AN ANALYSIS OF ALGEBRA CONTENT IN THE FINLAND AND TAIWAN ELEMENTARY SCHOOL MATHEMATICS TEXTBOOKS

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The major purpose of this study was to examine the similarities and differences in the contents of algebra topic in selected elementary school mathematics textbooks of Finland and Taiwan. The mathematics curriculum market leaders of Finland and Taiwan, Laskutaito and Kang Hsuan, were examined using content analysis method. Results show that while both textbook series introduce the early algebra through arithmetic and numerical reasoning, they differ from each other in terms of the use of patterning tasks. Furthermore, there is a significant difference in the representation forms of problems used in these textbooks and the use of contextual problems. Implications related to mathematics textbooks' design and teaching are discussed along with suggestions for future studies.

Keywords: curriculum analysis, algebra and algebraic thinking, elementary school education

Textbooks play a significant role in mathematics classrooms worldwide (Schmidt, McKnight, Valverde, Houang, & Wiley, 1997). They are commonly considered as a source of explanation and exercises for students to learn and for teachers to teach (Reys & Reys, 2006), and an important indicator for the opportunities to learn mathematics (Stein, Remillard, & Smith, 2007). The primary purpose of this study was to examine the similarities and differences in the contents of algebra topic in selected elementary school mathematics textbooks of Finland and Taiwan.

This study is significant in terms of the choice of the target countries, topic focus, and grade levels. Students in Finland and Taiwan are high achievers in PISA 2006 and 2009 (Organisation for Economic Co-operation and Development [OECD], 2010). It is important to find out what opportunities to learn are afforded to the students in these two countries. Furthermore, there is a growing recognition of the important role early algebra learning plays in the overall success with algebra (Common Core State Standards Initiative, 2010). A detailed analysis of the algebra materials in elementary mathematics textbooks from Taiwan and Finland will shed light on the content and characteristics of the mathematics problems used to support the development of early algebra reasoning in these two countries.

Prior Studies on Textbook Analysis

Prior studies on textbook analysis have focused on two primary issues. The first one is content coverage. This includes learning goals, list of topics, meanings and representations of key concepts, sequence, pacing, and learning trajectories (Charalambous, Delaney, Hsu, & Mesa, 2010; Yang, Reys, & Wu, 2010). The second focus is classifying the textbook exercises and problems by various kinds of constructs and schemes. For example, Charalambous, Delaney, Hsu, and Mesa (2010) examined the fraction constructs (e.g., part-whole, measure, ratio, operator, quotient) used in elementary mathematics textbooks from Cyprus, Ireland, and Taiwan.

Early Algebra Learning

Carraher and Schliemann (2007) identified three entry points for early algebra learning. The first one is through arithmetic and numerical reasoning that includes the field of axioms, such as

commutative, associative properties that result from performing arithmetic operations. The second entry point is arithmetic and quantitative reasoning. The most prominent approach to early algebra learning based on quantitative reasoning is centered on the work developed by Davydov and his colleagues from the Vygotskian perspective (Schmittau, 2005). The third entry point is based on arithmetic and function (Carraher, Martinez, & Schliemann, 2008). Students are introduced to the numerical and geometrical patterns and are asked to predict the next one in informal language. Textbooks in certain parts of the world introduce formal algebraic notation and simple symbolic manipulations in the upper elementary grades. For example, Wong (2005) reported that students in Hong Kong are expected to solve simple equations involving fractions and decimals such as $x - 2.2 = 1.8$ or $y \times 2/3 = 4$ in sixth grade.

Based on the work summarized above, the following two research questions guide the design and analysis of this study:

1. What are the similarities and differences in the types of problems used in the selected textbook series in Finland and Taiwan in terms of content-dependent characteristics such content coverage, sequencing, pacing, and the emphasis in terms of arithmetic, quantitative, functional, or symbolic reasoning?
2. What are the similarities and differences in the types of problems used in these two selected textbook series in terms of content-independent characteristics?

Because of the space limitation, only selected findings will be discussed.

Method

Compulsory education in both Finland and Taiwan shares similar basic structures. Both are 9 years in length; during the first 6 years, a homeroom teacher is in charge of most subjects, while the subject specialists teach different subjects during the last 3 years of schooling. Also, they both have national curriculum guides (Ministry of Education in Finland, 2008; Ministry of Education in Taiwan, 2003).

The selected mathematics textbooks, Laskutaito (Saarelainen, 2006) and Kang Hsuan (KH, Kang Hsuan Educational Publishing Group, 2010), both have the highest market share in Finland and Taiwan, respectively. Both series have one booklet for each semester; thus, there are 12 booklets in each.

This study applied the content analysis method to analyze and code the problems presented in the mathematics textbooks (Yang et al., 2010). The data analysis of this study involved several stages. First, we went through all the problems in the mathematics textbooks and coded each one as to whether it was an algebra problem. The problems include the worked example (solutions were provided) and exercise problems following the worked examples (no solutions were provided) in the student textbooks. Two of the authors did all the coding and compared their results. The reliability was calculated using (shared algebra problems)/(total identified algebra problems) for each textbook (Ding & Li, 2010). The reliability for coding the Finland textbooks was 92%. The reliability for coding the Taiwan textbooks was 95%. The discrepancies between the coders were discussed to reach agreement before data analysis.

Second, we went through all the algebra problems in both textbook series and noted the main characteristics of these problems, in terms of both the content-independent and dependent ones. To confirm our observations, we coded all the algebra problems along these two sets of characteristics. To check the reliability for each of the aforementioned categories of the algebra problems (context vs. non-context, and different forms), all of the algebra problems were randomly selected and coded independently by the first and second authors for each textbook. Both coders discussed any discrepancies among the problems to reach agreement for the final

results. Finally, several themes emerged during these iterations of analyses when comparing similar problems from both textbook series, which will be discussed in the next section.

Selected Findings

The primary focus of the algebra problems in the Finland elementary textbook starts with developing arithmetic and numerical reasoning, and pattern recognition ability. As each operation is introduced, students first learn to write and evaluate expressions for given contextual problems, then solve number sentence problems with the unknown represented with "☐," and finally with the letters "x" and "y" in sixth grade. Properties of operations such as commutative laws and order of operations are introduced with concrete examples and plenty of exercises to consolidate the ideas. The design of the Finland elementary math textbook is spiral. Old ideas are revisited briefly before they are used with larger whole numbers or rational numbers with more complex computations. In addition, logical patterning tasks are first introduced at the first grade level and revisited with increased complexity throughout the remainder of the five grades.

Similar to algebra problems in the Finland curriculum, Taiwan textbooks encourage students to learn how to write the expression with the unknown represented with "( )" for given contextual problems, and to learn how to solve them with various algebraic properties first, and then with "x" and "y" in grade six. However, the algebra problems in the Taiwan elementary textbook do not focus on patterning recognition tasks as much as the Finland textbook in the early grades. The Kang Hsuan series does contain exercises that encourage students to find the regularity in the expression for the total number of chairs when the number of tables is "n." This kind of numerical patterning task is not found in the Laskutaito series.

There are a few other marked differences. KH tends to focus on a single unknown quantity, while Laskutaito includes ample exercises that require students to reason with multiple quantities. KH includes far more worked-out examples than Laskutaito. In addition to showing step-by-step instructions, KH provides examples and subquestions that guide students through the reasoning process. For example, KH asks students the following series of questions: "A fruit basket weighs 50 g. How much would it weigh all together after putting in fruit that weighs 100 g, 200 g, 300 g, x g?" After students have plenty of experience in writing expressions with a single operation, they then are given problems that involve multiple operations, for example, "Ji-Hua and three friends went on a picnic. They spent 160 NT dollars to buy bus tickets and x NT dollars to buy snacks. Write an expression to show the amount of money each person should pay. If x = NT 120 dollars, what is one person's share? If x = NT 200 dollars, what is one person's share?" Finally, the textbook illustrates the additive and multiplicative axioms and how they can be used to solve equations such as $y \div 0.8 - 8 = 12$ by first adding 8, and then multiplying by 0.8 to both sides of the equal sign.

Discussion and Implications

This research reports the results of an analysis of elementary mathematics textbooks that are used in Finland and Taiwan. Both textbook series introduce the early algebraic concepts to their students through arithmetic and numerical reasoning, with an emphasis on a conceptual understanding of various principles and properties for the four basic operations. In addition, Laskutaito provides logical patterning tasks throughout all grade levels, while KH includes discussion of numerical patterning tasks that prepares students for a functional approach. The problems in KH tend to focus on one unknown quantity, while Laskutaito provides students an opportunity to reason with multiple quantities. Finally, both textbooks include some worked-out

examples; the solutions in Laskutaito include only computation steps, while those in KH include the reasoning behind those steps.

This difference reflects the different types of algebraic reasoning afforded by each textbook series. The early and systematic approach of algebraic reasoning through arithmetic and numerical reasoning might have contributed to the good performance of Finnish and Taiwanese students in the international assessment. How and to what extent students' algebraic reasoning is influenced by the different curricular emphasis is a topic worthy of future study. For example, will middle school students in Taiwan perform better on the numerical patterning tasks because of their experience with such tasks in the elementary grades? With less explicit reasoning and more variety of representations included in the algebraic problems, teachers and students in Finland are freer to explore and to make connections between concrete and abstract ideas through logical and pattern reasoning than their Taiwanese counterparts. However, such differences in the textbooks' approach may increase the importance of the teacher's knowledge and experience in facilitating students' learning through the given curriculum (Stein et al., 2007). This could potentially lead to more variation in terms of curriculum implementation. Future studies can be conducted to test this conjecture.

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