INDIVIDUAL ADHERENCE TO INQUIRY-ORIENTED NORMS OF DEFINING IN ADVANCED MATHEMATICS

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This study investigates students' individual patterns of adherence to a norm for creating and assessing definitions in an undergraduate real analysis classroom. Our findings indicate that, though all students understood the expectation to define, students differed in their individual adherence to the norm depending upon their perception of the nature of the defining activity and their associated role in the classroom. While some students primarily attributed the expectation to define to the teacher (taken-as-expected), others understood the expectation as truly shared as a means toward classroom learning (taken-as-beneficial). A third group of students internalized the expectation as a means of their membership in the mathematical community (taken-as-meaningful) and were therein willing to challenge the professor's defining choices in light of their values for mathematical defining.

Keywords: Advanced Mathematical Thinking, Reasoning and Proof, Post-Secondary Education

Introduction

For some time, the Emergent Perspective framework (Cobb & Baursfeld, 1995; Yackel & Cobb, 1996) has provided a fruitful set of tools for documenting collective learning phenomena within mathematics classrooms. One of the primary strengths of their framework is a refusal to dichotomize psychological and sociocultural phenomena, instead viewing each as an analytical perspective for which the other provides a necessary context. As a result of this balanced approach, research points to strong connections between collective constructs (classroom norms, sociomathematical norms, or mathematical practices) and individual constructs (mathematical beliefs or conceptions; Yackel & Rasmussen, 2002).

However, the university classroom structure places a larger onus of mathematical learning on students' time outside of class. This indicates that the research community also must consider exactly how classroom norms and practices influence students' mathematical activity outside the classroom. Thus, this paper seeks to address the following questions:

1. In an undergraduate real analysis course characterized by inquiry-oriented norms for defining, what are students' individual patterns of participation in the norms during class meetings and while working on class assignments outside the classroom?
2. How do students' patterns of participation interact with their beliefs about the purpose(s) of shared defining activities and students' role in advanced mathematical activity?

The current investigation draws from classroom observations and interviews with volunteer students from two different inquiry-oriented real analysis classes to answer these questions. These classes, taught by the same professor, adopted an inquiry-oriented approach to defining in the sense that the class treated definitions as "under construction." The class consistently discussed how and why mathematicians define concepts in the way they do. My findings indicate that even after certain mathematical expectations became "taken-as-shared" norms, students display individual differences and shifts in their perception of the source of the expectation: from external, to generalized, to internal. These categories extend the psychological lens of the Emergent Perspective as it relates to students' mathematical activity beyond the classroom.

The Emergent Perspective framework

The Emergent Perspective is especially useful for providing rigorous and actionable means of defining what it means for a collective (members of a class or small group) to "know" something as well as characterizing essential differences between classroom cultures. Drawing from the interactionist perspective of sociology, the framework recognizes that the behavioral regularities of culture are constituted through interpersonal interactions (Blumer, 1969; Prus, 1994). While no one person creates culture, participation in cultural practices simultaneously reinforces those cultural elements. The emergent framework identifies three broad categories of cultural elements pertinent to the mathematics classroom:

1. taken-as-shared expectations for participating in the classroom are *classroom norms*,
2. taken-as-shared criteria by which participation is deemed acceptable are *sociomathematical norms*, and
3. taken-as-shared mathematical activities and associated meanings are *classroom mathematical practices* (Cobb, Stephan, McClain, & Gravemeijer, 2001).

The current study focuses on the first two categories. Examples of classroom norms include explaining and justifying one's reasoning (Cobb, Wood, Yackel, 1993) or presenting proofs and answering questions about them (Fukawa-Connelly, 2012). Examples of sociomathematical norms include mathematical criteria for determining what constitutes acceptable explanation, justification (Yackel & Cobb, 1996), or proof (Fukawa-Connelly, 2012).

Classroom norms begin as expectations for mathematical activity that are endorsed by members of a classroom. Once members of a classroom community: (a) act in accordance with the expectation, (b) respond against breaches of the expectation, and (c) cease to question adherence to the expectation, then that expectation is "taken-as-shared" among that community. Norms that satisfy these criteria can be considered "known" within the classroom not because of their presence or establishment within individual minds, but rather because of their interactively legitimized functionality in classroom activity.

The Individual Lens of the Emergent Perspective Framework

Many researchers link elements of collective understanding to elements of individual understanding such as students' beliefs and community values (Cobb et al., 2001). Fukawa-Connelly (2012) investigated classroom norms in an undergraduate abstract algebra class both via classroom observation and delayed student interviews, and found a large level of agreement between students' descriptions of classroom norms and his analysis of classroom videos. Levenson, Tirosh, and Tsamir (2009) instead noted differences between teacher and student perceptions of classroom norms. However, fewer studies compare classroom mathematical culture to students' individual activity outside the classroom. This is because the Emergent Framework was developed for classroom instructional design (Yackel & Cobb, 1996; Cobb et al., 2001). But keeping with the Emergent Perspective's resistance to dichotomizing individual and collective learning, social constructs must inform or situate the investigation of individual learning. The current investigation employs the Emergent Framework in this alternative way.

One question not clearly addressed in prior literature is, "What is a norm prior to it becoming normative?" By standard definitions it would be a contradiction to say one person introduced a norm because norms must be collectively constituted. I use the term *expectation* for a not-yet-taken-as-shared norm, which is somewhat novel though it appears in the Emergent Perspective literature along with terms such as "obliged" (Cobb et al., 2001; p. 133). If students were "obliged" (a form of "obligated") to act in a certain way, this expresses a mutually understood expectation. Once an expectation is mutually understood, it is taken-as-shared. However, another question that arose from analysis of my data is, "By whom is one expected to act in certain ways?" Even if students act to satisfy an expectation and/or reinforce the expectation to one another, they may attribute the expectation to a particular source. I thus distinguish the source of an expectation (or "locus of expectation") from its "taken-as-shared" status in the classroom.

Reform-oriented norms and student autonomy

The literature on sociomathematical norms consistently points to a link between inquiry-oriented norms and students' sense of autonomy and authority in their own learning (Cobb et al, 2001; McClain & Cobb, 2001; Yackel & Cobb, 1996). When students are allowed to negotiate the criteria by which their activity is mathematically assessed, they may become a "community of validators" (Yackel & Cobb, 1996) rather than relying fully on the external sources. In such cases, students often display greater intellectual autonomy (McClain & Cobb, 2001), which is a direct goal of many instructional reform efforts (Yackel & Cobb, 1996).

Relevant literature

The current study was conducted in undergraduate, real analysis classrooms and thus within the instructional tradition of "advanced mathematical thinking" (Tall, 1991). Being a proof-oriented course, real analysis is part of mathematics majors' enculturation to the mathematical community and its associated processes (i.e. defining, conjecturing, proving). In the classroom, these processes are expressed and guided by norms of classroom activity. The following literature provides insights into the teaching and learning of these mathematical processes.

Students' use of definition

A survey of the definition literature reveals a recurrent dichotomy between two primary ways in which students reason about categories: either a definition describes a preexistent category or it constitutes the set of all exemplars satisfying its conditions. So, either a category suggests a defining property or the property determines the category members. Several studies (Edwards & Ward, 2008; Alcock & Simpson, 2002) observe that while mathematicians act as though definitions are in in the latter category (*stipulated* definitions), many students instead rely primarily on intuitive notions or prototypes (*extracted* definitions). Mathematicians' emphasis upon reasoning from the formal definition stems from the fact that "appropriate use of the definition means that any correct deductions he makes will be valid for all members of the mathematical category" (Alcock & Simpson, 2002, p. 32).

Engaging students in producing or assessing defining activities

Though a student's untrained use of definitions differs from a mathematician's, several studies indicate that students asked to produce or assess definitions often identify standard values for defining. (Zaslavsky & Shir, 2005; Zazkis & Leikin, 2008). These values include clarity, elegance, non-redundancy, and arbitrarity (non-uniqueness). These studies suggest that defining activities simultaneously reveal and help reorganize students' conceptions about defining.

Methods

As this study intended to relate elements of classroom culture to students' independent work outside of class, I simultaneously developed models of classroom activity and students' individual activity. Toward this end, I gathered data including: detailed notes from all class meetings, bi-weekly professor interviews, and weekly student interviews with a group of 4-6 volunteers per semester (two semesters of data are analyzed).

Professor and instructional context

The professor observed in this study is a tenured mathematician at a large, comprehensive, public university in the Southwestern United States. The professor received multiple teaching awards based on student and colleague nominations, including from her previous real analysis students. The classes met for 75 minutes twice per week for 15 weeks. Professor interview questions generally related to: her intentions and expectations for class sessions, her reflections upon class discussions, and her understanding of students' reasoning (to which she paid a great deal of attention). These interviews informed my emerging model of her instructional practice based on her class activities and teaching actions. The flow of the classroom discourse would best be described as a highly interactive lecture with the professor consistently guiding the conversation (Dawkins & Roh, 2011). Though extended student-to-student conversations were less common, the professor consistently re-voiced and championed students' ideas or played devil's advocate for ideas that she found reflective of commonly held misconceptions.

The classroom norms were reform-oriented in the sense that the professor treated elements of the real analysis theory as "under construction". I identified three types of activities by which she invited students to create and assess definitions: defining (a) *portrayed*, (b) *discussed*, and (c) *enacted*. In *defining portrayed* activities, the professor asked students to compare provided definitions against intuitive ideas or groups of examples. The group accepted or rejected the definitions based on the emerging shared criteria and examples. In *defining discussed* activities, the professor prompted students to reflect on how and why they defined (see Dawkins, 2012 for a detailed description of one such activity). She invited students to question her and the mathematicians "who wrote the books" to reflect upon definers' intentions. In *defining enacted* activities, students had to produce definitions to turn in or present to the class, often with heavy scaffolding from a previous definition. Through these activities, the professor endorsed the norm that "Students are obliged to create and assess mathematical definitions or conditions within definitions" (or the "norm of defining" for brevity).

Interview participants and analysis

During two semesters of one professor's real analysis course, I solicited a small group (4-6) of volunteers to participate in a sequence of 7-9 weekly interviews regarding their learning in the course. I analyze the activity of seven students (5 from semester 1 and 2 from semester 2), based on the number of participants who provided sufficient data for categorization relative to the research questions. The interviews (a) invited students to reflect on and explain parts of the classroom discussion and (b) documented student work on homework tasks to simulate their independent learning. The professor's *defining discussed* activities aided the study because the first type of interview question naturally extended the classroom dialogue. For instance, after spending a class period on the notion of cluster point without providing a ratified definition, the professor asked whether the class preferred that she provide a definition up front. She drew attention to her intentions in discussing these conceptual issues rather than dispensing pre-formulated knowledge. When I asked questions about this same practice, students appeared to report their reflections about the class dialogue in response to the classroom discussion (or in response to the professor) rather than as an artificial product of the interview environment.

All interviews were audio recorded, transcribed, and coded using a grounded theory open-coding protocol (Strauss & Corbin, 1998) in the NVivo program for qualitative data analysis. I analyzed and coded all cases of students' classroom and interview activity related to the process of mathematical defining. The categories related to a student's: (a) personal concept definitions (PCD), (b) understanding of examples or concepts related to key definitions, (c) ability to produce novel limit definitions (of the form $\lim_{x\to\blacksquare} f(x) = \blacksquare$), (d) choices regarding the professor's defining activities or challenges to her defining choices, (e) perceptions of the values of mathematical defining, and (f) sense of vestment in defining. The emergence of the last three categories motivated the current report. Categories (d) and (f) revealed some students' sense of intellectual autonomy and authority. The students' autonomy often appeared in tandem with particular patterns in their perception of the defining activity as shall be discussed.

Results

Analysis identified three primary patterns of students' adherence to the norm in their individual mathematical activity. These patterns of adherence are not mutually exclusive, being that one student shifted patterns over time, but they differ according to students' (a) understanding of the intent of defining activities, (b) beliefs about the nature of mathematical definitions, and (c) their locus of expectation for the norm of defining. I define *adherence* as a student's individual behavior in relation to a classroom norm, which is the psychological correlate to the collective notion of *participation*. By "behavior," I include cognitive activity in the classroom and isolated activity outside the classroom.

Non-adherence

Vincent was the only participant who clearly displayed a pattern of *non-adherence to the norm of defining*. *Non-adherence* is defined as a willing abstinence from mental or enacted defining despite understanding of the norm of defining. Soon after a lecture in which the class discussed defining function limits without a ratified definition, Vincent and the interviewer (also the author) had the following interchange (March 26):

I: Usually we talk one class period about a subject, and then we define it the next class period […] Why do you think she does that?

V: Maybe cause she wants us to all try and develop our own definition of it, then once we come to the next class, she is going to show us the definition that is accepted by the people who argue over the definition. I don't know… I mainly just wait for her to define it for me.
I: Why is that?
V: Because, I don't know. I guess, lack of motivation to do it on my own because I know she is going to do it. […] When she finally gets us set with a solid definition, sometimes I will look at the other stuff that she was talking about and try to relate 'em and everything. I don't go and find out on my own so much.

Vincent understood the expectation for students to reason about defining, but his perception of the activity kept him from adhering to the norm. Vincent's locus of the expectation was the professor. Though the professor invited students to consider various definitions, Vincent thought of "the definition that is accepted" as a single answer he wanted the teacher to provide. This is more in line with traditional norms of advanced mathematics instruction (Weber, 2004). Vincent located the source of mathematical authority outside of himself, not identifying himself with the "people who argue over the definition." He also expressed lack of motivation or necessity for adhering to the norm. Vincent was the only study participant who had to retake the course.

Peripheral adherence

Over time, Vincent shifted his pattern of adherence to the norm. On April 8, he reported (after a *defining enacted* activity), "She had us try to define in class, which I thought was pretty interesting […] And that was kind of fun, I liked that cause […] I don't really sit there and try to define it because I would rather just look it up, and it was actually different trying to define it and getting pretty close to what was in the book." Vincent consciously noted his shift from *non-adherence* to *peripheral adherence* to the norm. Peripheral adherence is defined as mental or enacted participation in defining as a pedagogical activity toward the end of learning pre-existing mathematical content. Vincent maintained an external sense of authority comparing his defining activity "to what was in the book", but Vincent showed a strong attitudinal shift toward adhering to the class' expectation on his behavior. He later reported that his shift toward independent defining grew from frustrations with having to depend upon his classmates' understanding.

Four of the study participants (Aerith, Celes, Tidus, & Vincent) exhibited *peripheral adherence* to the norm of defining. These students interpreted the defining activity as didactical in nature. Accordingly, they often retained a view that the formal definition was fixed by outside authority, and that they needed to understand those definitions. However, *peripheral definers* generalized the locus of expectation to the classroom. For instance, Tidus described the teachers' intentions in defining portrayed and enacted activities saying: "Instead of just being given information and trying to regurgitate, you're [trying to] see how you got to that point and then it seems to stick in your head more when it's done that way" (Dec 12). Later, he described the class' defining activities saying,

> "We didn't really have anything concrete that we knew without a doubt was true, we were just thrown some ideas, given some actual applications or examples. […] And then all of a sudden we get to the definition, and now it's not just this bland definition sitting on the board. We actually see why it's, why that's being defined, what it's used for, and there's a different appreciation for it. [When the professor poses T/F questions] we are gonna' actually test our and see if we actually understood it or not. And then the homework, of course we get to work out of class and use it some more, try to understand it some more. And then those [expletive] tests to see if we actually got it or not." (Dec 12)

Tidus' comments exemplify peripheral adherence in several key ways. First, he centers his justifications of the activity in terms of improved understanding and recall. He interprets the sequence of classroom activities (discussing, defining, true-false questions, homework, exams) as a connected means of developing and assessing learning. Tidus thus frames (Goffman, 1974) the activity within a didactical structure. Next, though he used a singular pronoun during the first quote ("you") to describe recall, he

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also regularly used a plural pronoun ("we") indicating a sense of communal activity and expectation in the classroom. Though the norm was *taken-as-shared,* Tidus' locus of expectation was generalized to the whole class. This differs from Vincent's identification of the professor as the source of expectations. However, *peripheral definers* tended to maintain an external sense of authority, consistent with the didactical frame. Their perceived role in defining is as learners of mathematics known or developed by experts.

Authoritative adoption

Three of the study participants (Cyan, Edgar, & Ronso) displayed *authoritative adoption* of the norm of defining. Authoritative adoption is defined as mental and enacted participation in the norm in accordance with an internal sense of authority in their mathematical activity. These students framed defining as their participation in advanced mathematical activity, implying that they were members of the mathematical community. Authoritative adoption evidenced itself by students' tendency to use the personal pronoun ("I") with respect to their defining activity and their willingness to contest the professor's mathematical claims. Both such behaviors suggest an internalized locus of expectation. All three authoritative adopters explicitly questioned the professor's defining choices based on some meta-mathematical criteria. For example, both Cyan and Ronso expressed dissatisfaction with the professor's choice to include the textbook's requirement that function domains must contain some interval (a, ∞) to have a limit of the form $\lim_{x \to \infty} f(x) = L$. Both students asked why functions defined on the rational numbers or the integers could not have such a limit. They argued instead that the book's definition violates the criterion that mathematical definitions should be sufficiently general within the local body of theory. These students also seemed to properly coordinate between treating definitions as *extracted* while defining and *stipulated* while proving. Like peripheral adherers, adopters said defining helped them learn. However, they acted primarily within a mathematical frame rather than an instructional frame. Adopters were the top-performing study participants in the courses.

Discussion

Summary of categories and constructs

While all students appeared to perceive the shared nature of the norm of defining, students differed in their locus of expectation, sense of authority, and their frame for the defining activity. The non-adherer Vincent located both the expectation and mathematical authority with the professor or other experts. Peripheral adherers like Tidus generalized the locus of expectation to the classroom viewing defining as a shared practice. However, they maintained an external sense of authority consistent with their pedagogical frame for the shared defining activity, which positioned them as learners of the mathematical understandings held by the professor. Authoritative adopters like Ronso or Cyan exhibited an internalized locus of expectation and source of authority, evidenced by their willingness to challenge the validity of the professor's defining choices. Table 1 organizes the characteristics of each category.

To clarify this diversity of adherence to the norm, I offer three new sub-constructs. For non-adherers, the practice is *taken-as-expected* (TAE), meaning the norm's value is extrinsic (imposed) in the sense of being a means toward the goal of satisfying the professor. For peripheral adherers the practice held intrinsic, secondary value, which I call *taken-as-beneficial* (TAB). In the case of TAB, the norm holds intrinsic (personal) value for the individual, but the benefit is derived from the value for learning and is thus secondary. For authoritative adopters, defining held an intrinsic, primary value, and is thus *taken-as-meaningful* (TAM). Once a student feels personally vested in defining as part of their mathematical identity, they intrinsically value the process itself in addition to its other benefits. These constructs are likely inclusive or nested inasmuch as authoritative adopters acknowledged that defining helped them learn (TAB) and understood the professor's expectations. However, intrinsic values generally prove stronger and more durable than extrinsic values suggesting why authoritative adopters displayed more detailed defining activity outside the classroom context. These finer distinctions extend and clarify the Emergent Perspective literature's claims that inquiry-oriented norms help promote students' development of compatible individual mathematical beliefs (Cobb et al., 2001; Yackel & Cobb, 1996).

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Table 1: Characterizations of Individual Adherence to the Norm of Defining

Category	*Non-adherers*	*Peripheral adherers*	*Authoritative adopters*
Locus of expectation	Professor	Generalized	Internalized
Source of mathematical authority	Professor/ textbook	Professor/ textbook	Self
Frame for defining	Pedagogical (non-beneficial)	Pedagogical (beneficial)	Mathematical
Individual status of the collective norm	TAE	TAE, TAB	TAE, TAB, TAM

Practical contributions

The university classroom structure assumes that a large onus of learning falls on students' time outside of class (traditionally in a 3:1 ratio). This suggests that understanding classroom activity and the culture therein only accounts for a portion of students' mathematical engagement. It thereby seems important to understand how classroom norms for mathematical practice influence students' mathematical activity beyond the classroom. My findings suggest that the classroom environment strongly influences students' individual activity, but the relationship between collective norms and individual action is more complex than direct correspondence. Factors of affect and identity seemed to strongly influence students' adherence to norms. Vincent's shift in adherence may reveal the positive influence of the classroom environment on his mathematical identity, but further evidence is required in this regard.

I also maintain that advanced mathematics instruction should enculturate students into the mathematical community. Therefore, it is important that students engage in and understand key mathematical practices such as defining. While almost every student reported learning from participating in defining, authoritative adopters also identified values of defining by which they contested the professor's defining activity. As such, they displayed rich understandings of mathematical definitions and expressed personal identification with the community of mathematicians. I hypothesize that these students who internalize the locus of expectation are most likely to adhere to and endorse the norm of defining in their future mathematical activity either as students or teachers (the majority of these students proceeded to graduate studies in mathematics or to secondary mathematics teaching). The ability of students to adhere to such practices beyond the influence of the original locus of expectation (the professor or class) seems a pertinent factor for the "durability of sociomathematical norms" (Van Zoest, Stockero, & Taylor, 2011), and thus warrants further study. The individual sub-constructs to *taken-as-shared* identified in this study should facilitate the continued investigation of such phenomena.

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