discuss what it looks like for STs and CTs to talk about mathematics. We present these results for two main reasons. First, given that the literature seldom reports significant mathematics conversations between STs and CTs, it is important for the field to get a feel for the kinds of mathematical conversations that can indeed happen. Second, we end this section by discussing statements that involve both mathematics and behavior. Although rare, these examples illustrate how even the nature of discussions about behavior can be altered when the underlying focus of conversations is students' mathematics.

Although the frequency of individual PSM codes provides some general indication of *what* reflection meeting conversations focused on, the individual statements, taken out of context, do little to help one understand the nature of these conversations. We have found it helpful to look at excerpts of conversations and then to discuss how the various statements (and their associated PSM codes) help us to capture the nature of the overall conversation.

The following excerpt is taken from a pre-algebra lesson toward the end of the 2006 student teaching experience. The goal for the lesson was to formalize the rule that "adding a negative is the same as subtracting a positive" (ST Emily). This excerpt contains examples of all of the PSM codes that involved mathematics—M, SM, PM and PSM. (The codes are included in the transcript following each statement.) ST Christina and ST Emily planned together and taught individually the lessons that were observed. As was often the case in these reflection meetings, this discussion begins with a statement about students' mathematics. (Note: It is helpful to know that the student thinking under consideration in this excerpt actually came up in both classes—by a girl in ST Christina's class and by a boy in ST Emily's class.]

ST Ashley: When you did your True/False questions at the beginning—I don't know whose class it was; I think it was yours [points to ST Christina]—one of the students came up and it was like 15 = 7 + 8. She came up and rewrote it 7 + 8 = 15. (SM) Is there a reason you didn't focus on that? I mean, did you want to make a statement about that at all, because I think that's an interesting idea that a student would bring up: you can just switch the things on the equals sign and it means the same thing. But then also that she has to rewrite it for her to make sense. Does that make sense? (PSM)

ST Christina: Yeah, no, I thought that her rewriting it just solidified even more that these are both the same thing. (PSM) I don't know, maybe I didn't understand that she was thinking that you had to rewrite it. (PSM) Is that what you were thinking, that she thought that you had to rewrite it to make sense? (SM)

ST Megan: I was thinking more about [the] commutative property that they did in the one right before—that they were just building on that. (PSM)

ST Jennifer: It's not the commutative property though; it's the property of the equals sign. (M)

ST Ashley: It's the property of equality. (M)

ST Jennifer: Equality. (M)

ST Megan: What am I thinking? (M)

ST Christina: I think the commutative was before that—it was the 3 + 4 = 4 + 3. (M)

ST Megan: Right, and that's what she did. She switched it because she knew that adding could commute and give you the same answer, right? Am I doing that wrong? (SM)

US Karl: Well, what you're talking about is moving something from left and right to right and left across the equals sign, and the commutative property is the across the operation. (M)

ST Megan: Well, I can see where they're coming from, thinking that when you've just talked about the commutative property. (SM)

ST Christina: Oh, right.
ST Jennifer: I think his understanding was the fact that, "Okay, I work from left to right." You know, "I do the operations on the left and then I write them on the right side of the equals sign." (SM) So I think that's where his understanding was. It's like, "I have to have the 8 + 7 on the left first and then it equals 15". (SM)
ST Emily: And didn't I ask—I said, "Is 7 + 8 = 15 the same thing as 15 = 7 + 8?" But I don't know if I really got them going through the discussion. (PM)
ST Christina: Well, I think the simple fact that they rewrite and they say it's true—that they understand that it's equivalent already. (SM)
ST Jennifer: Have you emphasized the fact that—. (PM)
ST Christina: No, we haven't. Like, we just barely did commutative and equalities now. (PM) And I thought about, "Should I state that?" But it was not a focus, a goal we had for the day, so I didn't state it. (PM)

ST Ashley begins her contribution to the discussion by presenting a particular instance of student mathematics that she noted during the day's lesson (coded SM). Her purpose in presenting the student' mathematics, however, was to pose a question about a pedagogical decision ST Christina made with respect to that student mathematics (coded PSM). ST Christina responds by first answering the question—explaining how she interpreted the students' mathematics in the moment (coded as PSM)—then questioning that interpretation (coded as PSM) and finally asking ST Ashley if she had correctly understood her interpretation of the student's mathematics (coded as SM). This return to talking about the students' mathematics prompts a mathematical discussion about the mathematics embodied in the student's actions. The next few turns focus on making sense of the mathematics itself, no longer in reference to the student (coded as M). ST Megan then ties the mathematical conversation back to the student's mathematical work. At this point US Karl chimes in to give his take on the two different properties that are under discussion—the symmetric property of equality and the commutative property of addition. ST Megan indicates that she sees how students might confuse the two properties (SM). It is in this context that ST Jennifer redirects the conversation back to the original analysis of ST Ashley—namely that this student might think that one actually *should* write the addends on the left and the sum on the right, an analysis that is related to the student not fully understanding (or at least appreciating) the symmetric property of equality. ST Emily then analyzes her own teacher move when a student did the same thing in her class (coded as PM). ST Christina provides her own interpretation of the student's mathematics (coded as SM), although there seems to be an implied pedagogical purpose in her statement—that ST Emily's teacher move would be justified with this particular interpretation of the student's thinking. ST Jennifer raises the question of whether that particular aspect of equality had been emphasized in their class (coded as PM), which seems to imply that ST Christina may have been justified in not pursuing this student's mathematical thinking. ST Christina follows up by explaining that these ideas are new and that she had not intended for them to be a focus of that day's lesson (coded as PM), providing yet another rationale for not pursuing the thinking during her lesson.

The SM statement that began this discussion provided inroads to discuss mathematical ideas (e.g., the symmetric property of equality and the commutative property of addition), students' mathematics (e.g., possible confusion about the symmetric property of equality), and pedagogical issues related to students' mathematics (e.g., why one would or would not pursue this particular students' rewriting of a number sentence). In this portion of the reflection meeting the CT and US were mostly silent, allowing the STs to work through their ideas. These ideas, however, were on the table now and could become fodder for discussion later in the meeting. The purpose of this paper, however, is not to analyze what the STs did or did not learn in these conversations. Rather, the purpose is to describe the nature of these conversations as a means of illustrating the content of the experience—what these STs had the opportunity to learn. We posit that conversations such as these gave the STs opportunities to learn mathematics, to learn about the nature of students' mathematical thinking, and to learn about effectively using students' mathematical thinking for mathematically-relevant pedagogical purposes. As mathematics teacher educators, these are the kinds of opportunities to learn we want for our STs.

We conclude this discussion by illustrating what behavior-related conversations can look like when they occur in the context of focusing on students' mathematics. In this meeting one of the 2007 clusters of STs was reflecting on lessons planned and taught by ST Katie and ST Jake.

> *ST Jane*: For [the] Tony's pizza [problem], ST Jake kind of had them work on the equation in a group, whereas you did it as a whole class. And I was just wondering—. (PSM)
>
> *ST Katie*: Our plan was to have them do it as a group, but again, to me, it seemed like when they go into their groups, it takes a long time to get them back, and I was already pressed for time. (PS, B)
>
> *ST Jake*: And there wasn't really a big diversity in what students came up with. They were all pretty much the same. (SM)
>
> *ST Jake*: We kind of talked about it in between [lessons] and said, "You know what, if we just let them go, most of them are going to be right on anyways." (PSM)
>
> *ST Katie*: So that's why I figured I could probably do it as a class, because we had just done a problem where they had written the equation, so I thought that maybe as a class, they could see this, and most of them would get it anyways. (PSM) So it was just—. For me, it was just another way to save time. It takes too much time to get their attention back, I think. (P, B)

This discussion begins with ST Jane questioning a difference in how ST Katie and ST Jake engaged their students in working on a particular mathematics problem (coded as PSM). ST Jake taught the lesson first and, as planned, had the students work on the problem in groups. All of the STs, including ST Katie, had observed that lesson. When it came time for ST Katie to teach the lesson, she altered the plan and solved the problem in a whole-class discussion. ST Katie explains that student behavior is often an issue when it comes to transitioning from group work back to whole class discussion and that she did not feel that she had time to deal with those behavior issues (coded as PS, B). ST Jake adds that the two of them had discussed the lack of diversity in the students' mathematics that had emerged in the group work in his class (coded as SM) and thus that they might get the same mathematical thinking out there through a whole class discussion (coded as PSM). ST Katie then expounds on this rationale, adding that the sequence of tasks seemed to be a contributing factor to the lack of diversity in solving the pizza problem (coded as PSM), then returns to her original behavior-based justification (coded as P, B).

Although the actual student mathematics is not visible in this discussion, clearly the STs consider students' mathematics as they discuss these lesson variations. The STs are able to discuss students' behavior (and how to deal with it) while connecting it to concerns about the mathematics learning. The point we wish to make here is that the focus on students' mathematics not only increases the frequency of mathematics related conversations, but also alters conversations about student behavior to be less about "controlling" students in general and more about managing that behavior for more productive mathematical discussions. These STs had the opportunity to learn about effective ways to elicit, understand and use students' mathematical thinking in addition to learning how classroom management might influence this practice.

Conclusion

Mathematics-related conversations among STs and between STs, CTs and USs were prevalent in this student teaching structure, whereas classroom management conversations were infrequent and were typically related to facilitating students' mathematics learning. These results provide evidence of movement toward the goal of making the student teaching experience more about making shoes and less about running the shoe store. More important than the frequency of mathematical conversations is the nature of them—conversations that, as mathematics teacher educators, we long for our STs and CTs to have.

Several aspects of our student teaching structure seemed to contribute to this prevalence of mathematics-related conversations. First, the presence of multiple observers lessened the likelihood of classroom management issues arising during the lessons, providing space for reflection on other topics. Second, the STs were engaged on a daily basis with learning-to-teach activities that focused their attention on student mathematical thinking. It seems likely that this focus carried over into these reflection meetings. And finally, the structure of the reflection meeting itself, wherein the leading questions asked the STs to reflect on the goals of the lesson and how the lesson tasks were designed to accomplish those goals of the task, seemed to set the stage for a conversation that began and remained focused on student mathematical thinking.

Because of how seldom mathematics-related conversations seem to take place during student teaching, STs and CTs may not understand just what such conversations might entail. Peterson and Williams (2008) reported that one ST in their study "felt that because he taught comparatively low level mathematics (Algebra and Pre-Algebra), he wasn't learning anything about mathematics, nor did he and [his CT] need to discuss it" (p. 470). He explained, "Especially since we teach pre-algebra and algebra, we rarely talk about mathematics content" (p. 470). This ST did not seem to have a clear image of how mathematics teachers can productively talk about students' mathematics. The present study demonstrates that conversations about how students were making sense of the mathematics and about how a teacher might help students make mathematical connections, regardless of the level of mathematics being taught, can be very productive. As STs and CTs discuss their students' mathematics they come to realize that such conversations are not a rehashing of "simple" mathematical procedures but rather involve in-depth analysis of mathematical connections and sense that students are or could be making. Such conversations also tend to lead to mathematical conversations that provide the opportunity for STs, CTs and USs to deepen their own mathematical understanding. In our opinion, such analysis and conversation is at the heart of learning to teach and should be an integral part of every student teaching experience, thus providing STs the opportunity to learn not just how to run the shoe store, but how to make shoes.

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