IDENTIFYING SPACES FOR DIVERSE LEARNERS' MULTIPLE MATHEMATICAL KNOWLEDGE BASES IN EXISTING CURRICULUM

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This project examines how prospective elementary teachers (PSTs) framed the idea of drawing on multiple mathematical knowledge bases (MMKB)—children's mathematical thinking and funds of knowledge— for diverse learners, in the context of adapting curriculum. We analyzed 47 written reflections of PSTs' analyses of an existing mathematics curriculum. Using inductive analysis, we identified four themes related to how PSTs evaluated the curriculum and identified possible spaces for small adaptations. Findings describe how these four themes related to incorporating MMKB. We discuss implications for mathematics teacher education.

Keywords: Equity and Diversity; Teacher Education-Preservice; Elementary School Education

The field of mathematics teacher education has begun to address how the cultural, linguistic, and socioeconomic positionality of students impacts their learning opportunities (Zevenbergen, 2001). Unfortunately, the focus on children's sociocultural identity in the classroom often appears as a separate subset of study from the more traditional focus on the psychology of mathematics education (Aguirre et al., 2012). Typically, children's home- and community-based knowledge receives much less attention in teacher education than children's mathematical thinking (e.g., problem types, solution strategies, etc.; Carpenter, Fennema, Peterson, Chiang & Loef, 1989) (Aguirre et al., 2012). In mathematics teacher education, the emphases on children's mathematical thinking and on children's home- and community-based knowledge and experiences remain largely disjointed, leaving prospective teachers (PSTs) ill-equipped to meaningfully integrate both important sources of mathematical knowledge and learning.

Research continues to reveal how a majority white, female, middle class teaching force struggles to effectively teach a diverse student population (Sleeter & Milner, 2011). This enduring challenge in education, more broadly, has serious implications for mathematics teacher education at all levels. The purpose of this paper is to examine how elementary PSTs make sense of addressing the needs of historically underrepresented populations in school mathematics during their mathematics methods course. In particular, we examine PSTs' work on a curriculum analysis assignment to better understand how PSTs frame meeting the needs of historically underrepresented populations through mathematics curriculum adaptation.

Theoretical Perspectives

Theoretical perspectives from two strands of research on elementary mathematics teaching and learning guide this project. First, the extensive body of research on mathematics instruction that centers on children's mathematical thinking (e.g., *Cognitively Guided Instruction*, Carpenter, et al., 1989) provides a basis for developing PSTs' knowledge of children's mathematical thinking in ways that change beliefs and shape classroom practices. Second, research that documents the benefits of drawing upon the cultural, linguistic, and community-based knowledge of historically underrepresented groups (Ladson-Billings, 2009; Turner, Celedón-Pattichis & Marshall, 2008) guides PSTs' development of leveraging home- and community-based knowledge in mathematics instruction. In particular, this project draws on the theory of *funds of knowledge*(FoK) for teaching. FoK refer to the "historically accumulated and culturally developed bodies of knowledge and skills essential for household or individual functioning and well-being" (Moll, Amanti, Neff, and

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Gonzalez, 1992, p. 133). Using students' FoK for mathematics teaching means that classroom instruction utilizes the cultural, linguistic and cognitive resources from home or community settings to promote students' learning of the standard mathematics curriculum in school settings (Moll et al., 1992).

Although both strands of research are well developed, they remain disconnected in mathematics teacher education, as mentioned above. As a result, the field of mathematics education lacks a deep understanding of how teachers might learn to integrate the focus on children's mathematical thinking with the emphasis on home- and community-based knowledge. This project aims to bridge these two bodies of research by guiding K-8 PSTs to use children's *multiple mathematical knowledge bases* (MMKB) to support student learning. In this paper, we refer to MMKB as the integration of children's mathematical thinking and children's FoK (Aguirre et al., 2012). More specifically, the research question for this project is: How do K-5 PSTs frame the idea of drawing on MMKB, specifically for historically underrepresented student groups, in the context of adapting curriculum?

Methods

The research presented in this paper is part of a larger project, TEACH Math. In this section, we briefly discuss the goals and methods of the larger project and provide details about the specific data collection and analysis that produced the findings presented here.

Research Overview

The TEACH Math project aims to transform elementary mathematics teacher preparation so that new generations of teachers will be equipped with powerful tools and strategies to increase student learning and achievement in mathematics in our nation's increasingly diverse public schools. The project involves iterative refinement of three instructional modules for elementary mathematics methods courses designed to explicitly develop teacher competencies related to mathematics, children's mathematical thinking, and community and cultural FoK. Across these three modules, PSTs develop specific knowledge, beliefs, and dispositions related to MMKB.

Research has occurred at six university sites across the United States, with data on PSTs' work in all three modules collected from elementary mathematics methods courses at each of these sites. We analyzed data collected at one university site, a large university in the Midwest located near a small city with an increasingly diverse population. For this analysis, we used data collected from an activity in two K-5 mathematics methods courses at this university, each with a different co-principal investigator (PI) as course instructor.

In the activity selected for this analysis, Analyzing Curriculum Spaces, PSTs analyzed an existing elementary mathematics curriculum to identify opportunities for accessing, building on, and integrating children's mathematical thinking and children's home and community-based mathematical FoK (i.e. MMKB). We refer to places in the curriculum where teachers can make these types of small adjustments as curriculum *spaces*(Drake et al., 2015). This activity is one of four in the Classroom Practices Module in which PSTs learn to analyze classroom practices through four lenses: teaching, learning, task, and power and participation. In this specific activity, PSTs use a tool, Curriculum Spaces Table, designed by the co-PIs, to guide PSTs' identification and adaptations of curriculum materials that create spaces for eliciting and building on children's MMKB.

The Curriculum Spaces Table has three sections. In the first section, PSTs identified the central mathematical goal or ideas of the lesson. In the second section, they answered questions about the different phases of the lesson (e.g. launch, explore, summarize). The questions included: (1) What makes the task(s) in each phase good and/or problematic?; (2) What are opportunities for activating or connecting to family/cultural/community knowledge in each phase of the lesson?; (3) How does each phase of the lesson open spaces for making real-world connections?; (4) What are opportunities for students to make sense of the mathematics and develop/use their own solution strategies and approaches?; (5) What kinds of spaces exist for children to share and discuss their mathematical thinking with the teacher and the class?; (6) Where does the mathematical authority reside in the lesson? In the final section, PSTs proposed possible adaptations for the lesson phases or the overall lesson.

Data Collection and Analysis

Data for this paper included PSTs' written reflections on their use of the Curriculum Spaces Table to analyze the *Stickers: A Base-Ten Model* lesson from Grade 3 *Investigations in Number, Data, and Space* (TERC, 2008, p. 26-33). We analyzed a total of 47 written reflections. Participating

Primary Theme	Definition	Subthemes & Definitions (if applicable)
Learning Supports	Aspects of the lesson/teaching provide supports to facilitate student learning of mathematical content, including ***scaffolding*** (i.e. gradually decreasing the need for learning aids as students' comfort with language, concepts, etc. increases); ***teacher questioning***(i.e. various forms of questioning recognized to support learning (Boaler & Brodie, 2004)); or ***differentiation*** (i.e. individualized adaptations of lessons/tasks).	**General Learning Supports:** Considerations that will arise in essentially every classroom setting; no reference to FoK.
		Learning Supports for Diverse Learners: *give diverse learners access to the mathematical content*; particularly important when the class included culturally and linguistically diverse students or special education students; may or may not reference FoK.
Prior Knowledge	Aspects of the lesson/teaching are ***relevant to the students' prior knowledge***. The teacher or students can provide this connection to prior knowledge.	**School-Based Knowledge:** Prior knowledge that arose in a school-based setting.
		Funds of Knowledge: Prior knowledge that arose from community/family/cultural knowledge or experiences.
		Not-Specified: Source of prior knowledge is not specified
Motivation	Aspects of the lesson/teaching are ***relevant or familiar to students*** for the purpose of ***engaging or motivating students***.	
Children's Mathematical Thinking	Aspect of the lesson/teaching accessed, built on, or integrated children's mathematical thinking, including references to: (1) students' mathematical ***explanations and justifications***; (2) orchestration of mathematical ***discussion*** (Smith & Stein, 2011); and (3) specific mathematical ***features of the task*** (e.g. manipulatives, multiple representations, multiple strategies).	

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Figure 1: Coding Themes & Definitions for Content of Written Reflections

PSTs were reflective of national demographics (i.e. mostly white, middle-class females). In the written reflections, PSTs discussed strengths and limitations of the lesson, spaces they identified for eliciting and building on children's MMKB, and the ways in which using the Curriculum Spaces Table aided in their analysis.

We analyzed the written reflections through an iterative coding process. The first two authors of this paper began the coding process separately, analyzing two of the written reflections and noting themes. We compared our initial impressions and used the themes to develop an initial codebook. We continued coding separately, comparing analyses, and revising the codebook until we produced a final (seventh) version of the codebook (Figure 1). Throughout our development of the codebook, we continually looked for confirming and disconfirming evidence of the identified themes (Erikson, 1986).

Using the final version, we coded the remaining written reflections together, discussing discrepancies and reaching consensus on coding. Written reflections were coded at the paragraph level (as denoted by the participant or roughly 10-15 lines) because surrounding sentences (or turns) provided important context for identifying themes. The codebook provided exhaustive codes (i.e. every paragraph received at least one code), but primary codes were not mutually exclusive.

We used the codebook described above to identify major themes related to the *content* of PSTs' written reflections, and we created a second coding stream to identify themes related to PSTs' *evaluation* of the curriculum. This secondary coding stream represented three major themes in PSTs' analyses: (1) Strength; (2) Weakness; and (3) Curriculum Space (Figure 2). Coding in these two streams allowed us to examine both the specific aspects of the lesson/teaching that PSTs identified in their curriculum analysis and whether PSTs framed those aspects of the lesson as strong/weak or spaces for adaptation. We linked codes in this second stream directly to codes in the first stream. For example, sometimes PSTs discussed ways in which the lesson provided learning supports, and they clearly identified those learning supports as strengths of the lesson. Such a paragraph would receive a "learning supports-strength" code. In some cases, PSTs did not clearly evaluate lesson aspects as a strength/weakness or a space for adaptation, and we did not use the second coding stream. Codes in the second stream were not mutually exclusive because some paragraphs included a discussion of both strengths and weaknesses of the same aspect of the lesson and because curriculum spaces were generally identified alongside weaknesses.

Primary Theme	Definition	Subthemes & Definitions (if applicable)
Strength	PSTs evaluated some aspect of the lesson/teaching as strong.	
Weakness	PSTs evaluated some aspect of the lesson/teaching as weak or limited.	**Too little** – Weakness or limitation resulted from too little of an aspect (e.g., too little support for diverse learners)
		Too much – Weakness or limitation resulted from too much of an aspect (e.g., too much explaining/telling by the teacher)
Curriculum Space	PSTs identified an aspect of the lesson/teaching where the curriculum could be adapted. This code was used both when the PST made a specific adaptation suggestion (e.g., I would let the students try the problem on their own first.) or when the PST talked more generally about spaces in the curriculum (e.g., The lesson needs to be adapted for English Language Leaners (ELL).)	

Figure 2: Coding Themes & Definitions for Evaluation by PSTs

Findings

In this section, we share the major themes that emerged across the written reflections. We found that attending to children's mathematical thinking represented the most dominant theme (Table 1), and we present these findings first. Attention to the needs of historically underrepresented populations of students, specifically, emerged under several different themes - learning supports, prior knowledge, and motivation. We present findings related to each of these themes separately. Overall, FoK and diverse learners received lesser attention in PSTs' curriculum analysis.

Table 1: Percent of total codes (n=1086) by specific code/theme

Children's math thinking	Learning supports		Prior knowledge		Motivation	Other
35.6%		17.7%		11.8%	7.8%	27.1%
	General	15.2%	School	4.2%		
	Diverse	2.5%	FoK	4.7%		
			Unspecified	2.9%		

Attention to Children's Mathematical Thinking

PSTs paid considerable attention to aspects of the lesson that accessed, built on, or integrated children's mathematical thinking (Table 1). PSTs identified both strengths and limitations in the ways the lesson attended to children's mathematical thinking, and they recognized spaces for adapting lessons to integrate, build on, or elicit children's mathematical thinking beyond opportunities already offered in the lesson. Consider the following excerpt:

> The launch of the lesson is very important in making the lesson effective in promoting students' learning…Students might only use one method of solving the task if they were not taught multiple ways to see the numbers. The lesson provides opportunities for students to explore but the lesson could summarize more in a group discussion format. In doing a discussion students could share their solutions and allow other students to ask questions, compare, or justify their own thinking…

In this reflection, the PST identified aspects of the launch as strong but also recognized limitations in and suggested adaptations for eliciting student thinking, particularly in the summary discussion.

Learning Supports

Among all relevant themes for our research question, learning supports, both general and specific to diverse learners, represented the second most common theme in written reflections (Table 1). Learning supports for diverse learners represented 2.5% of all codes. This means that fewer than 15% of all reflections on learning supports focused on diverse learners, specifically. In other words, overwhelmingly, PSTs seemed to focus on general learning supports in their analysis of the lesson. The following PST identified general learning support as a strength.

> This lesson…gives [students] a way to visualize the patterns and fully understand what it means to add by ten as opposed to just giving them a problem and hoping they figure out the answer. Giving students ways to remember patterns is more beneficial in my opinion.

When PSTs did discuss learning supports for diverse learners, a greater proportion of the codes occurred alongside codes for weaknesses than alongside codes for strengths in the lesson. The following excerpt is from a rare instance where a PST identified strengths of the lesson in regards to supporting the needs of diverse learners:

> This lesson states, "As frequently as possible, refer to strips as *strips of 10* to reinforce the groupings of 10s and 1s". When teaching I think it is important to pay attention to the vocabulary you are using and try to keep it the same throughout. Especially when thinking about ELL students…using the same words to describe something throughout a lesson can help decrease confusion of word meanings and phrases.

More commonly, PSTs expressed that the lesson inadequately addressed the needs of diverse learners: "This lesson also limits ELLs. At no point in the lesson do I see any talk of ELLs so that's something that could limit their learning." Despite being able to identify learning supports for diverse learners as a weakness in the lesson analysis, however, only approximately 11% of all suggested adaptations related to learning supports focused on the needs of diverse learners, specifically.

Prior Knowledge

Prior Knowledge represented one of the most identified categories relevant to how K-5 PSTs framed the idea of drawing on MMKB, specifically for historically underrepresented student groups, in the context of adapting curriculum (Table 1). PSTs discussed home- and community-based FoK with roughly the same frequency as school-based prior knowledge. Figure 3 provides example excerpts of school-based prior knowledge, funds of knowledge, and prior knowledge with an unspecified source.

Examples of Prior Knowledge Themes	
School Based	"The lesson connects back to Sticker Station done in second grade so it utilizes [students'] background knowledge."
FoK	"Aspects of the lesson plan that stand out as especially important for making the lesson effective in promoting students' learning are…that the lesson opens spaces for making connections to their family/cultural/community knowledge."
Non-Specified	"I do believe that it is possible to be responsive to students' thinking and background knowledge while also using the curriculum materials."

Figure 3: Examples for Each Category of Prior Knowledge

Only a small percentage of PSTs identified prior knowledge (combined) as a limitation in their lesson analysis; however, when PSTs did identify prior knowledge as a weakness, they overwhelmingly focused on FoK, specifically. Consider the following excerpt:

> I thought the lesson was limited at making a connection to cultural, community, and/or family knowledge. It was great that the lesson used stickers which students were familiar with from the sticker station store in 2^{nd} grade. This would hook student's interest and motivate them for the lesson.

As the above quote illustrates, school-based prior knowledge was rarely identified as a weakness in the analysis of this lesson. PSTs who identified FoK as a weakness in this lesson also generally discussed adaptations related to FoK. The focus on adapting the lesson to attend to FoK, however, represented less than 10% of all identified spaces for adaption. For example:

> The teacher could have had students create their own problems using something from their culture, home, or of interest that could be bundled in a set of 10 or used individually; for example, my case study student was very interested in basketball and played it at home with family often. He could create a problem having one team of basketball players represent the 10's place and an individual basketball player representing the 1's place. They could then share these

> problems in partners or with the whole class. I feel if students could connect with the problem better, it would better promote students learning.

Motivation

Attention to engagement and motivation in the lesson plan was common throughout the reflections, receiving about 8% of the codes. PSTs identified aspects of the lesson that attended to motivation in both strong and limited ways. Nonetheless, about 10% of codes related to curriculum adaptations identified motivation as a space for improving upon the lesson, suggesting an overall desire to improve engagement and motivation in the lesson.

> If students are forced to complete tedious, uninteresting tasks, their motivation for learning the material will decrease…However, if a teacher is allowed to use the requirements of the curriculum materials as a guide to the objectives students are expected to learn and revise the material to the interests of the students, both motivation and success in learning the material will increase.

Additionally, as a specific focus of the study, students commonly referred to engagement and motivation when discussing the purpose of integrating students' FoK. The quote in the prior knowledge section above offers one example of this connection, but the following excerpt also illustrates how PSTs linked FoK to motivation.

> [This analysis] helps me to think how I should engage real-life objects with my lesson plans and how it would help [students] to have clearer understandings…[students] not only easily learn how to add and subtract multi-digit numbers, but also learn how to engage their class lessons in their real life.

We interpreted this PST's use of "engage" as evidence of attention to students' motivation or interests.

Discussion and Conclusion

Even though the mathematics methods courses in the TEACH Math project aimed to integrate a focus on children's mathematics thinking and children's FoK, the emphasis given by the PSTs still seems unbalanced. Of all the themes discussed by the PSTs in their curriculum analysis, less than 10% focused on children's FoK specifically (i.e. prior knowledge: FoK or learning supports for diverse learners). PSTs overwhelmingly attended to aspects of the lesson focused on children's mathematical thinking (over 30%). These findings suggest that the two emphases, children's mathematical thinking and children's FoK, remain disjointed. Other teacher educators/researchers have observed similar trends when PSTs use lesson analysis tools that attend to both children's mathematical thinking and FoK (Aguirre, Zavala, & Katanyoutanant, 2012). Even though FoK received less attention than we might have expected given the methods course goals, the fact that some PSTs emphasized FoK and suggested lesson adaptations, while still attending to children's mathematical thinking, offers hope of balancing these two important emphases in mathematics teaching.

The equal attention to school-based prior knowledge and FoK offers some hope of guiding PSTs to consider MMKB in their lesson analysis, but prior knowledge, overall, received only limited attention in PSTs' analyses. These findings suggest a need to understand how PSTs frame FoK. In the rare instances when PSTs attended to children's FoK, they commonly discussed the practice in terms of motivating or engaging students. In science education, teachers' rationale for attending to FoK ranges from motivating students to increasing access to the content to changing the content itself (Barton & Tan, 2009). A deeper understanding of the reasons PSTs focused on FoK in the context of

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mathematics curriculum analysis could shed light on more effective ways of balancing the emphasis on children's mathematical thinking with FoK. The range in motivations for attending to FoK raises questions about which framings promote consideration of MMKB and challenge deficit views of historically underrepresented students.

Findings from this study also have implications for research on how teachers learn to adapt lessons to attend to the needs of historically underrepresented students, once they have identified the need for such adaptions. Those PSTs who attended to FoK in their reflections overwhelmingly identified a need for incorporating more focus on children's home- and community-based knowledge into the analyzed lesson. The ability of some PSTs to identify spaces for and to recognize the importance of incorporating MMKB suggests that this type of analysis holds promise. Nonetheless, PSTs did not necessarily suggest specific lesson adaptations or specific spaces in the lesson where adaptations might occur. This finding suggests that PSTs might need more support in thinking about how to adapt mathematics lessons to integrate, meaningfully, children's mathematical thinking with children's FoK. Further research is necessary to support PSTs in analyzing lessons and adapting them in order to support the needs of diverse students.

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