# EXPLORING ROLES OF COGNITIVE STUDIES IN EQUITY: A CASE FOR KNOWLEDGE IN PIECES

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*Underrepresentation of people of color in mathematics at the postsecondary level warrants more focus on equity issues. The prevalence of cognitive studies at the undergraduate level is met with the call for critical analysis about the kinds of knowledge that get privileged in mathematics education. Connecting to the Funds of Knowledge work, this paper discusses the utility of diSessa's Knowledge in Pieces cognitive framework to uncover productive informal knowledge in learning formal mathematics. Seeing the valorization of knowledge as related to issues of power, a case of a Chicana student's productive sense making about the formal definition of a limit illustrates the way diSessa's framework can help challenge what counts as productive mathematical knowledge and reasoning.*

Keywords: Learning Theory; Advanced Mathematical Thinking; Equity and Diversity; Cognition

> Whether parents, teachers, students, or researchers, we all bring valorization of knowledge to our views of what counts as "proper" or "better" approaches to doing mathematics.
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> –Marta Civil (2014, p. 12)

The President's Council of Advisors on Science and Technology (PCAST) called for 1 million additional college graduates in science, technology, engineering, and mathematics (STEM) fields based on economic forecasts (Executive Office of the President, PCAST, 2012). Within STEM, the number of mathematics degrees being conferred continues to be generally low. People of color, in particular, continue to be severely underrepresented in mathematics. A recent National Science Foundation report shows that historically marginalized population accounts for 20% of the mathematics degrees conferred (NSB, 2014).

Gutiérrez (2002) characterizes equity research as one that explicitly focuses on efforts to understand and mitigate systematic differences in how people experience educational opportunities, particularly differences that privilege one group over another. In the broader area of undergraduate STEM education, some studies have begun to explore issues of marginalization of women and students of color and ways that students manage it in their everyday lives (e.g., McGee & Martin, 2011). However, there has not been a sustained and concerted effort to focus on equity considerations in undergraduate mathematics education research (Adiredja, Alexander & Andrews-Larson, 2015), like there has been in K-12 mathematics education research in recent years (Gutiérrez, 2013).

The need to understand how students make sense of challenging topics in undergraduate mathematics contributed to the prevalence of studies about students' and teachers' individual cognition and practices (Adiredja et al., 2015). Several scholars have argued for critical analysis of the kinds of knowledge and practices that we privilege in teaching and learning of mathematics (Civil, 2014; Gutiérrez, 2013). Considering the socio-political nature of education, Gutiérrez (2013) emphasized the interconnectedness of knowledge and power:

> Knowledge and power are inextricably linked. That is, because the production of knowledge reflects the society in which it is created, it brings with it the power relations that are part of society. What counts as knowledge, how we come to "know" things, and who is privileged in the process are all part and parcel of issues of power.

For example, children of immigrant parents at times would discount their parents' mathematical knowledge as a result of the way things are taught in US schools (Civil and Planas, 2010). This

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example illustrates one explicit implication of power vis-à-vis the kinds of knowledge students learn to be valued in the classroom. The question then becomes how can we begin to unpack issues of power behind our valorization of mathematical knowledge and approaches, particularly at the undergraduate level?

At the K-12 level, some researchers have responded to this question by trying to leverage and build on cultural aspects of the students' communities in designing curriculum (Moll, Amanti, Neff & Gonzalez, 1992). The goal with the Funds of Knowledge project (Moll et al., 1992) is to help teachers explore and document the often invisible but productive knowledge of non-dominant students. While it is tempting to assert that the nature of mathematics at the undergraduate level is different from that of K-12 mathematics, I posit that the spirit of the Funds of Knowledge work can also be productive at the post-secondary level.

The kind of mathematics that is being learned at the postsecondary level does become increasingly abstract and are often built on prior formal mathematical knowledge. Moreover, the Euro-centricity of mathematics (Joseph, 2010) becomes even more privileged at the post-secondary level. As such one might argue that these realities of mathematics at the post-secondary level limit opportunities for teachers to connect to students' informal knowledge and experiences. For example, in the context of limit in calculus, some researchers have argued that students' intuitive knowledge is an obstacle in learning (Davis & Vinner, 1986), despite the prevalence of such knowledge student thinking about the concept (Monaghan, 1991).

While the Euro-centricity of advance mathematics is a reality, students' intuitive knowledge about mathematics, and more importantly ways that such knowledge might be productive in learning formal mathematics, are largely underexplored in undergraduate mathematics education. Wawro et al. (2012) have shown a case where incorporating students' intuition about using different modes of transportation in traveling is productive to explore concepts in linear algebra. This study suggests that there is room for intuitive knowledge in learning mathematics at the undergraduate level. Recognizing the potential utility of students' informal knowledge, and its potential in disturbing power distribution in the classroom, where can we find such knowledge? What might be able to assist us in uncovering many of the invisible productive knowledge students bring with them to learn formal mathematics?

This paper attempts to offer a response to those questions, particularly in the context of undergraduate mathematics education. This paper argues that the theoretical perspective on epistemology and learning in cognitive studies plays an important role in uncovering knowledge resources students bring into learning formal mathematics. In particular, this paper argues for the utility of the Knowledge in Pieces cognitive framework (diSessa, 1993) to recognize productive knowledge resources students might use in learning formal mathematics, particularly in the face of non-normative language. The analysis shows how this framework has a potential of challenging the existing distribution of power vis-à-vis what counts as productive knowledge.

## Theoretical Framework for Cognition

The Knowledge in Pieces (KiP) theoretical framework (diSessa, 1993; Smith, diSessa and Roschelle, 1993) models knowledge as a system of diverse elements and complex connections. The nature of the elements, their diversity, and connections are typical interests for studies using this framework. Characterizing knowledge using generic ideas like "concept" or the commonly used idea of "misconceptions" is viewed as uninformative and unproductive (Smith et al., 1993). Instead, KiP focuses on the context specificity of knowledge to maintain the *productivity* of the particular piece of knowledge. KiP also pays particular attention to the continuity of knowledge, i.e., ways that knowledge gets used or built upon in new contexts. It is common for studies using this framework to uncover productive sense making behind students' use of non-normative language to describe their reasoning (e.g., Campbell, 2011; diSessa, 2014).

One of the main principles of KiP is that knowledge is context specific (Smith et al., 1993). This means that the productivity of a piece of knowledge is highly dependent on the context in which it is used. Context variation can happen as a result of change in the literal problem context, the passage of time, or simply as knowledge is assessed more or less carefully. For example, the knowledge that "multiplication makes a number bigger" is productive in the context of multiplication with numbers larger than one. The knowledge is not productive in the context of multiplication with all real numbers. In contrast to studies that focus on identifying students' misconceptions, KiP focuses on building new knowledge on students' prior knowledge, instead of focusing on efforts to "replace" students' misconceptions (Smith et al., 1993). Adopting this theoretical framework implies that the analysis in this paper will focus on ways that students build on their prior ideas while suspending judgment about their correctness. KiP also posits that students have a lot of intuitive ideas that can be leveraged in instruction. KiP was developed in the context of physics where students have a diversity of intuitive ideas about physics originating from their everyday experience. Some studies have shown that intuitive ideas can also be found in mathematical reasoning as well (e.g., Campbell, 2011).

## Mathematical Context and Literature

The formal definition of a limit of a function at a point, also known as the epsilon-delta ($\varepsilon$-$\delta$) definition, is an essential topic in mathematics majors' development that is introduced in calculus. The limit of a function $f(x)$ as $x$ approaches $a$ is $L$ and is written as $\lim_{x \to a} f(x) = L$ if and only if, for every positive number $\varepsilon$, there exists a positive number $\delta$, such that all numbers $x$ that are within $\delta$ of $a$ (but not equal to $a$) yield $f(x)$ values that are within $\varepsilon$ of the limit $L$. This defining property is often written as "for every number $\varepsilon > 0$, there exists a number $\delta > 0$ such that if $0 < |x - a| < \delta$ then $|f(x) - L| < \varepsilon$". Informally, one might say, "If $L$ is the limit, then for however close one wants $f(x)$ to be to $L$, one can constrain the $x$-values so that $f(x)$ would satisfy the given constraint." We return to this intuitive idea shortly.

The formal definition provides the technical tools for demonstrating how a limit works and introduces students to the rigor of calculus. Yet even thoughtful efforts at instruction leave students, including intending and continuing mathematics majors, confused or with at most a procedural understanding about the formal definition (Cottrill, Dubinksy, Nichols, Schwingendorf, and Vidakovic, 1996; Oehrtman, 2008). Many studies assert that students' dynamic conception (the limit is the number that $f(x)$ approaches as $x$ approaches $a$) is an obstacle in learning the formal definition (Parameswaran, 2006; Williams, 2001). These studies largely focused on the unproductivity of students' prior conception and their sense making.

In the meantime, a small number of studies that focuses on students' sense making of the formal definition (Knapp & Oehrtman, 2005; Roh, 2009; Swinyard, 2011) suggest that students' understanding of a crucial relationship between two quantities featured in the formal definition, epsilon ($\varepsilon$) and delta ($\delta$), warrants further investigation. Davis and Vinner (1986) used the term *temporal order* to describe their relationship. While studies have shown the existence and prevalence of this particular difficulty, its nature is largely underexplored.

The relationship between the quantities $\delta$ and $\varepsilon$ in the definition can be described using the idea of quality control in manufacturing an item. The conceptual structure at issue can be described as follows: given a permissible error in the measurement of the output ($\varepsilon$), one determines a way to control the input to achieve that result. One does so by determining the permissible error in the measurement of the input ($\delta$) based on the given parameter for the output ($\varepsilon$). In this way, the error bounds follow the following sequential order, error bound for the output, then the error bound for the input. This is because the error bound for the output is given. In some ways, the error bound for the input could be seen as being dependent on the given error bound for the output. Epsilon can be seen as the error bound of the output whereas delta is the error bound for the input. Therefore, $\delta$ and

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$\varepsilon$follow the order of $\varepsilon$ first, and then $\delta$, or $\delta$depends on $\varepsilon$. In this paper, the student discussed this idea of quality control in the context of working at a pancake house that is known to make 5-inch diameter pancakes. Students are given a permissible error for the size of the pancakes, and they were responsible to control the error in the amount of batter.

## Data Collection and Analysis Methods

The data presented in this report is a case study from a larger interview study investigating the role of prior (and intuitive) knowledge in student understanding of the temporal order of epsilon and delta within the formal definition (Adiredja, 2014). Participants of the study were calculus students at a large Western public research university. Students were interviewed about their understanding about the temporal order. They were asked a series of questions about the temporal order before and after engaging with the instructional intervention. The instructional intervention, the Pancake Story uses the context of working at a pancake house to leverage the idea of quality control in discussing the formal definition, as explained in the previous section. A video recording of the interview was transcribed following Ochs' (1979) guidelines. Transcripts were organized by turns, marked changes in the speaker. They included non-verbal behaviors, including relevant gazes, laughter and gestures. Turns that discus one mathematical argument make up an episode. The transcript was modified to facilitate reading. Many hedges, and uh-huh's and um-hm's from the interviewer were removed.

Adriana, the focus of the analysis of this paper, was a mathematics and Chicano studies major. She ethnically identified as Chicana. Adriana received an A in her first semester calculus course in high school and in college. She was selected because despite her strong academic background, even after engaging with the Pancake Story, she still initially (and incorrectly) argued that $\varepsilon$ depended on $\delta$. Ultimately, Adriana adopted many of the productive resources from the story and used them to reorganize her knowledge and modify her claim. The analysis was interested in understanding how and why she did so. The analysis did not explore ways that Adriana's identity as a Chicana influenced her reasoning about the topic. Her ethnicity and gender were included to better represent her as a student and challenge the common unintended assumption with cognitive studies that the student is a White male student (Nasir, 2013).

The analysis focused on identifying a knowledge entity called *knowledge resources* (Adiredja, 2014), which is defined as a single or a collection of knowledge elements that might be involved in making a single claim from larger ideas that the student used to make her claim. Knowledge resources were assumed to be neutral; they are not correct or incorrect. This theoretical assumption distinguished knowledge resources from larger ideas that were combinations of several knowledge resources. To identify knowledge resources, the analysis exploit any relevant data (e.g., gestures, other parts of transcripts) that might inform the aim to optimally understand the activation of knowledge resources in various contexts. The analysis then generated multiple models (interpretations) of the student's argument in each episode. The analysis then put these models of student thinking in competition with one another. This process of *competitive argumentation* (VanLehn, Brown, & Greeno, 1984) was used to refine interpretations of student thinking. In this paper I only present the final model of each episode that was the result of the process of competitive argumentation.

## Results

This paper only presents two of the four episodes of Adriana's sense making: the first and final episode. These episodes illustrate the changes in Adriana's thinking and salient ideas from the Pancake Story. They also show the initial conflict that Adriana faced in aligning the ideas from the story with her prior knowledge. The four episodes occurred on the span of 14 minutes.

The first episode started with the interviewer's asking Adriana about the dependence between$\delta$and $\varepsilon$ after they discussed the Pancake Story. Adriana responded with the same [incorrect]

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claim she made before she engaged with the story. She argued that $\varepsilon$ depended on $\delta$ because epsilon was with $f(x)$ and delta was with $x$ and $f(x)$ depended on $x$. The bolded texts marked the ideas that from which knowledge resources were identified.

> *Adriana*: [They kinda depend on each other], yeah in a sense because, but more whatever you're getting, like $f(x)$ is always gonna depend on what $x$ you're inputting it. But then, if you want to get something that's within delta [*marks a small interval on the x axis with two fingers*] you need to see if /.../ for example here [*points to the pancake story*] our epsilon here was already set, then that [*points back and forth between* 4.5 *and* 5.5 *in the inequality* $4.5<f(x)-L<5.5$] kind of depended on what we were putting in for $x$ [*points at the same interval around x on the graph*] but..but mostly whatever you're putting in to your $x$ is gonna determine what you get for $f(x)$ [*pause*]. So I'm still saying the same thing like delta depends on epsilon but=
>
> *Interviewer*: =Delta depends on epsilon? Or epsilon depends..
>
> *Adriana*: No, yeah, epsilon depends on delta. But, /…/ if epsilon's already set then you'll manipulate your /…/ delta so it's within an error bound and /…/ then continue to manipu- wait [*long pause*] wait, so you're… hm.

*Final model of episode 1*: Adriana focuses on her prior claim that epsilon depends on delta. She justifies the claim using *functional dependence* and *function slots* resources. She simultaneously brings up many of the productive resources from the story: *domain constraint for a limit, the givenness* of epsilon and *quality control.* However, these resources are in conflict with her prior conception. Adriana's use of the knowledge resource of *functional dependence* can be seen in her statement, "whatever you're putting in to your $x$ is gonna determine what you get for $f(x)$." Analysis of the final episode revealed that in this episode, Adriana thought about epsilon and delta as a range of errors, i.e., $x$ and $y$ values, instead of error bounds for those values. This suggests that along with *functional dependence,* Adriana uses the knowledge resource *function slots*, i.e., the assumption that when two quantities share a functional relationship, one quantity is the $x$ and the other is the $f(x)$ or the $y$.

Separate from her previous argument, Adriana also mentioned the idea that the epsilon (the error bound for the pancake) was given (*givenness* knowledge resource) in the story, and that she wanted to get "something" that was within delta. That statement reveals Adriana's preference of only considering $x$ values that are close to $a$ in discussing limit problems. She would control that closeness by choosing $x$ values that were within a small delta. This suggests her use of the knowledge resource *domain constraint for a limit*. The last line of the episode suggests that Adriana might have also taken up the idea of *quality control*: for a given specification on the output, one would manipulate the input so the output would be within the specified error bound. The sentence also reveals her use of the *dynamic conception of a limit* when she talked about continuing to manipulate the delta to get $x$ closer and closer to $a$. She also erroneously talked about wanting delta to be within an error bound, which is consistent with the interpretation that delta in this episode as a range of $x$ values.

In the final episode, Adriana repurposed the *functional dependence* resource to describe the relationship between the errors but not the error bounds. This productive move helped Adriana to align productive resources from the story with Adriana's prior knowledge. She then prioritized the idea of *givenness* of epsilon and *quality control*, which she already knew since episode 1,to conclude that delta depended on epsilon. She also adopted the story's language.

> *Interviewer*: So, do they depend on each other, is it just one way now?
>
> *Adriana*: Um, see cus I was looking at it like /…/ **the f of x [$f(x)$] depends on the $x$** and that's how I was like saying that epsilon depends on delta because **epsilon is related to the f of x**

[$f(x)$]/…/. But that's just saying **the error of the *L* and the f of x [$f(x)$] depends on the *a* and *x*** but that's not to say that epsilon depends on delta.

*Interviewer*: Ok, so?

*Adriana*: So, I think that delta depends on epsilon now [*laughs*]. Just cus if it's given like this [*reference unclear*] and **you're trying to aim at getting /…/ within a certain error bound**, then you're gonna **try to manipulate your entries /…/ to be within a certain error bound** [*gestures a small horizontal interval with her palms*]

*Interviewer*: Ok. Alright, so and so you changed your mind it seems? Um, so how did that happen? Why did you change your mind?

*Adriana*: Because I was **given an epsilon** [*points at the inequality* $4.5<f(x)–L<5.5$] and that's kinda like the main goal. **The main goal is to get the pancake, /…/ and they gave me a constraint** /…/ and /…/ they **didn't give me an error bound for the batter** or for like the *a* or *x*, they didn't give me an error bound. But I know I **want to make it small so that it's within the error bound, the epsilon**. So then **I would kinda base my delta on what was epsilon**.

*Final model of episode* 4: Adriana uses *functional dependence* to describe the relationship between the errors ("But that's just saying the error of the *L* and the $f(x)$ depends on the *a* and *x,* but that's not to say that $\varepsilon$ depends on $\delta$."). To determine the dependence relationship between $\varepsilon$ and $\delta$, she prioritizes the resource *givenness* of epsilon and *quality control*, as seen in her statement, "you're trying to aim at getting within a certain error bound, then you're gonna try to manipulate your entries to be within a certain error bound." Adriana also made the productive observation that delta was not given, showing her use of the *givenness* resource with delta as well. More importantly, not only did Adriana treat delta as an error bound, she also stated that she wanted delta to be *small*. By making delta small, Adriana was no longer using the idea of smaller and smaller or "continuing to manipulate" the input errors that suggests the use of *dynamic conception of a limit* in the first episode.

## Discussion

In summary, the analysis revealed that Adriana took up many of the productive resources from the story despite its initially looking as if her understanding seemed unchanged. It took effort for Adriana to align the new productive resources from the story with her prior knowledge. After she repurposed the *functional dependence* resource to describe a relationship between the errors, she prioritized the productive resources from the story. Adriana also used the story to make a novel observation about the temporal order (delta was not given). That move and the adoption of the language of the Pancake Story suggest that the story was a rich learning context for Adriana. The story was able to leverage Adriana's prior knowledge about *functional dependence* and *quality control* while reasoning about the temporal order.

The Knowledge in Pieces framework guided the analysis in revealing knowledge resources from the story that were salient to Adriana, as well as those that existed as part of her prior knowledge. Adriana's language in describing her conception in the first episode was not clear. However, analysis of the structure of her knowledge revealed these productive resources. The analysis was also able to recognize the productivity of Adriana's moves because KiP takes seriously the process of reorganization of knowledge. The theoretical assumptions about epistemology and learning make KiP particularly sensitivity to subtle changes in sense making and potential productive roles that students' prior knowledge can play in learning. It challenges the deficit perspective of student thinking and challenges what counts as productive mathematical knowledge and reasoning. More broadly, researchers studying student thinking wield a great deal of power in deciding what kind of knowledge is valuable, and particularly in suggesting implications to practice from the findings of the analysis. For example, cognitive studies that focus on pathologizing students' thinking might have simply characterized Adriana's return to her prior argument about the temporal order as a result of a persistent misconception. Moreover, a lot of the subtle changes and her adoption of many of the productive resources might have been easily overlooked. Thus, not only would it position her and her thinking in a deficit way, it would also fail to recognize her contribution.

The findings also show the utility of intuitive knowledge in building a conceptual understanding of formal mathematics. The spirit of the Funds of Knowledge work can be seen in the way that the Pancake Story leveraged the intuitive notion of quality control to learn about the temporal order within formal definition of a limit. At the same time, the story was designed with the KiP framework's assumptions about the potential productivity of prior knowledge and ways that knowledge is reorganized. In addition to Wawro and colleagues' (2012) work, we see another case where intuitive knowledge can be productive in learning formal mathematics.

In sum, I argue that cognitive studies can contribute to equity issues more directly by addressing issues of power vis-à-vis valorization of knowledge. In this paper, I made a case for the KiP framework, and recognize that there might be other frameworks that can help uncover non-normative but potentially productive ways of thinking with formal mathematics. This type of work would benefit all students, but would particularly benefit non-dominant students whose knowledge are often devalued or unrecognized at the post-secondary level. In the face of underrepresentation and marginalization of non-dominant students more broadly, cognitive research can play an important role in challenging issues of power in mathematics education.

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