BUILDING PEDAGOGICAL CAPACITY THROUGH TASK DESIGN AND IMPLEMENTATION

Ali Fleming
Ohio State University
fleming.72@osu.edu

Amanda Roble
Ohio State University
roble.9@osu.edu

Xiangquan Yao
Ohio State University
yao.298@osu.edu

Patricia Brosnan
Ohio State University
brosnan.1@osu.edu

We traced the impact of a sequence of five research-based professional development sessions on a cohort of mathematics teachers' Mathematical Knowledge for Teaching. The sessions focused on task design and implementation as a means of building teachers' pedagogical capacity. Findings revealed that teachers' pedagogical knowledge pertaining to student thinking, if not their practice, was influenced by the activities they experienced.

Keywords: Teacher Education-Inservice; Mathematical Knowledge for Teaching; Instructional Activities and Practices

Introduction

Improving the quality of Mathematical Knowledge for Teaching (MKT) among teachers has been at the forefront of mathematics education reform agenda for quite some time. Within the last three decades, advances have taken place in defining accurately and precisely what mathematical knowledge for teaching might mean and the various dimensions that are embedded in the construct (Ball, Thames, & Phelps, 2008), developing theoretical models that inform how teacher learning of this body of knowledge might be best grounded (Borko, 2004), and identifying features of effective professional development programs that facilitate such learning for teachers (Garet, Porter, Desimone, Birman, & Yoon, 2001).

Less clear however, are the specific domains of MKT of teachers that are enhanced by their participation in research-based professional development programs or empirical data that support changes as the result of their newly acquired knowledge (Sztajn, 2011). In this paper, we will describe the findings of an exploratory research project in which we traced the impact of a series of professional development sessions focused on task design and implementation on teachers' pedagogical content knowledge.

Background

Building on two of Shulman's categories of teacher knowledge (content knowledge and pedagogical knowledge), Ball, Thames, and Phelps (2008) defined six domains of Mathematical Knowledge for Teaching (MKT), which they defined as the "mathematical knowledge needed to carry out the work of teaching mathematics" (p. 395). While Ball and colleagues distinguished between content knowledge (Subject Matter Knowledge) and pedagogical knowledge (Pedagogical Content Knowledge) as Shulman did, they more specifically designated three domains within each. Within Subject Matter Knowledge, they defined Common Content Knowledge (CCK), Specialized Content Knowledge (SCK), and Horizon Content Knowledge (HCK). The CCK domain includes knowledge of mathematics that students must learn, while the SCK domain includes knowledge of mathematics that is specific to the classroom environment (e.g. analyze student errors). HCK is described as “a view of the larger mathematical landscape that teaching requires” (Hill & Ball, 2009, p. 70).

Within Pedagogical Content Knowledge, Ball and colleagues (2008) distinguished between Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and Knowledge of Content and Curriculum (KCC). In the KCS domain, teachers must understand how students may come to understand a concept, while in the KCT domain teachers must make instructional decisions that best facilitate student learning. KCC requires that teachers know the standards and curriculum not only for a specific mathematics course, but also across grade levels, courses, and subject areas.

Guskey (2003) argued that the primary goal of professional development (PD) programs is to “bring about change in the classroom practices of teachers” (Guskey, 2002, p. 381). In order to achieve this goal, researchers have identified characteristics of teacher PD programs that tend to advance teacher learning and shifts in practice. Among them include opportunities for sustained interactions over time, collective participation by participants, and activities that are content-focused and grounded in the teachers’ everyday practice (Garet et al., 2001).

The use of mathematics tasks in PD sessions has been found to influence teachers’ teaching knowledge and their instructional practices. Through activities with mathematics tasks, teachers can increase their capacity to implement the curriculum with coherence across multiple grade levels (Ferrini-Mundy, Burrill, & Schmidt, 2007), improve their problem solving skills (Guberman & Leikin, 2013), and increase their content knowledge of mathematics (Silver, Clark, Ghousseini, Charalambous, & Sealy, 2007).Boston (2013) found that teachers’ capacity to identify cognitive demands of tasks, as well as identify the opportunities particular tasks provided to elicit student thinking, was increased by participation in a PD program that focused on the cognitive demand of tasks. What remains to be learned about the use of tasks in PD programs is how the modification of traditional tasks and their implementation in the classroom builds teachers’ pedagogical capacity.

Drawing from this body of scholarly ideas, a series of 5 PD sessions for mathematics teachers who were expected to serve as instructional leaders in their respective schools was designed. Prominently, relying on Cognitively Guided Instruction (Carpenter, Fennema, & Franke, 1996), we intended to engage teachers in building their own knowledge of mathematics through investigating children’s understanding of mathematics content and using that knowledge to guide instruction. In implementing the sessions, we capitalized on five practices recognized to be pivotal to orchestrate productive mathematics discussions (Smith & Stein, 2011). The primary focus of our work was facilitating knowledge development of the teachers through the creation and implementation of rich mathematical tasks, and then considering different solution strategies (appropriate or inappropriate) students might use on the tasks as a way of anticipating what may need to be addressed in instruction.

The Five Practices for Orchestrating Productive Mathematics Discussions includes the planning and selecting of appropriate tasks for classroom activity (Smith & Stein, 2011) since this venue has been recognized to have an impact on the mathematics students engage with (or not) in the classroom (Hiebert & Wearne, 1993). Coupled with the understanding that textbooks and teacher resources are often limited in their offering of tasks that require reasoning of children (Thompson, Senk, & Johnson, 2012), two of the PD sessions were focused on task selection and design in motivating mathematical thinking among school learners. Brown and Walter’s (1983) problem posing framework served as the primary guide for organizing teachers’ activities during these sessions. The teachers were first presented with a task, and asked to list its attributes. They then removed constraints from the task and rephrased it in a way that could substantially extend learners’ thinking. The goal was for the modified tasks to reflect characteristics of rich mathematical questions: tasks that provide multiple entry points, multiple solution strategies, and opportunities for students and teachers to develop deeper mathematical connections (Stein, Grover, & Henningsen, 1996). Our research goal was to determine the knowledge teachers may have gained from such experiences based on their classroom implementation.

Methodology

Setting & Participants

The participants in this study consisted of 12 mathematics teachers expected to serve as instructional leaders in their own respective schools using a coaching model. The participants' prior mathematics teaching experience ranged from 1-31 years, with a mean of 13.9 years. All teachers engaged in a year-long PD program that was grounded in principles of Cognitively Guided Instruction (Carpenter et al., 1996). This approach was used to help teachers develop capacity towards shifting classrooms from teacher-centered orientation to a more student-centered environment. The PD sessions that served as the basis for our exploratory inquiry consisted of series of five sessions that lasted approximately 3 hours each (a total of 15 hours). The fourth session in this series used the problem posing framework of Brown and Walter (1983) to assist teachers in learning about how to modify mathematics tasks to advance students' mathematical thinking. The teachers were asked to revise a task that already existed in their practice or curriculum materials, implement it the classroom, and then reflect on what they learned from doing so.

Data Collection

Upon the creation of the new tasks to be implemented in the classrooms, the teachers were asked to identify mathematical and pedagogical goals they intended to meet using the new tasks. They were also asked to collect samples of student work (correct or incorrect) and to comment on what they may have gained from the experience. The participants submitted their reflections/assessments electronically. For this particular study, the teachers' original and revised tasks, as well as their responses to four questions served as the primary sources for data analysis: (1) How is your revised task different from the original task? Describe the process you used to adapt the task from the original task.; (2) What additional questions did (or would) you or your teacher ask to elicit, scaffold, or extend student thinking?; (3) What insights into your students' mathematical thinking and understanding did the revised task provide that the original task may not have been able to provide?; and (4) How would you adapt or implement this task differently in the future and why?

Data Analyses

The original and revised tasks submitted by the teachers were coded using Stein, Smith, Henningsen, and Silver's (2000) categories of level and kind of cognitive demands and/or thinking processes in mathematics classrooms: high-doing mathematics; high-use of procedures with connections to concepts, meaning, and/or understanding; low-use of procedures without connections to concepts, meaning, and/or understanding; and low-memorization. An example of our coding procedure is show below, illustrating the original and the revised tasks submitted by one teacher:

> *Original*: There are twenty-one shells. The shells are evenly divided among three students. How many shells will each student get? A. 6 B. 7 C. 8 D. 9
>
> *Revised*: Susie has 24 gumballs. She wants to share them with some friends. She wants to make sure each friend gets the same amount of gumballs. How many different ways can you come up with for Susie to share her gumballs with friends? Show all the ways below.

The original task was coded as low-use of procedures without connections to concepts, meaning, and/or understanding, and the revised task was coded as high-use of procedures with connections to concepts, meaning, and/or understanding. Once the codes were generated, further analysis determined strategies the teachers used to revise the original task.

The teachers' responses to the four reflection questions were coded first using the Mathematical Knowledge for Teaching framework (Ball et al., 2008). The unit of analysis was one sentence. If two or more sentences referred to the same idea, the two sentences were coded once, and if a sentence contained two or more phrases that fell into different codes or ideas, the phrases were coded separately. Three independent researchers coded the teachers' responses. Coding results were compared among the researchers; if disagreement occurred, discussions were held to reach agreement on the coding for each response. Disagreement on the coding occurred and was remedied five times.

The following excerpt from one of the teachers' responses to question (1) serves as an example of our coding procedure:

> The original task was simple math. There was not a lot of higher level thinking involved. The students had worked on this type of equivalent fractions for some time. It came pretty easy to them. We needed to create a task that would challenge the students and take the math they learned and apply it to a real life situation.

The first two sentences were coded once as Specialized Content Knowledge, the second two sentences were coded once as Knowledge of Content and Students. The last sentence was coded twice as Knowledge of Content and Teaching (one code for "challenge the students" and one code for "take the math they learned and apply it to a real life situation").

The codes that were identified for the responses to each question were tallied corresponding to each participant. Since question (2) primarily elicited responses in the MKT domain of Knowledge of Content and Teaching, the question-type framework as described by Boaler and Brodie (1994) served as a second level of analysis for the teachers' responses for question (2). The responses to question (4) also overwhelmingly fell into the KCT domain, so they were coded on a second level as either related to content or pedagogy.

Results

Original and Revised Tasks

The coding results for cognitive demand level and type of the original and revised tasks are summarized in Table 1.

Table 1: Cognitive demand level and type for original and revised tasks (n=12)

	High-doing mathematics	High-procedures with connections	Low-procedures without connections	Low-memorization
Original task	0	0	10	1
Revised task	0	7	5	0

Note that all of the original tasks were low level cognitive demand (one of the tasks did not provide enough information to be coded), and that 7 of the 12 revised tasks increased the cognitive demand level to high. The teachers who increased the cognitive demand from low to high did so by removing constraints in the task to allow for multiple solutions, asking students to provide explanations and visual representations, introducing new mathematics content, and requiring students to give non-examples.

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Five teachers submitted original and revised tasks that were both categorized as low-procedures without connections. While they did not increase the cognitive demand of the task, these teachers added a context or changed the language of the original problem to revise their task. For example, one teacher's original task, a worksheet of 3-digit subtraction problems, was revised to include three 3-digit subtraction problems placed in a context, one of which included "There are 300 dogwood trees currently in the park. Park workers cut down 115 dogwood trees today. How many dogwood trees are left in the park?".

Reflection on the Task and its Implementation

Six teachers responded to all four questions, and six teachers responded to questions (1) and (2) only. Table 2 is a summary of the MKT codes, specifically in the domains of SCK, KCT, KCS, and KCC that were generated from the teachers' responses to each question, illustrating the knowledge domains that may have been influenced as a result of the activity.

Table 2: Domains of teacher knowledge elicited by four reflection questions

	Specialized Content Knowledge (SCK)	Knowledge of Content and Teaching (KCT)	Knowledge of Content and Students (KCS)	Knowledge of Content and Curriculum (KCC)	Total
Question (1)	11	32	7	4	54
Question (2)	0	57	0	0	57
Question (3)	11	14	5	0	30
Question (4)	0	13	0	0	13
Total	22	116	12	4	154

For question (1), in which the teachers were asked to compare their original and revised tasks, 11 of the 12 teachers made statements that compared the tasks in terms of the different levels or types of thinking the tasks may (or may not) elicit from children. For example, teachers stated that the revised task was more open-ended than the original and would thus require students to use their own thinking to solve the problem, or that the new task would require a higher level of thinking than the original task. Six teachers explicitly stated that they removed a constraint or piece of information in the original task to make the revised task more open-ended. Five teachers responded that the revised task would require different mathematics content than the original task.

For question (2), the teachers stated questions they asked or would ask to elicit, scaffold, or extend student thinking. In total, the teachers stated 57 questions, all coded as KCT. 66.7% of the questions asked students to go deeper than the revised task, categorized in the Boaler and Brodie (2004) question-type framework as extending thinking (n=18); probing, getting students to explain their thinking (n=14); exploring mathematical meanings (n=4); and linking and applying (n=2). Questions posed by the teachers in these categories included "How might your ideas change if Susie includes herself in the equal sharing?" (extending thinking) or "Do you think those represent the same solution or different solutions?" (exploring mathematical meanings). The remaining 33.3% of the questions geared students toward finding the answer to the posed task. These questions fell into the categories of orienting and focusing (n=12), gathering information (n=5), inserting terminology (n=1) and establishing context (n=1). Questions posed by the teachers in these categories included

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"What is this problem asking you to do?" (orienting and focusing) and "How did you know this was a subtraction problem?" (gathering information).

For question (3), the 6 teachers who implemented the task reported what they learned about student thinking as a result of implementing the revised task. All six of the teachers claimed to gain insight into their students' thinking, their misconceptions, and where they may struggle with the content. For example, one teacher stated "it was apparent which students recognized that there was a missing addend and that an efficient way to find the missing addend is to subtract." Four of the six teachers reported improving their SCK as a result of implementing the task, stating that the task provided opportunities for students to use multiple representations and solution strategies, and for the teacher to interpret student work and student thinking on their particular topic.

For question (4), the 6 teachers who implemented the task reflected on how they would implement the task differently in the future. The teachers' responses to this question generated 13 KCT codes, 5 related to content, and 8 related to pedagogy. For an example of a pedagogy modification, one teacher stated that she might "have different students solving the same problem but with different numbers of gumballs. This may allow for more connectedness about numbers being divided up in different ways." Pedagogically, a different teacher thought he would "give students more time to work through the task, provide manipulatives, encourage collaboration and communication, and strategically have students share ideas."

Summary

All of the teachers revised a traditional task to be implemented in the classroom with the intent to offer more open-ended venues for student explorations. Of the 6 teachers who revised a low cognitive demand task to a high cognitive demand task, 3 implemented the revised task in their respective classrooms. The reflections of these 3 teachers revealed that their revised task gave them more insight into student thinking, the reflection comments offered by these individuals indicated that they felt the new questions allowed their students to learn more mathematics and make deeper connections as a result. These 3 individuals also reported that they felt more efficacious in guiding the classroom discussions in a manner that deviated from additional tell, show, and correct pattern.

In their reflections, all 12 teachers reported ways in which their Mathematical Knowledge for Teaching may have been influenced by their participation in the PD sessions and related activities. Overwhelmingly, the knowledge the teachers claimed to gain could be described as pedagogical, as 75.3% (n=116) of the teachers' responses described ways their Knowledge of Content and Teaching was impacted as a result of the activity. The teachers claimed to have gained knowledge of strategies they could utilize to modify a traditional task to create a task that is more open-ended, allows for multiple solution strategies, and requires students to use their own thinking. The 6 teachers who implemented the task cited ways the new task enabled them to better facilitate mathematical discussions that focused on student thinking and making mathematical connections.

Six teachers who implemented their revised task in the classroom cited ways that this process influenced their Knowledge of Content and Students. One articulated her heightened awareness of student thinking as she stated "it was clear that the students did understand the relationship between multiplication and division. This is something that the original task would not have brought out about student understanding." Two of the teachers noted that students had difficulty finding new strategies that were different than the strategies their teacher had taught them. This leads us to conclude that the enactment of the revised task in the classroom, not just its revision as an isolated activity, is critical in impacting teachers' pedagogical knowledge.

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Conclusion

The main goal of our exploratory research was to determine what aspects of teachers' Mathematical Knowledge for Teaching could be influenced by PD sessions guided by the principles of CGI and with a focus on designing rich, open-ended tasks. The reports of these participants indicated that their pedagogical knowledge pertaining to their understanding of student thinking, if not their practice, was influenced by what they had learned as a result of implementing the revised tasks in their respective classrooms. Additionally, the teachers who revised the traditional task from low to high cognitive demand claimed their students learned different mathematics content and made richer connections as a result of engaging with the revised task.

The National Council of Teachers of Mathematics (2000) and the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010) require that children have the opportunity to engage with mathematics in ways that foster understanding, sense making, and reasoning. No longer is the traditional drill and practice classroom environment acceptable to meet these standards. But teachers must be given learning opportunities that may foster the pedagogical and content knowledge necessary to facilitate such a classroom environment. The PD program and sessions that informed this research gave teachers valuable tools and increased pedagogical capacity necessary to take their current classroom materials and adapt and implement them in ways that may help children reason more deeply about mathematics to meet these new standards.

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