HOW STUDENTS' INTEGER ARITHMETIC LEARNING DEPENDS ON WHETHER THEY WALK A PATH OR COLLECT CHIPS

Julie Nurnberger-Haag
Kent State University
jnurnber@kent.edu

In light of conceptual metaphor theory, historical mathematicians' and students' difficulty with negative numbers reveals that the collecting objects metaphor may be a cognitive obstacle to those first learning about negative numbers. Moreover, consistency of physical motions with targeted ideas is a factor of cognition. Thus, this pre-post-delayed post study randomly assigned 8 classes of initial learners to a collecting objects integer model (chip model) or a moving-along-a-path metaphor-based model (number line model) to learn integer arithmetic with the four primary operations during an eight-day mini-unit. The study investigated the questions: What do students demonstrate learning with each model and what, if any differences in learning are found between models? Findings did support theory that a motion-aligned model using a moving-along-a-path metaphor would likely support learning better than collecting objects.

Keywords: Number Concepts and Operations; Cognition; Learning Theories

In this paper I share results from a larger study designed to understand the effects of instructional models on students' learning about integers. This study contributes to resolving two enduring challenges in mathematics education: one practical and one theoretical. The first concerns improving the way that classroom-based research can inform teachers' practical decisions about teaching integer arithmetic. The second offers new insights into the theoretical and practical debate about whether and how physical experience supports learning mathematics. The study provides evidence about how students' physical motions or model-movements can support or interfere with mathematics learning.

Prior Research

Integer arithmetic with negative numbers is counterintuitive, yet essential to most mathematics beyond middle school. It has been extensively studied (Gallardo, 2002; Küchermann, 1981; Liebeck, 1990; Linchevski & Williams, 1999; Vlassis, 2008), yet recommendations are contradictory about how to help students adapt their arithmetic concepts to embrace negative numbers (Star & Nurnberger-Haag, 2011). Most integer research has assessed students' learning during or immediately after instruction and focused on addition and subtraction only. Given that students need to understand negative number arithmetic to build on and use in complex ways in formal algebra (Vlassis, 2008), research that investigates a larger set of integer knowledge, including all operations, along with longer-term implications of instructional experiences is crucial.

Küchermann (1981) categorized three types of integer instruction models as (a) cancellation models in which two opposites cancel, (b) number line, or (c) abstract models. Although integer thinking and learning has been extensively studied, investigations of integer learning with particular models, particularly those most accessible in classrooms, have yet to be conducted. In spite of this lack of research about student learning with different models and theoretical critiques of models offered (e.g. Star & Nurnberger-Haag, 2011; Freudenthal, 1973; Vig, Murray, & Star, 2014), multiple integer models are promoted in methods textbooks for prospective teachers as well as school textbooks, particularly cancellation and certain number line models. One study did compare a cancellation model and a number line model (Liebeck, 1990) but it did not involve parallel instruction and post-tests nor did it include a pretest. For educators to make effective instructional decisions, it is important to compare methods (Nunez, 2012) and to understand how different methods might offer similar or different learning opportunities. Consequently, the present study experimentally compared students' initial learning of negative number arithmetic using a cancellation model or a number line model.

Theoretical Perspectives

Documentation of historical mathematicians and of modern students has described cognitive obstacles for negative numbers (e.g., Fischbein, 1987; Pierson, et al., 2014). A reading of these works in terms of conceptual metaphor theory reveals that the *object collection metaphor* may be a cognitive obstacle to those first learning about negative numbers (hereafter called *initial learners)*. The conceptual metaphors identified by Lakoff and Nunez (2000) applicable to integer arithmetic include the *object collection*, *motion along a path*, and *measuring stick* metaphors. Flexible use of more than one metaphor may be necessary for expert understanding of negative numbers (Chiu, 2001). Yet, research of integer arithmetic via conceptual metaphors is in its infancy and has focused on what metaphors people used while thinking (Nurnberger-Haag, 2013; Chiu, 2001; Kilhamn, 2011), rather than the impact of metaphor-based physical motions on initial learning.

Conceptual metaphor theory offers educational researchers valuable insights to identify the ways that different models invite students to conceive of numbers. *Cancellation models* treat numbers as objects (using an object collection metaphor for negative number concepts). In schools, the cancellation model most commonly used is an integer model that uses chips of opposite colors to represent opposite numbers. In research additional models have also been promoted and studied that draw on an object collection metaphor (Nurnberger-Haag, 2013; Kilhamn, 2011). At first glance Küchermann's (1981) characterization of cancellation versus number line models might offer a sufficient framework. Or one might suspect that Fischbein (1987) and Freudenthal's (1973) explanations that thinking of numbers as objects renders use of conceptual metaphor theory moot, because this neatly aligns with the object collection metaphor. Using conceptual metaphor theory to investigate integer arithmetic learning, however, affords considering the differences between the mathematical representations and the ways we think about these representations. For example, *number line models* all use a commonly accepted representation of a number line, but there is not a single number line model. A number line representation can be thought of using a measuring stick metaphor as Descartes did historically (Berlinghoff, & Gouvêa, 2002; Lakoff & Nunez, 2000) in which numbers are found at the end of a positive length in a particular direction. Number line representations can also be thought of with a motion-along-a-path metaphor (Lakoff & Nunez, 2000; Nurnberger-Haag, 2007; Kilhamn, 2011), or as a combination of both of these metaphors to be discussed in later studies. Moreover, considering conceptual metaphor theory aides recognition that the differences of model metaphors might matter because of how these ways of thinking fit with the ways humans think more generally. In other words, conceptual metaphor theory assists with explaining and unifying mathematical thinking to show that and in what ways mathematical thinking is part of the varied abstract thinking humans do (Lakoff & Nunez, 2000).

Lakoff and Nunez treat conceptual metaphor as an object of thought that results from physical experiences, which grounds how we think about abstract ideas. Whereas other research has referred to conceptual metaphors similarly using nouns (Chiu, 2001; Kilhamn, 2011), I use the verb forms (*collecting objects*, *moving-along-a-path, and measuring*) to emphasize and transform the original claims to consider the metaphorical mechanism as grounding the patterns of interacting with the world as part of an on-going dynamic system (Nurnberger-Haag, 2014).

Furthermore, when promoting one metaphor with students, such as the moving-along-a-path metaphor on a number line, we must realize that this metaphor does not prescribe how students move on a number line. The typical number line model promoted in curricular resources directs students to move backwards or forwards depending on the sign of a number and to move in a particular direction

Bartell, T. G., Bieda, K. N., Putnam, R. T., Bradfield, K., & Dominguez, H. (Eds.). (2015). *Proceedings of the 37th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. East Lansing, MI: Michigan State University.

depending on the operation. In other words, these models inform students *which* direction to move. The walk-it-off model, in contrast, was designed to promote the opposite operator meaning of the negative sign necessary for algebra, but not afforded by other models (Nurnberger-Haag, 2007). Rather than written symbols indicating *which* direction students travel, the written symbols in the walk-it-off model emphasize to change direction by turning the *opposite direction* or not to change direction for addition or positive values.

Purpose of Study

Although broadly, hands-on learning or manipulatives have been extensively studied in mathematics education, this research has not attended to the ways the physical model-movements may represent or misrepresent the mathematics students learn. Yet, research from psychology has shown that consistency of physical motions with targeted ideas is a factor of metaphor comprehension (Glenberg & Kaschak, 2002). Thus, this study examined how two different conceptual metaphors and students' physical motions and related explanations affect initial learning of integer arithmetic in classrooms. With a goal of practical impact, I chose to compare models with which students physically represent integer arithmetic that could be or are easily implemented in schools and what I thought would be the best case of each metaphor. In order to capture the complexity of students' initial learning of integer arithmetic on all four basic operations and assess longer-term learning, this pre-post-delayed posttest study used multiple methods to address the questions: After using either a chip model, or a number line model that emphasizes opposites and magnitude, what do students demonstrate knowing about integers and what, if any, differences in learning are found between students who used each model?

Method

The most common *collecting objects* metaphor-based model (a chip model) or a *moving-along-a-path* metaphor-based model (a number line model see Nurnberger-Haag, 2007) was randomly assigned to eight intact classes of initial learners (four classes per model). Here I report findings from written pre/post/delayed posttests.

Settings, Participants, & Research Personnel

A power analysis indicated that including at least 50 students per integer model should detect a medium effect. To study initial learning, district sites that met the following criteria were recruited: curriculum had not yet addressed integer operations in the recruited grade and all students in this grade attended the same school with the same mathematics teacher. Two public rural districts in a Midwest state participated. According to the grade level data on the state website, 45% of the students I instructed had free or reduced lunch and were primarily European American. After removing students from analysis due to absences, 78 chips and 76 walk-it-off students remained in the analysis. The study instruction occurred in the grade prior to when integer arithmetic is typically taught in the district (School A first semester sixth grade, School B second semester fifth grade). As the researcher-teacher, with approximately 20 years of experience teaching mathematics (including integer arithmetic to K-16+ students), I taught all students in the targeted grade. The classroom teacher remained in the classroom to ensure safety of students, but not to teach. Only those students who themselves assented and whose guardians consented participated in the research by giving their written work and assessments to the researcher-teacher for an incentive the equivalent of a university folder and pencil.

Instruction and Measures

Each class experienced parallel instruction with the same tasks and activities, differing only by the integer model used (two colors of chips or ten-foot long empty number lines, the language about

Bartell, T. G., Bieda, K. N., Putnam, R. T., Bradfield, K., & Dominguez, H. (Eds.). (2015). *Proceedings of the 37th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. East Lansing, MI: Michigan State University.

how to use those representations and model-movements). During each lesson students worked on tasks and played games in assigned trios or pairs and had the opportunity to participate in whole-class discussions. I planned and implemented eight approximately 50-minute lessons about negative numbers, ordering numbers, and operations with negative integers (addition, subtraction, multiplication, and division) including sums of additive inverses, hereafter called opposite sums. The written measures assessed these constructs as well as opposite operators (transfer problems) that were not taught during the lessons (e.g., -(-4) and – (6-8)). The data reported here are from a 46-item open response skill-based test *Integer Arithmetic Test* (IAT) and one of the items from a seven-item *Explain and Draw Test* (EDT). These measures were developed through several phases of piloting and analysis including factor analysis to remove items that did not perform as expected. Students could only use pencils when completing these written tests (neither chips nor number lines were provided).

Data Analysis

The IAT data reported here included item accuracy and qualitative assessment of student reasoning on the EDT Opposite Sums Item. The IAT total test score was scaled to a 100-point test by weighting the subtotals of the following constructs: ordering numbers (20%), addition and subtraction (35%), multiplication and division (35%), and opposite operations (10%).

Multiple methods were used to determine each student's level of opposite sums knowledge at pre, post, and delayed posttest. Two types of IAT questions (calculation problems such as -19 + 19= and generative problems ___ + ___ = 0) , were each separately subtotaled for accuracy 0 to 2. An example of the EDT Opposite Sums Item follows:

> Trina and Jaleesa are students in your grade at another school.
> Trina said that $^-8 + (^-7 + 7)$ does <u>not</u> give the same answer as $^-8 + (^-5 + 5)$.
> Jaleesa said they will. Circle who is right: Trina or Jaleesa.
> Draw and write an explanation in words to convince a friend that this student is right.

I coded student explanations with a qualitative coding scheme using a constant comparative approach (Glaser, 1965). A second trained coder assessed 20% of the randomly selected tests with 92.7% agreement. A K-cluster analysis informed determination of Leveled Opposite Sums knowledge profiles (no/low, moderate, or strong) using the three ways of demonstrating opposite sum knowledge (calculation problems, generative problems, and EDT Opposite Sums Item).

Statistical controls were built in to the study design and analysis including a pretest, whole number fact test, gender, and preconceptions of negative number multiplication and division. No significant differences were found between the eight classes or between integer models at pretest. Although I planned to include district and class as statistical controls, the model would not run with both and a statistical model that accounted for either accounted for 48.4% of the variance, so district, which had significant differences at pretest was included in spite of it not being a significant predictor of the statistical model. Scaled data (overall test and subtest scores) were analyzed using multivariate analysis of covariance (MANCOVA). I used an ordinal regression to analyze leveled data.

Results and Analysis

Which, If Either, Model Supports Better Overall Learning?

Multivariate analysis of covariance (MANCOVA) was conducted on the IAT post and delayed posttest total scores to compare student learning with the walk-it-off model to learning with a chip model controlling for pretest IAT, whole number fact test, gender, and preconceptions of integer

Bartell, T. G., Bieda, K. N., Putnam, R. T., Bradfield, K., & Dominguez, H. (Eds.). (2015). *Proceedings of the 37th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. East Lansing, MI: Michigan State University.

multiplication and division. Both integer models supported statistically significant integer learning from pre to posttest, indicating that both models were reasonable models for integer learning. Statistically significant integer model differences were found between the overall learning of students using walk-it-off compared to chip models: $F(2, 146)=11.414$, $p<0.001$, $\eta\rho^2=.14$. On average in the short-term students using the walk-it-off integer model for initial instruction scored 11 points higher on this 100-point test than students using chips (posttest $\beta=-10.8$, 95% CI [-15.7, -5.9]) and 13 points higher in the longer-term (delayed posttest $\beta=-12.7$, 95% CI [-18.3, -7.0]).

Does the Way Model-Movements Represent Mathematics Impact Learning?

The IAT problems that I argue require chips students to move in ways that are inconsistent with the targeted integer operations, were grouped into Inconsistent Model-Movement Problems and those consistent into Consistent Model-Movement Problems. Inconsistent Model-Movements for example $^-4 \times {}^-3$ and $^-2 - {}^-5$ require students to put in enough chips to represent zero with sufficient numbers of chips to be able to remove 4 groups of 3 negatives or 5 negatives, respectively. Multivariate analysis of covariance (MANCOVA) was conducted on the IAT operation post and delayed post Consistent Model-Movement and Inconsistent Model-Movement problems (22 and 14 respectively) controlling for these scores at pretest and the rest of the controls used in the total test analysis. The differences were again statistically significant, but with more than twice the practical effect, ($\eta\rho^2 =.34$): $F(4, 145)=18.358$, $p<0.001$. When the chip model movements were consistent with the mathematics of integer arithmetic operations, no significant differences were found between the integer model groups' test performance. Statistically significant differences were found, however, for those problems for which I argue that the chip model required inconsistent movements ($p<.001$).

Opposite Sums Knowledge

Ordinal regression analysis on the students' levels of opposite sums knowledge showed that the chips group did demonstrate greater learning at posttest than the walk-it-off model ($p=0.002$), but this difference was not maintained five weeks later. No significant differences on opposite sums knowledge were found between students who learned with the chip model or walk-it-off model on the delayed posttest ($p=0.090$).

To test if students who did not have strong opposite sums knowledge prior to instruction had more difficulty learning with this chip model than the walk-it-off model, multivariate analysis of covariance (MANCOVA) was conducted on the post and delayed post IAT scores only for the 118 participants whose opposite sums knowledge was not strong at pretest. These students assigned to the chip model did significantly worse overall on the IAT than students assigned to the walk-it-off model: $F(2, 110)= 13.35$, $p<0.001$, ($\eta\rho^2=.20$). Students using chips scored lower on the posttest -12.7, 95% CI [-18.4, -7.0] and delayed posttest -15.8, 95% CI [-22.1, -9.6].

Discussion & Conclusions

These overall results fit the theoretical reasons drawn from cognitive science, analysis of mathematical processes, and practical classroom experience. These issues converge to predict that the walk-it-off model should better support student learning, which the data confirmed. Several reasons for these results were identified: the walk-it-off model uses model-movements consistent with mathematical ideas (whereas the chip model is often inconsistent) and the chip model visually violates the meaning of zero students would expect when thinking of numbers using a collecting objects metaphor.

Consistency with Mathematics Matters: Model-Movements

Some theoretical work referred to the mathematical alignment of integer models as breaking or requiring model-rules that differ from the mathematics (Star & Nurnberger-Haag, 2011; Vig, Murray, & Star, 2014). This study offers evidence to support these theoretical arguments that incongruent mathematical alignment does impact students' learning outcomes. Moreover, this study offers reasons related to human cognition why these breaks are likely problematic. One predicted reason was that students move differently in order to interact with the representations with these models. If this difference and the consistency of the ways model-movements represent mathematics did not matter for student learning, then there should be no significant differences in performance on the Consistent and Inconsistent Model-Movement problems. This analysis, however, did show that when the chip model required model-movements that contradicted or were extraneous to the mathematical processes and ideas, this interfered with learning. In contrast, the walk-it-off model-movements consistently represent the mathematical ideas, so I classify this model as a Motion-Aligned-Model. When approaching integer arithmetic problems with model-movements consistent with the mathematics, the results demonstrated that either model could be equally effective. This lack of significant differences on Consistent Model-Movement problems further supports the claim that model-movement alignment with mathematics could be a factor in students' learning with models.

Consistency with Mathematics Matters: Conceptual Metaphor

In order to calculate almost every integer problem with chips, students need to represent the idea of cancelling opposite values (e.g., -9 and 9). Thus, due to repetition of the underlying metaphor of cancelling opposite things, one might suspect that a benefit of the chip model might be to better support student learning opposite sums. The results did fit this prediction, but the benefits of using a cancellation model for this purpose were only in the short-term significantly different from the number line model used in this study. Longer-term on average students performed equally well after learning with either model.

Opposite sums knowledge may actually be required in order to learn integer arithmetic with the chip model, because this model requires students to sum opposite values to calculate almost every integer problem. The findings seem to support this, because students without strong cancellation knowledge at pretest who used this collecting objects metaphor model, scored the equivalent of about 1.5 grades lower than students who used this particular moving-along-a-path metaphor model. A more global reason for these initial learners' challenges with the chip model may be because it visually violates a central feature of applying the collecting objects metaphor to numbers. Rotman (1993) articulated, that when thinking of numbers using a collecting objects metaphor, zero should be visually represented as nothing or "no thing." Yet, chip models require students to use multiple things to represent no thing.

Implications

This study that tested several aspects of integer learning, including all four basic operations, suggests that the walk-it-off model may be a parsimonious model for initial integer instruction. It also reveals for which aspects of integer knowledge a collecting objects metaphor (in the form of a chip model) might add richness to student thinking. The delayed posttest results reflect longer-term learning, which although rare in educational experiments, is crucial to make claims about educational impact that matters in students' lives.

Practical Implications

Given that the walk-it-off model was more effective overall and improved learning even more for those who have less integer knowledge prior to instruction, this model is likely the most

Bartell, T. G., Bieda, K. N., Putnam, R. T., Bradfield, K., & Dominguez, H. (Eds.). (2015). *Proceedings of the 37th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. East Lansing, MI: Michigan State University.

parsimonious model with which teachers might begin instruction that meets the diverse range of learning needs in real classrooms. The sample was economically diverse (45% free and reduced lunch), but were primarily European American fifth and sixth grade students in a rural district, so the study should also be replicated with other populations to ensure that these results appropriately inform instruction for all students. Anecdotal evidence suggests the walk-it-off model, which a teacher developed and has shared with hundreds of other teachers, is a feasible model for teachers to implement (Nurnberger-Haag, 2007). Nevertheless, this study was conducted by an experienced researcher-teacher, so future investigations should confirm that students using these models with typical classroom teachers experience similar results.

Research could investigate beginning integer instruction with a moving-on-a-path metaphor in the form of the walk-it-off model, which works for every integer problem, and then integrating other conceptual metaphors in the real-life contexts in which these metaphors make sense (as well as have students assess in which contexts these metaphors make sense). The study reported here used a number line model designed to encourage students to move in ways that represent opposite operators, which differs from other approaches, so the findings of this study should not be generalized to other number line models. To further consider how learning with a model affords and constrains integer learning, other collecting objects, measuring, and moving-along-a-path metaphor-based models could be experimentally compared.

Theoretical Implications

Humans are always moving. Research in cognitive science shows that these movements influence what and how we think (Antle, 2013; Glenberg & Kaschak, 2002). Important work about moving to learn mathematics has begun (e.g., Gerofsky, 2012; Roth & Thom, 2009). Yet, more is needed, and it is crucial that mathematics education research attend to the *ways* that students move due to instructional models, instead of *whether* they move during instruction. This study contributed to this theoretical goal specific to integer arithmetic and findings suggest further investigating if motion-aligned models are more parsimonious instructional models across mathematics topics.

Acknowledgements

This research was supported by the National Academy of Education, the National Academy of Education/Spencer Foundation Fellowship Program, and Michigan State University Fellowships.

References

Antle, A.N. (2013). Balancing justice: Comparing whole body and controller-based interaction for an abstract domain. *International Journal of Arts and Technology*, 6, 388 – 409.

Berlinghoff, W.P., & Gouvêa, F.O. (2002). Something less than nothing? Negative numbers *Math through the ages: A gentle history for teachers and others.* (pp. 81-86). Farmington, MA: Oxton House Publishers.

Chiu, M. (2001). Using metaphors to understand and solve arithmetic problems: Novice and experts working with negative numbers. *Mathematical Thinking and Learning, 3*(2), 93-124. doi: 10.1207/S15327833MTL0302&3_01

Fischbein, E. (1987). *Intuition in Science and Mathematics: An Educational Approach*. Boston: D Reidel Publishing Company.

Freudenthal, H. (1973). *Mathematics as an Educational Task*. Dordrecht, Holland: D, Reidel.

Gallardo, A. (2002). The extension of the natural-number domain to the integers in the transition from arithmetic to algebra. *Educational Studies in Mathematics, 49*, 171-192.

Gerofsky, S. (2012). *Seeing* the graph vs. *being* the graph. In G. Stam & M. Ishino (Eds.), *Integrating Gestures: The Interdisciplinary Nature of Gestures* (245-256). Amsterdam/Philadelphia: John Benjamins Publishing.

Glaser, B.G. (1965). The constant comparative method to qualitative analysis. Social Problems, 12, 436-445.

Glenberg, A.M., & Kaschak, M.P. (2002). Grounding language in action. *Psychonomic Bulletin & Review, 9*(3), 558-565.

Kilhamn, C. (2011). *Making Sense of Negative Numbers* (Vol. 304). Gothenburg: University of Gothenburg.

Küchermann, D. (1981). Positive and negative numbers. . In K. M. Hart (Ed.), *Children's understanding of mathematics: 11-16* (pp. 82-87): John Murray London.

Lakoff, G, & Nunez, R.E. (2000). *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being*. New York: Basic Books.

Liebeck, P. (1990). Scores and forfeits-an intuitive model for integer arithmetic. *Educational Studies in Mathematics, 21*, 221-239.

Linchevski, L., & Williams, J. (1999). Using intuition from everyday life in 'filling' the gap in children's extension of their number concept to include the negative numbers. *Educational Studies in Mathematics, 39*(1/3), 131-147.

Nunez, R.E. (2012). On the science of embodied cognition in the 2010s: Research questions, appropriate reductionism, and testable explanations. Journal of the Learning Sciences, 21, 324-36.

Nurnberger-Haag, J. (2014, August). Noticing verbally and physically expressed conceptual metaphors for mathematics: A method for conceptual metaphor analysis. Poster presented at the *9th International Conference On Conceptual Change, Special Interest Group of the European Association of Research on Learning and Instruction*. Bologna, Italy.

Nurnberger-Haag, J. (2013, November) Metaphors students express about integers after using multiple models. Poster presented at the *International Group of the Psychology of Mathematics Education-North American Chapter Conference*, Chicago, IL.

Nurnberger-Haag, J. (2007). Integers made easy: Just walk it off! *Mathematics Teaching in the Middle School*, 13, 118-121.

Pierson, J.B., Lamb, L.L., Philipp, R.A., Whitacre, I., Schappelle, B.P., & Lewis, M. L. (2014). Obstacles and affordances for integer reasoning: An analysis of children's thinking and the history of mathematics. *Journal for Research in Mathematics Education*, 45, 19-61.

Roth, W, & Thom, J.S. (2009). Bodily experience and mathematical conceptions: from classical views to a phenomenological reconceptualization. *Educational Studies in Mathematics, 70*(2), 175-189.

Rotman, B. (1993). *Signifying nothing: The semiotics of zero*. Stanford, CA: Stanford University Press.

Star, J. & Nurnberger-Haag, J. (2011, April). Toward a research agenda on mathematical models. Paper presented in symposium entitled "Models in elementary mathematics teaching and learning" chaired by Jon Star at the *Annual Meeting of the American Educational Research Association*, New Orleans, LA.

Vig, R., Murray, E., & Star, J.R. (2014). Model breaking points conceptualized. *Educational Psychology Review*, 26, 73-90.

Vlassis, J. (2008). The role of mathematical symbols in the development of number conceptualization: The case of the minus sign. *Philosophical Psychology, 21*, 555-570.