THE EFFECT OF WORKED EXAMPLES ON STUDENT LEARNING AND ERROR ANTICIPATION IN ALGEBRA

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The present study examines the effectiveness of incorporating worked examples with prompts for self-explanation into a middle school math textbook. Algebra 1 students (N=75) completed an equation-solving unit with reform textbooks either containing the original practice problems or in which a portion of those problems were converted into correct, incorrect, or incomplete examples. Students completed pre- and posttest measures of conceptual understanding, procedural problem-solving skill, and error anticipation. Results indicate the example-based textbook assignments increased students' procedural knowledge and their ability to anticipate errors one might make when solving problems. Differences in students' anticipation of various types of errors are also examined.

Keywords: Algebra and Algebraic Thinking, Cognition, Middle School Education

Introduction

Over the past few decades, a significant body of work in cognitive science has provided evidence for the Worked Example principle, which argues for having learners spend some of their practice time studying worked out examples of problem solutions rather than solving all of the problems themselves (e.g., Sweller & Cooper, 1985; Sweller, 2006). Several studies have confirmed the effectiveness of studying worked examples in computer-based classrooms (Booth, Lange, Koedinger, & Newton, 2013; Kim, Weitz, Heffernan, & Krach, 2009; Paas, 1992) and in traditional mathematics classrooms (Booth, Cooper, Donovan, Huyghe, Koedinger & Pare-Blagoev, 2015; Booth, Oyer, Pare-Blagoev, Elliot, Barbieri, & Koedinger, 2015), though none to date have examined its effect in reform classrooms. Further, studies have also expanded on the worked example principle to suggest that having students self-explain the examples (Renkl, Stark, Gruber, & Mandl, 1998), gradually fading the support provided in correct examples (Atkinson, Renkl, & Merrill, 2003), and having students also consider incorrect examples (Siegler, 2002) can have enhanced learning benefits for conceptual and procedural knowledge, both of which are critical in mathematics (NMAP, 2008).

Despite this solid body of support, these practices are not typically undertaken in real world classrooms; the incorporation of incorrect examples is looked upon as a particularly foreign practice. In the United States, teachers tend to shy away from talking about errors (Lannin, Townsend, & Barker, 2006) at least in part due to the fear that their students will adopt the errors in their own problem solving (Santagata, 2004). However, students can learn a great deal from considering errors. Two established theories provide explanations for how studying errors can be useful for learning. Ohlsson's (1996) theory suggests that explaining why an error is wrong can help learners identify the particular features of the problem that make the solution incorrect; this can lead to refinement of problem solving skills and remediation of misconceptions. In addition, Siegler's (1996) overlapping waves theory maintains that individuals know and use a variety of (correct and incorrect) strategies for solving problems, and those strategies compete for use each time a problem is encountered. Studying errors is thought to be an effective way of helping learners accept that those particular strategies are wrong and prompting them to construct and strengthen other, correct strategies (Siegler, 2002).

Consistent with these theories, recent research has found that error reflection is indeed beneficial to learning. For example, having students think about and correct their own errors can lead to greater engagement and improved problem solving skill (Cherepinsky, 2011; Henderson & Harper, 2009);

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studying the errors made by others may be even more effective (Yerushalmi & Polingher, 2006), in part because it exposes students to multiple perspectives other than their own (Siegler & Chen, 2008). Incorporating incorrect examples into class assignments and prompting students to explain why they are incorrect has been found to be particularly beneficial for improving conceptual understanding (Booth et al., 2013)

Thus, there is growing consensus that students can learn effectively from explaining errors. However, textbooks don't often include incorrect examples, and creating materials and lessons that include incorrect examples can be very time consuming for teachers. Some curriculum materials have recently been developed and tested and are available for use (e.g., see Booth et al., 2015). However, these efforts are only useful for the content they have explicitly been developed for—the considerable work that goes into such efforts does not translate directly for other content areas. One possible solution would be to ask students to think about the kinds of errors that might be made and explain to themselves why those solutions would be incorrect. But are students able to anticipate the types of errors other students might make? And might explaining worked examples improve students' ability to anticipate errors? These questions are a main focus of the present study.

The Present Study

There were two main purposes in this study. First, it aimed to replicate and extend previous results showing that the use of worked examples with self-explanation prompts is effective for students learning mathematics in real world classrooms. Prior results have found that in traditional mathematics classes, where procedural knowledge is typically emphasized and conceptual knowledge may be lacking, assignments containing correct and incorrect examples to explain along with problems to solve were more effective for improving both conceptual and procedural knowledge than assignments in which students solved all of the problems themselves. In the present study, correct, incorrect, and partially completed examples were incorporated into reform mathematics classrooms, where conceptual knowledge is typically emphasized through rich problem-based activities, but procedural knowledge may suffer (NRC, 2004). We hypothesize that worked examples will still be effective in a reform classroom and investigate differences in effectiveness for conceptual vs. procedural knowledge.

The second purpose of this study was to examine students' ability to anticipate the types of errors other students might make when solving equations, and determine if this ability is improved after experience explaining correct and incorrect examples, which highlight features of problems for which students might have misconceptions and demonstrate incorrect methods for solving the problems. We will examine the types of errors that students anticipate and determine how the anticipated types of errors change after instruction in general and/or the worked examples intervention.

Methods

Participants

Seventy-five 8th grade Algebra I students from an inner-ring suburban middle school in the Midwestern United States participated in the study (55% female; 59% African American, 21% White, 15% American Indian/Alaskan, 4% Asian, and 1% classified as other ethnicities). The study employed a quasi-experimental design where students were assigned to the treatment and control groups according to their rostered section of Algebra I. The four sections of Algebra I were taught by two teachers, with each teacher having one treatment and one control class. In all, 37 students (49%) participated in the experimental group while 38 students (51%) participated in the control group. All of the Algebra I classes utilized the Connected Mathematics Project 2 Curriculum (CMP2; Lappan, Fey, Fitzgerald, Friel, Phillips, 2006) CMP2 includes rich, problem-based investigations during classroom lessons, and provides a variety of practice problems for students to solve afterward as classwork and/or homework. This study took place during the *Say It With Symbols* unit, which focuses on understanding symbols in algebraic equations.

Measures and Coding

A single, experimenter-designed paper-and-pencil test was utilized for this study and administered as both pretest and posttest. The test included three types of items: conceptual knowledge, procedural knowledge, and error anticipation.

Conceptual knowledge. To examine students' conceptual knowledge about problem features, we used 21 items focused on the meanings of different terms in an equation, identification of equivalent expressions, and categorization of functions as linear, quadratic, or exponential. The percentage of these items answered correctly was computed for each student at pretest and at posttest.

Procedural knowledge. To examine procedural skill, we used 9 items which asked students to solve multi-step equations, simplify expressions using the distributive property, and evaluate formulas at given values. The percentage of these items answered correctly was computed for each student at pretest and at posttest.

Error anticipation. To evaluate ability to identify errors that others might make when solving multi-step equations, we utilized one item which asked students what mistakes they thought a seventh-grader might make in solving the equation $5x - 2 = 8$. In total, they were asked to identify two potential errors. Student responses were coded first in terms of whether or not the provided responses were reasonable, and then by the type of error referenced: mistakes involving variables (e.g., handle the coefficient separately from the variable), like terms (e.g., subtract 2 from 5x), negative signs (e.g., subtract 2 from both sides instead of adding 2), equals sign (e.g., perform an operation to one side and not the other), operations (e.g., adding two numbers instead of multiplying), other reasonable errors, or unreasonable errors. For each type of error, students were scored in terms of whether at least one of their responses fit in that category.

Procedure

Prior to beginning the *Say It With Symbols* unit, all students took the paper-and-pencil pretest. Instruction for the treatment and control classrooms was kept constant within teacher (e.g., Teacher A provided the same lesson to both her treatment class and her control class). The only difference between conditions is that when students were to work on their practice problems, treatment classes were given an adapted version of the *Say It With Symbols* book. In the adapted book, approximately 26% of the practice problems were replaced with a correct, incorrect, or partial example of a solution to that problem. Teachers could assign as many or as few practice problems as they desired as long as the same items were assigned to both their treatment and control groups. When the unit was complete, teachers administered the paper-and-pencil posttest to all students.

Results

To examine the effectiveness of the treatment for improving students' conceptual and procedural knowledge, we conducted a 2 (condition: treatment vs. control) x 2 (time: pretest vs. posttest) x 2 (measure: conceptual vs. procedural) RMANOVA, with repeated measures on time and measure. The analysis yielded a main effect of time (F (1, 73) = 23.50, $p < .0.001$, $\eta_p^2 = 0.24$), with students performing better at posttest (M = 52%) than at pretest (M = 41%). The main effect of measure was also significant (F (1, 73) = 52.97, $p < .0.001$, $\eta_p^2 = 0.42$), with students performing better on conceptual items (M = 52%) than on procedural items (M = 41%). There was a significant time by measure interaction, F (1, 73) = 15.43, $p < .0.001$, $\eta_p^2 = 0.17$), with students improving more from pretest to posttest on procedural items (31% to 48%) than conceptual items (51% to 55%). Finally,

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there was a significant interaction between time and condition, F (1, 73) = 4.66, p = .0.034, η_p^2 = 0.04, revealing that students in the treatment group improved more from pretest to posttest (42% to 57%) than students in the control group (40% to 46%). No other main effects or interactions reached significance.

To examine the types of errors anticipated by students and how this anticipation changes with instruction and worked examples intervention, we conducted a 2 (condition: treatment vs. control) x 2 (time: pretest vs. posttest) x 6 (error type: variable, like terms, negative signs, equals signs, operations, and other reasonable errors) RMANOVA, with repeated measures on time and error type. The analysis yielded a main effect of time, F (1, 73) = 23.92, p < .0.001, η_p^2 = 0.25), with a greater likelihood of anticipating reasonable error responses at posttest (23%) than at pretest (14%). There was also a significant interaction between time and condition, F (1, 73) = 4.77, p = .0.032, η_p^2 = 0.06), with the likelihood of anticipating reasonable errors increasing more from pretest to posttest for the treatment group (12% to 24%) than for the control group (17% to 22%). The main effect of error type also reached significance, F (5, 69) = 10.51, p < .0.001, η_p^2 = 0.43). Post-hoc tests with Bonferroni correction revealed that variable, like terms, and negative sign errors were more likely to be anticipated than equals sign and other reasonable errors; variable errors were also more commonly anticipated than operations errors (see Figure 1). Finally, there was a significant interaction between time and error type F (5, 69) = 2.47, p = .0.041, η_p^2 = 0.15). Follow-up paired-sample t-tests revealed that the likelihood of anticipating three types of errors increased from pretest to posttest: Like terms (12% to 39%; t(74) = 4.37, p < .001), operations (8% to 19%; t(74) = 2.04, p = .045), and other reasonable errors (4% to 15%; t(74) = 2.38, p = .020).

Discussion

Results from the present study replicate and extend prior studies on the effectiveness of worked examples in mathematics learning in two important ways. First, they demonstrate that incorporating worked examples into students' practice in reform classrooms is a useful practice. Even though such classrooms are more focused on conceptual understanding than traditional classrooms, studying and explaining worked examples leads to improved learning over problem-solving practice alone. In particular, improvement in procedural knowledge as a result of worked examples may be especially helpful in reform classrooms.

The second type of extension provided by the present study is that practice containing worked examples, many of which prompt students to reflect on incorrect procedures, leads to an increased likelihood that students will be able to anticipate errors that other students might make. If teachers want to make learning from errors a more prominent part of their classrooms but do not have access to—or time to create—relevant error-centered lessons, it may be desirable to have students that can think about potential errors on their own and reflect on why those anticipated errors are problematic. Introducing assignments with incorrect examples earlier in the process may train students to anticipate such errors on their own. However, further research is needed to examine the mechanism underlying this improvement and determine whether and how error anticipation skill transfers from one particular type of content to another. For example, if students' error anticipation skill improves after worked example assignments in a linear equations unit, would they be more likely to anticipate errors for solving quadratic equations? For graphing linear functions? For geometry? The answers to these questions are likely different depending on whether error anticipation improves because students get used to reflecting on errors in general or because they are exposed to examples of errors specific to that content area.

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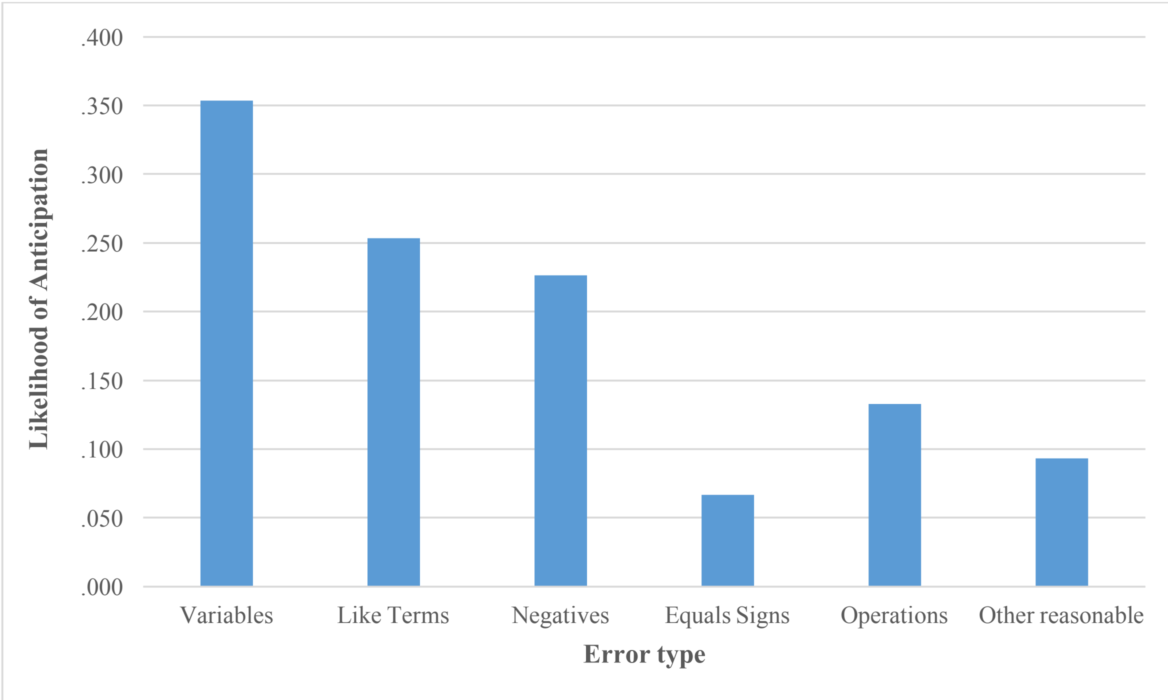

Figure 1: Likelihood of anticipation by error type.

The present study also revealed differences in the types of errors students tend to anticipate. Errors dealing with variables, like terms, and negative signs were the most frequently anticipated across time points, and anticipation of like terms, operations, and other reasonable errors were most likely to increase after students gained more knowledge about the content area. This is interesting for several reasons. For instance, prior research has identified the types of algebraic errors that are the most prevalent for Algebra I students (Booth, Barbieri, Eyer, & Pare-Blagoev, 2014). This work suggests that one of the types of highly-anticipated errors in the present study—those involving negative signs—are highly prevalent in equation-solving activities, but that the other two highly-anticipated error types—variables and terms errors—are not among the most prevalent in that content area. This indicates that students may be likely to anticipate negative sign errors because they see (or make) them frequently. However, it is not clear why students would be likely to anticipate variables or terms errors if they are not frequently made. Further study, perhaps with think aloud data collection, is necessary to determine how students come up with the errors they anticipate. A combination of think aloud and classroom observation data would also enable us to examine differences in these anticipations at pretest vs. posttest and determine why certain types of errors become easier to recognize after instruction on the topic. Knowing what the teachers are highlighting in their lessons may help explain why anticipation of certain error types increases while other types do not. If a teacher aims to get students to anticipate a wider variety of errors, further intervention targeted at helping students notice less anticipated errors may be necessary.

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