TOWARD A FRAMEWORK FOR ATTENDING TO REFLEXIVITY IN THE CONTEXT OF CONDUCTING TEACHING EXPERIMENTS

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In this paper, we present an analytical framework for attending to reflexivity in the context of conducting teaching experiments and rationalize its components by appealing to the constructivist foundations on which the methodology is based. To illustrate the importance and utility of this framework, we discuss our analysis of recent mathematics education literature that has employed the teaching experiment methodology. In so doing, we reveal the extent to which researchers' presentations of their models of students' mathematical thinking and learning reflect these researchers' attention to reflexivity. We conclude by reflecting on the implications of researchers attending to reflexivity in the context of conducting teaching experiments.

Keywords: Research Methods; Design Experiments; Learning Theory; Cognition

Steffe and Thompson's (2000) elaboration of the teaching experiment methodology has gained traction in recent years as a scientific methodology for constructing models of students' mathematical thinking and learning. The teaching experiment methodology is grounded in radical constructivist epistemology, and the methods and procedures that comprise it reveal its constructivist roots. However, the constructivist foundations of both the experimental and analytical aspects of the methodology are often overlooked or, at a minimum, are not explicitly addressed in reports of studies that employ teaching experiments. We have especially noticed that while researchers may strictly adhere to the experimental practices of teaching experiments, the analytical practices essential to the methodology are less rigorously observed. We conjecture that researchers' inattention to particular analytical procedures of teaching experiments derive, at least in part, from their lack of attention to, or understanding of, the constructivist foundation on which these analytical procedures are based. A particularly common analytical practice of teaching experiments that researchers often overlook is the necessity of attending to *reflexivity*, a critical discernment and communication of the researcher's role in the constitution of his or her model of students' mathematical thinking and learning. Accordingly, the aim of the present paper is to present a framework for what it means to attend to reflexivity in the context of conducting teaching experiments. We take the position that justifying the need for attending to reflexivity, and conceptualizing what is involved in doing so, results from an understanding of specific constructivist premises that underlie the teaching experiment methodology.

Teaching Experiment Methodology

The principal aim of a teaching experiment is to construct a model of another's mathematical thinking and learning. It is important to note that such a model is not a direct representation of another's mathematics, but is rather a characterization of plausible conceptual operations from which his or her observable actions may have derived. It is thus the goal of a teaching experiment for a researcher to construct a model of another's mathematics that is viable with the researcher's interpretation of his or her observable behaviors. To achieve this goal, Steffe and Thompson (2000) designed the teaching experiment methodology to provide researchers with opportunities to experience and make sense of students' mathematical learning and reasoning, both in real time and retrospectively.

A teaching experiment, Steffe and Thompson explain, is "primarily an exploratory tool, derived from Piaget's clinical interview and aimed at exploring students' mathematics" (2000, p. 273). While the intent of a clinical interview is to understand students' current knowledge, teaching experiments, in contrast, are aimed at investigating students' progress over multiple teaching episodes. Teaching experiments, therefore, allow researchers to investigate student learning, which involves the modification of students' current cognitive schemes, as they engage in mathematical activity. In the context of a teaching experiment, the schemes that students construct through spontaneous development are brought forth through exploratory teaching, and the interest of the researcher is to discern how students modify their cognitive schemes as they encounter specific teaching actions.

In a teaching experiment, the researcher generates a major hypothesis at its outset and returns to this major hypothesis retrospectively at the conclusion of the teaching episodes. In addition to testing a main research hypothesis, the researcher continually generates and tests sub-hypotheses within and among teaching episodes. These sub-hypotheses are tentative models of students' mathematical realities that seek to explain the specific actions or utterances the researcher observes. Accordingly, the researcher develops these sub-hypotheses by attending to students' language and actions, and abductively postulating meanings that may lie behind them.

"Teaching" in the context of the teaching experiment methodology is a dynamic interaction informed by an evolving model of students' mathematics. Accordingly, learning how to productively interact with the participant is an instrumental component of conducting a teaching experiment. There are two complimentary types of interaction between the teacher-researcher and the student in a teaching experiment: (1) *responsive and intuitive interaction*, and (2) *analytical interaction*.

In responsive and intuitive interactions, the teacher-researcher is usually not explicitly aware of how or why she acts as she does and the action appears without forethought; the researcher acts without planning the action in advance of the action (Steffe & Thompson, 2000, p. 278).Steffe and Thompson define analytical interaction as "an interaction with students initiated for the purpose of comparing their actions in specific contexts with actions consonant with the hypothesis" (2000, p. 281). Between teaching episodes, the teacher-researcher develops a hypothetical model of student thinking and defines initial goals. The teacher-researcher interacts responsively and intuitively prior to constructing this hypothetical model of student thinking. In the teaching episode that follows the teacher-researcher's development of the hypothetical model of student thinking and the initial goals, he or she interacts in an analytical manner, extending and articulating the initial goals and revising the hypothetical model of student thinking. This process repeats itself until a mature living model of students' mathematics emerges.

Framework for Attending to Reflexivity

Constructing models of students' mathematical thinking and learning in the context of conducting a teaching experiment requires the researcher to construct a model of the mathematical knowledge students bring to the instructional context and to design and/or select mathematical tasks that will allow students to construct the understandings the researcher envisions. Both of these aspects of constructing models of students' mathematical thinking and learning are fashioned by the researcher's theory of learning as well as his or her mathematical knowledge. It is therefore important that the researcher explicate these two aspects of his or her cognition. While conducting a teaching experiment, the researcher engages in responsive and intuitive interaction with students as they engage in the mathematical experiences in order to elicit observable products of their reasoning. Effectively eliciting observable products of students' reasoning requires that the researcher's actions be informed by a model of students' emerging ways of understanding and ways of thinking. The researcher constructs this provisional model through analytical interaction. The researcher's interaction with students plays a significant role in students' behaviors from which the researcher constructs his or her model of the students' mathematical thinking and learning. Explicating the demands of the researcher on constructing models of students' thinking and learning reveals the various ways that attention to reflexivity is warranted in the context of conducting teaching experiments. We illustrate these four areas of attending to reflexivity in Figure 1 and offer concrete

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recommendations for how a researcher may attend to reflexivity in each of these areas. These recommendations derive principally from the constructivist assertion that researchers do not explain phenomena, rather researchers explain their *interpretation* of phenomena.

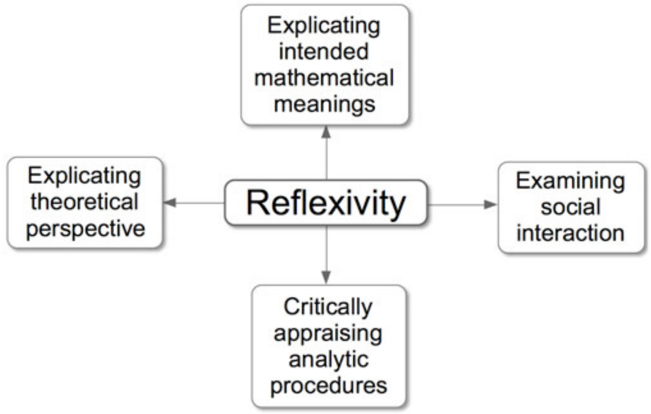

Figure 1. Four Areas of Attending to Reflexivity

Explicating One's Theoretical Perspective

von Glasersfeld (1995) developed the psychological learning theory of radical constructivism as an elaboration of Piaget's genetic epistemology (1971, 1977). The "radical" qualifier emphasizes von Glasersfeld's position that cognitive processing is the foundation of the only reality an organism may come to know. Accordingly, researchers do not have unfettered access to the phenomena they observe and thus must make sense of such phenomena through a variety of interpretative lenses. The purpose of defining and adhering to a theoretical perspective is to attempt to view the world in a systematic and disciplined way that can be communicated. It is important to note that researchers who do not conduct their work by adhering to a particular theoretical orientation have no more direct access to the phenomena they observe than those who do. Not adhering to a particular epistemological stance does not liberate one from perceiving phenomena through a number of subjective interpretative lenses; these lenses are simply not explicit and are thus unavailable to the researcher's conscious awareness. The utility of adhering to an explicit theoretical orientation, then, is that it allows one to become aware of at least some of the interpretative lenses through which he or she views the phenomenon under investigation, thereby giving one agency over these interpretative lenses.

Generally speaking, the role of theory in educational research is to orient and constrain the researcher's attention to those causal variables assumed to be most fundamental to explaining a particular phenomenon, thereby making the complex phenomenon under investigation accessible to empirical study. The theoretical perspective one assumes serves as a lens through which one is able to "control" specific aspects of the phenomenon he or she investigates so as to permit the construction of a viable characterization of a system in a particular state, or of a system undergoing transformation. In other words, researchers adopt theoretical perspectives in an effort to isolate what their theoretical perspective prescribes as the causal variables with the most explanatory power. In this way, our theoretical perspectives impose a set of assumptions and expectations about the phenomena we study that serve "to constrain the types of explanations we give, to frame our conceptions of what needs explaining, and to filter what may be taken as a legitimate problem" (Thompson, 2002, p. 192).

In order for others to ascertain the conceptual origins of a researcher's model of students' mathematical thinking and learning, the researcher must explicate the theoretical suppositions on which her or his work is based. Doing so allows others to discern the interpretive lenses through

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which the researcher views the phenomenon she studies and to evaluate the researcher's theoretical justification for what she is studying and how she studies it.

Our proposal is for researchers to make explicit their understanding of the background theoretical orientations (e.g., constructivism, sociocultural theory, situated cognition, cognitive information processing theory, embodied cognition) that orients them to framing their work in particular ways, and which conditions how researchers interpret the phenomena they study. It seems to us that, at a minimum, researchers constructing models of students' thinking and learning should address questions like, "What is knowledge? What is learning? What is the process by which one learns? What constitutes evidence of learning? How can one engender learning?" Making explicit one's answers to questions like these allows others to infer aspects of the researcher's role in the constitution of his or her model of students' mathematical thinking and learning.

Explicating Mathematical Meanings

As previously emphasized, a core consideration of conducting a teaching experiment is constructing models of students' initial and emerging mathematical knowledge. In a teaching experiment the researcher interprets what students say and do through the lens of his or her mathematical understandings and creates inferences about students' knowledge through those interpretations. It is in this way that one's own mathematical knowledge very much constitutes an interpretative framework. It is therefore important that a researcher specify what it means to understand the mathematical concept for which he or she is attempting to model students' understandings and to anticipate a multiplicity of ways of understanding this idea. Doing so allows the researcher to expand his or her mathematical interpretative lens so as to accommodate for a variety of students' observable actions in order to construct a viable model of students' mathematical thinking and learning.

Steffe and Thompson (2000) note that the goal of teaching experiments is not that students will come to see an idea as the researcher or teacher does. Instead, it is important that a researcher's model of students' learning represent a reasonable development of a student's thinking given his or her initial mathematical knowledge. At this stage, the researcher might consider a number of questions to explore these issues:

- What is my understanding of the idea that is the focus of the teaching experiment?
- In what way do I intend students understand this idea?
- What understandings do I assume students have at the onset of the teaching experiment? In what ways do I expect these initial understandings to support or inhibit the students' learning the idea that is the focus of the teaching experiment?
- What are the principles on which my design of the activities within the teaching experiment is based?
- How do I anticipate the activities I have developed will support students in constructing the meanings I intend?

Consideration of these questions allows the researcher to expand the interpretative lens through which he or she views the students' mathematics as well as recognize important mathematical understandings that might be surprising or different from his or her own. Addressing these questions also pushes the researcher to articulate the understandings he or she assumes students have at the beginning of the teaching experiment. By documenting these issues during the design of a teaching experiment (often prior to working with students) the researcher creates a record of his or her initial hypotheses. These hypotheses later serve as a means to consider how the researcher's ways of thinking changed in tandem throughout the teaching experiment with the students' mathematical thinking.

Critical Examination of Social Interaction

To suggest that researchers' examination of the social interaction between themselves and the students is an important aspect of attending to reflexivity in the context of conducting a teaching experiment is to suppose that developments in students' knowledge, and thus the models of students' learning that researchers construct to account for such developments, are conditioned by social interaction. We must therefore explain how a researcher's construction of a model of students' mathematical thinking and learning is fashioned by the social interaction that occurs between a researcher and students during the teaching experiment episodes.

From a constructivist perspective, an individual's enacted knowledge is fashioned by his or her understanding of the stimuli inherent to a particular environmental context—stimuli that are often mediated by social interaction. Accordingly, the model of students' mathematical thinking and learning a researcher constructs in the context of a teaching experiment is very much influenced by the social interaction that conditions the evocation of students' mathematical knowledge. Consistent with our recommendation for researchers to explicate their theoretical orientation, we now turn to providing a justification, grounded in constructivist epistemology, for the claim that students' knowledge, and thus the model a researcher constructs to account for its development, is fashioned by the social interaction that occurs in the context of a teaching experiment.

The evocation of specific knowledge is contingent upon an individual interpreting stimuli that activate particular cognitive schemes. In other words, certain knowledge is not made manifest until an individual interprets a certain stimulus in such a way that his or her construction of the stimulus serves as a cue for the enactment of a particular cognitive scheme or a network of related schemes. Therefore, while many believe knowledge is invariantly accessible across time and space, we consider knowledge to be the set of one's cognitive schemes that may become operational in the space of stimuli in which the individual is situated that may enact these schemes. Accordingly, one's ways of perceiving his or her environmental context constitutes a space of stimuli that maintain the potential to make a subset of one's knowledge operational. An individual's knowledge, then, can be thought of as the set of cognitive schemes that *may* become enacted as a consequence of the individual's interpretation of the stimuli inherent to a given environmental context. It is therefore appropriate to say that enacted knowledge is conditioned by the individual's interpretation of his or her environmental context. This is not to suggest that knowledge resides external to the knower since individuals *interpret* and *appraise* their environmental context—interpretations and appraisals that inform the knowledge one employs. To speak of enacted knowledge, then, is to speak of the cognitive schemes that become operational upon one's interpretation of stimuli in a given environmental context that serve to activate such cognitive schemes.

Given our view that knowledge is conditioned by an individual's interpretation of his or her environmental context, we contend that the observable actions students demonstrate in the context of a teaching experiment derive from enacted knowledge that is influenced by their interpretation of their interaction with the researcher. What students make of the researcher's language and actions constitute environmental stimuli that may support or inhibit specific cognitive schemes from being activated. For example, if a student interprets a researcher's questioning as suggesting that the researcher simply wants the student to recite the correct answer, the student may be disinclined to engage in sustained reasoning and sense making. We have ourselves observed students demonstrating very different mathematical knowledge after simply switching the interviewer in a teaching experiment session or task-based clinical interview (cf. Thompson & Thompson 1994). For this reason, it is important for a researcher to discern how students perceive his or her language and actions.

To this end, there are a couple of specific practices to consider. Regularly asking students to verbalize their interpretation of the researcher's questions and statements allows one to obtain artifacts of students' image of their social interaction with the researcher. For instance, asking students questions such as, “What is your understanding of what I just said?” and “Can you tell me in your own words what am I asking you to do?” allows the researcher to gain insight into how students are interpreting their interaction with the researcher. Discerning how a researcher’s instructional actions support students in attaining the understandings that the researcher seeks to promote is an essential to constructing viable models of students’ mathematical thinking and learning. It is important at this point to reiterate that a researcher’s instructional actions do not influence students’ thinking. Rather, students’ *interpretation* of a researcher’s instructional actions influence students’ thinking. It is for this reason that, in addition to attempting to elicit observable products of students’ reasoning, researchers should consistently attempt to provide occasions for the students to convey their interpretation of the researcher’s language and actions.

Critical Examination of Data Analysis

While conducting a teaching experiment, the model a researcher constructs of students’ emerging mathematical knowledge is fashioned by his or her ways of perceiving and conceiving students’ observable behaviors. Mason’s (2002) observation eloquently summarizes this point: “what we learn from an observation is something about the researcher, as well as, perhaps, something about the phenomenon” (p. 181). It is therefore imperative for researchers to document and explicate the decisions and interpretations they make throughout their construction of a model of student’s mathematical thinking and learning, and to detail the evolution of this model throughout the research process.

While conducting a teaching experiment, data analysis proceeds in a cyclic fashion whereby the researcher continually generates and refines hypotheses until a stable and viable inductively-derived theory regarding the process by which students construct a desirable understanding of some mathematical idea emerges. Refining provisional hypotheses requires purposeful data collection informed by ongoing analysis. Thus, a hallmark of the teaching experiment methodology is the reciprocal relationship between data collection and analysis; that is, while constructing models of students’ mathematical thinking and learning, the data a researcher collects influences the hypotheses he or she constructs, and the hypotheses a researcher constructs informs subsequent data collection. Hence, the boundary between data collection and analysis is necessarily blurred when one conducts a teaching experiment. For this reason it is important for researchers to critically examine how they interpret data during ongoing analysis so that they may ascertain their role on subsequent data collection and, ultimately, on the model of students’ mathematical thinking and learning they construct.

Researchers’ attention to the role of their interpretation of data on the construction of their model of the process by which students may understand a particular mathematics concept not only clarifies the phenomenon under investigation but also details researchers interaction with the phenomenon. Researchers’ documenting their decisions and interpretations during ongoing analysis is important because it produces a form of data about their interaction with students that they can then use in subsequent analyses to bring into conscious awareness the ways in which their interaction with the subject informed their interpretation of students’ language and actions.

To discern the role of one’s interpretation of data on the model one constructs to account for students’ mathematical thinking and learning, a researcher may consider creating artifacts of his or her thinking during ongoing analysis in the form of audio recordings or written memos that focus on the following:

1. Explicating hypothetical conceptual operations that may explain the researcher’s interpretation of the students’ language and actions throughout testing the viability of an emerging model of students’ thinking and learning;

2. Identifying students' specific utterances and actions that contributed to the researcher's construction of these hypothetical conceptual operations;
3. Explaining how the researcher interpreted these utterances and actions so as to make their contribution to his or her model of the students' conceptual operations explicit;
4. Justifying the researcher's instructional actions throughout implementing the instructional sequence and describing his or her interpretation of students' responses to these instructional actions.

It is clear that a researcher's responses to foci (2), (3), and (4) constitute a data set that he or she may retrospectively analyze for purpose of providing insight into the conceptual origins of his or her model of students' mathematical thinking and learning. This data set comprises a record of a researcher's interpretation of the primary data, and the inferences the researcher drew from this data, throughout ongoing and post analysis. One's retrospective analysis of these analytical artifacts is in the service of elaborating a chronology, presented in narrative form, of the development of his or her model of students' thinking and learning by explicating the interactional, institutional, emotional, discursive, theoretical, epistemological, and ontological influences that contributed to its construction (Mauthner & Doucet, 2003). This chronology allows a researcher to present his or her model of student's thinking and learning not just as a product but also as a process, and not by an impersonal machine but by a researcher abounding with subjective interpretative lenses.

Teaching Experiment Literature Analysis

To examine the utility of the analytical framework presented earlier, we examined nineteen of the most highly cited studies in mathematics education that employed a teaching experiment methodology, beginning two years after Steffe & Thompson's (2000) work was published. These articles came from all of the top journals in mathematics education, and covered a wide variety of mathematical topics. We constructed a coding scheme, the components of which corresponded to the four domains of our framework for attending to reflexivity. In our initial coding, we found that less than half (8/19) of these studies met the "explicating mathematical meanings" criterion; just over half (11/19) clearly identified a theoretical perspective; most (18/19) examined social interaction in some way; and roughly half critically examined the data analysis procedures (11/19). While this analysis is in its initial stages, it clearly illustrates important role these components of our framework play in the highest quality studies in our field, yet suggests that there are ways in which the analytical practices of the teaching experiment studies could be expanded. We also recognize that in some cases, the limitations of a journal space and the review process shape the focus of each paper. Yet we think it is important that the field begin to critically appraise what elements must be present in the presentation of results based on a teaching experiment and to include the four components we have presented in this paper.

Conclusion

In this paper, we have proposed four ways in which researchers may attend to reflexivity in the process of conducting teaching experiments in mathematics education. A reviewer of a previous version of this manuscript suggested that each of the four domains of reflexivity we discuss appear as analytical recommendations elsewhere. We respond by noting that such recommendations have not been rationalized with an appeal to the foundational theoretical premises on which specific qualitative methodologies are based. Such rationalizations are essential to supporting researchers in observing the analytical practices that comprise attention to reflexivity in non-superficial ways. It is for this reason that we consider the present paper a contribution, and encourage other researchers to explicate what it means to attend to reflexivity in the context of other qualitative methodologies informed by other theoretical orientations.

Attending to reflexivity is an essential component of the teaching experiment methodology and involves researchers in a systematic and disciplined investigation of how their interpretations and actions influence the models and theories they construct to explain students' mathematical thinking and learning. The recommendations we outlined in this paper assist researchers in becoming consciously aware of at least some of the subjective interpretative lenses through which they perceive the phenomena they study, thereby affording researchers agency over these interpretative lenses. When researchers bring into conscious awareness the lenses through which they make sense of data, they are positioned to communicate the products of their research in way that reveals that their results and conclusions are not about a particular phenomenon, but are instead about their *interpretation* of the phenomenon. Communicating the products of one's research in this way lends a transparency to the research process, thereby inviting others to scrutinize the origins of the models and theories presented in the literature, thereby fulfilling one of the necessary conditions for a scientific enterprise.

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