# COMPENSATION: REWRITING OUR UNDERSTANDING OF MATH LEARNING DISABILITIES

Katherine E. Lewis
University of Washington
kelewis2@uw.edu

Dylan M. Lynn
University of Washington
dylanml@uw.edu

*Research has yet to make measurable progress toward understanding how to help students with math learning disabilities (MLDs) overcome their persistent difficulties. Prior research has traditionally framed MLDs as cognitive deficits and studied these deficits by analyzing failing students' errors. In this paper, we provide an alternative. We explore a student with an MLD who has compensated so effectively that she was able to major in statistics. Eight videotaped interview session were conducted. We identify how symbolic notation was inaccessible for her and how she developed ways of compensating. This research pushes boundaries not only by breaking away from the traditional deficit model, but also by removing the delineation between researcher and participant. The case study participant (second author) was an active member of the research team collaborating in the design, analysis, and dissemination of this work.*

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How do you solve the problem: 8x3=? Most adults simply retrieve the answer from memory, requiring only a fraction of a second. This problem took "Dylan," a statistics major with a mathematical learning disability (MLD), over 10 seconds to solve. She later explained her calculation process, "two eights is sixteen, and then I'm adding [another] eight. Sixteen plus what equals 20? Sixteen plus four. Now the eight, minus the four, and there is four left over. Twenty plus four, twenty-four." Rather than retrieving this solution from memory[1], she was solving four independent calculation problems each with an intermediate sum. Although it is well documented that students with MLDs have difficulties solving basic number fact problems (Geary, 2004; Swanson & Jerman, 2006), research has rarely, if ever, examined the ways in which students, like Dylan, might be solving problems differently.

In this study we take an in depth look at Dylan's mathematical understanding and problem solving approaches. She is particularly worthy of study because despite having an MLD, she was incredibly successful in navigating upper division mathematics classes as a statistics major. By examining the ways in which she compensates, we can begin to understand the unique difficulties students with MLDs may experience and explore avenues to consider when designing instruction for students with MLDs. This paper provides a novel vantage point on MLDs by drawing upon a Vygotskian notion of disability and emancipatory research approaches used in disability studies. Through videotaped interviews, we explored the nature of Dylan's difficulties and the ways in which she compensated. In this paper we focus on one predominant category of compensatory strategies that emerged from the data that we termed "rewriting." Although there were various kinds of "rewriting," each involved representing numbers or symbols in an altered form, which enabled Dylan to remember, understand, or solve problems more effectively.

## Prior Research on MLDs

Math learning disabilities are neurologically based differences in how an individual processes numerical information, which lead to significant difficulties learning and doing mathematics (Butterworth, 2010). Although it is estimated that 5-8% of students have MLDs (Shalev, 2007), the field lacks methodological approaches to accurately identify students with MLDs (Mazzocco, 2007). Currently, researchers classify students as having MLDs if they fall below a researcher-established achievement threshold and conduct statistical analyses to establish the ways that the students

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classified as having MLDs are deficient as compared to their typically achieving peers (Lewis & Fisher, in press). Because of the focus on documenting deficits and the reliance upon low achievement as a proxy for disability, the field has made little progress towards understanding the characteristics of MLDs or identifying potential strategies to help students overcome their difficulties.

## Context of This Study

This research endeavor pushes on traditional borders between "researcher" and "participant". This work is aligned with the principles of "emancipatory research" – in which the individual with disabilities is an active member determining the goals, design, analysis, and dissemination of the research (e.g., Walmsley, 2004). Emancipatory research is an important step forward to address issues with the traditional researcher-participant dichotomy, which is oppressive to individuals with disabilities (Walmsley, 2004). Collaborating with individuals with disabilities is a major step forward because it shifts research "on" individuals with disabilities to research "with" individuals with disabilities (Charlton, 2000; Ginsburg & Rapp, 2001). This collaborative research project was initially undertaken 5 years ago by Dylan – an undergraduate statistics major with an MLD – and Katie – a math education graduate student. Although not the focus of the present analysis, it should be noted that Katie has a diagnosed language-based learning disability (i.e., dyslexia) and therefore, in some ways, also has an "insiders" perspective on disability.

Dylan (second author) initially contacted Katie (first author) in an attempt to learn more about research on adults with math learning disabilities so she could understand what strategies were available to help overcome her difficulties. Katie informed her that unfortunately there was almost no research on MLDs beyond elementary aged students engaged in basic arithmetic (Lewis & Fisher, in press). Furthermore, the field had no documented cases of an individual with a math learning disability who had majored in the field of mathematics. Dylan, therefore, had unique insight into the nature of difficulties she experienced across a range of mathematical topics (including upper division math classes) and had a wealth of compensatory strategies she employed to adjust for the atypical ways her brain processed numbers. Together, we decided to embark upon a research project to document both the nature of Dylan's difficulties and the ways in which she learned to compensate. Our shared goal for this research was to identify the particular compensatory strategies in order to inform the design of instruction for students with MLDs. Because the typical terms of "researcher" and "participant" are insufficient for this kind of collaborative and co-constructed work, Dylan is referred to as the "expert" and Katie is referred to as the "inquirer" (see Knox, Mok, & Parmenter, 2000 for similar terminology).

## Theoretical Framework

In this study, we draw upon a Vygotksian theoretical framing of disability, which stands in stark contrast to the deficit model predominantly used in research on MLDs (e.g., Geary, 2010). Vygotsky (1929/1993) argued that a child with a disability is not *less* developed than his/her peers, but has developed *differently*. Vygotsky's understanding of disability was aligned with his general theory of human development. He argued that human development progressed along two lines: biological and the sociocultural. In children without disabilities these two lines of development intertwine and are mutually constituted. In children with disabilities, mediational tools (e.g., language, symbols), which have developed over the course of human history, often do not serve the same function. For example, printed text may be inaccessible to a blind child, and therefore this standard mediational form does not serve the same function in the blind child's development of literacy as it would for a child who could see.

Central to Vygotksy's theory is that the disability creates the impetus for the development of *compensatory* processes. For example, a blind individual who cannot rely upon visual stimuli to navigate may naturally begin to echolocate (i.e., use clicking sounds to navigate; e.g., Thaler, Arnott, & Goodale, 2011). The biological difference (e.g., blindness), therefore has led to the recruitment of alternative resources and the same task is accomplished with compensatory processes. To understand a disability researcher must not only document the student's difficulties, but also the student's strengths and the ways in which a student compensates. In this study we use this Vygotkisan framing to explore MLDs. We specifically focus on identifying how standard mediational forms (i.e., numerals, symbols, representations) may be inaccessible to Dylan and the ways in which she compensates.

## Methods

### Classification of MLD

Given the difficulty involved in accurately identifying students with MLDs recent research that attempts to differentiate low achievement from MLDs have suggested tests of numerical processing (e.g., Dyscalculia Screener, Butterworth, 2003) or timed calculation tests (Mazzocco, 2009) be used to help identify students with cognitively-based numerical processing problems. Both of these measures were used to establish that the student, Dylan, met the qualifications for having a mathematical learning disability. On the Dyscalculia Screener (Butterworth, 2003), she received a classification of "dyscalculiac tendencies with compensatory aspects", and measures of her timed arithmetic performance indicated that she processed single-digit addition and multiplication problems slowly, (averaging 2.355 and 5.235 seconds/problem respectively). Given her performance on the Dyscalculia Screener and on the timed calculation test, Dylan is considered to have a MLD.

### Data Collection

Fourteen hours of videotaped interview data was collected during eight separate sessions. During these sessions we explored various mathematical domains, including: basic arithmetic, fraction operations, algebra, and statistics. We began each session by clarifying any outstanding questions from the previous session and making any needed modifications to the agenda we had planned in the previous session. As we worked through our agenda for the day, Dylan's role in the sessions was as an expert informant, someone who was able to reflect upon and demonstrate the kinds of difficulties she experienced and explain the ways in which she was able to compensate. Katie's role in the sessions was to listen attentively and ask questions to better understand the scope of the difficulties or compensatory strategies Dylan reported. We concluded each session by collaboratively deciding our agenda for the following session. After each session Katie wrote up notes from the session, which included scanned copies of all written artifacts and a general description of what was discussed. In many instances the production of these notes resulted in Katie posing several clarifying questions, which were discussed at the start of the next session. Dylan reviewed these notes to ensure the accuracy of Katie's descriptions.

### Analytic Approach

All videos were transcribed and all artifacts were scanned. The research team, comprised of the authors and one graduate research assistant, conducted an open coding on the transcripts then met to discuss preliminary coding categories. This analysis identified several predominant themes in the data. Several of these themes can be loosely classified as instances in which Dylan used the compensatory strategy of "rewriting." We explore the various ways in which Dylan employed this strategy and how it enabled her to compensate for the particular difficulties she experienced when using numeric or symbolic notation.

## Results

The results focus on several different ways in which Dylan used the compensatory strategy of "rewriting". Through different kinds of rewriting Dylan accomplished several different goals, which directly addressed her difficulty processing, manipulating, and remembering symbolic representations of numbers. The first kind of rewriting enabled Dylan to kinesthetically encode numerical information that she found difficult to remember. The second kind of rewriting enabled Dylan to connect the symbols to their underlying meaning by translating the symbols into words. The third kind of rewriting involved addressing notational ambiguity by rewriting the problem in a consistent form. In each case she reflected on why the particular compensation was needed. These episodes, therefore, illuminate both the ways in which standard mediational forms were inaccessible and also how Dylan learned to compensate through various kinds of rewriting.

### Rewriting For Memory

The first form of rewriting allowed Dylan to compensate for difficulties remembering numerical information. Dylan had significant difficulties using and remembering numbers throughout her life. She reported that she had difficulty remembering her pin number, the number and zip code of her street address, and historical dates. She recalled that when she was young, "I could remember what street name I lived on, but for the life of me I could not remember the house number." Dylan developed ways of compensating for her difficulties memorizing numbers. For example, to remember her address she described, "I would write it out a ton of times. And even if I couldn't actually remember it, I would remember the sensation of that movement, so that I could replicate that movement on a page," "so even if I couldn't recall it, if I had a piece of paper, and I wrote it out and then I could read it to you." She clarified that she was remembering the kinesthetic experience of writing the numerals, rather than the numerals themselves, "So you're not actually inherently remembering the thing, you are just remembering the feeling of creating it. And then once I see it again, then I remember, but it isn't until it's written."

This kind of rewriting involved kinesthetically encoding information represented with numerical digits. It is worth noting that this kind of compensatory strategy appeared to be used primarily in cases where the digits themselves did not have any quantitative property. For example, the house number for "1610 Main Street" does not represent one-thousand six hundred ten of anything. Similarly, dates and pin numbers use numerical digits but similarly do not represent quantities.

### Rewriting to Connect to Meaning

The second form of rewriting involved Dylan rewriting mathematical symbols in words in order to help her connect the mathematical symbols to their underlying meaning. Dylan reported that she had difficulty being able to "read" mathematical notation particularly when she was learning new mathematical content. When she was very young and learning arithmetic, she explained, "I'd write it out as a sentence, I guess, is like the best way to think about it. And in sentences you don't use symbols, you use words." For example, when learning to solve a problem like 3+2= she would translate the symbols into words and write underneath it "three plus two equals." She explained, "Because the word, like: T - H - R - E - E , has much more meaning to me than these two little backwards Cs laying on top of each other." She clarified that it was not the auditory word "three," but actually the written English word "T H R E E," that gave the symbol meaning. She described it as a sequence of steps she needs to go through to decode the numerical symbol, "If I can go from the symbol to the actual written word, then I can go to relating it to something in real life." "*Three* - I have that word and now I can think of three objects."

Although she no longer needed to translate arithmetic problems into words, she explained that, "in my higher division math courses I will actually still write these things out." She gave an example from her probability class, "this (writes "P(A|B)") is so incredibly short but it actually translates to

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the probability of event hap – event A happening, given event B has happened. I actually have to physically write that down every time I do one of these." These examples suggest that this kind of rewriting was necessary not only for the numerical symbols zero through nine, but for other symbols that had a mathematical or quantitative meaning.

**Rewriting to Remove Ambiguity**

The third form or rewriting involved rewriting problems in a standardized form to remove notational ambiguity. Dylan identified symbolic notation as being particularly problematic for her. Although many students experience difficulties as symbols take on new meaning, these notational issues caused persistent issues for Dylan and her ability to engage with the problem. For example, when solving a problem presented as: $\frac{1}{2} \times \frac{1}{5} =$ (see Figure 1), she immediately rewrote the problem using parentheses and said, "Because I have taken algebra, and I know that *x* can in fact be a variable and not necessarily multiplication… I always use parentheses now for multiplication." Because of the ambiguity around the meaning of the "x" she found it necessary to rewrite the problem before solving it.

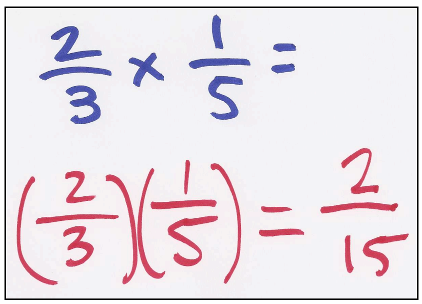


**Figure 1.** Scanned artifact of the presented problem "1/2x1/5=" and Dylan's rewritten problem form.

Dylan also found that she frequently needed to rewrite parts of the problem while in the process of solving it. For example, as she was solving the problem 123-47=, she rewrote the problem in multiple parts to be able to clearly "see" the borrowed values (see Figure 2 and Figure 3). She noted that the standard superscript notation was ineffective and problematic for her, which resulted in her rewriting parts of this problem three times in the process of solving it.

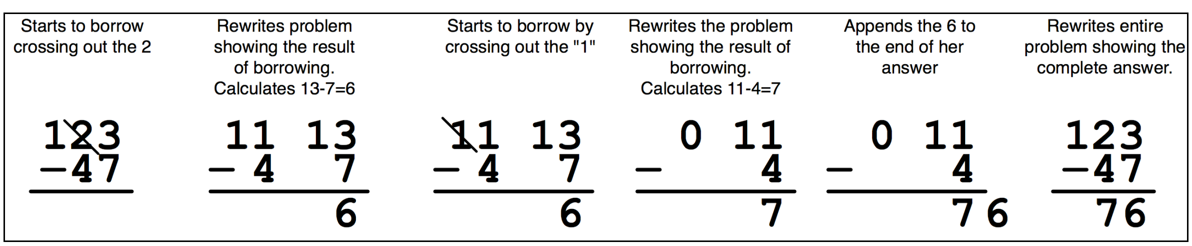


**Figure 2.** Step-by-step illustration of Dylan's solution process for the problem "123-47=".

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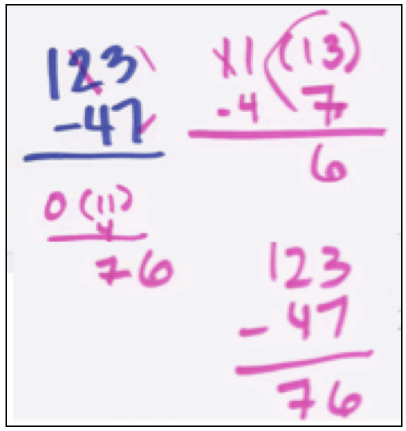

**Figure 3.** Scanned artifact showing Dylan's work for the problem "123-47=".

**Commonalities of Rewriting**

Dylan used the compensatory strategy of rewriting in several different contexts with different goals (memory, meaning, and resolving ambiguity). In each case her compensatory strategy of rewriting required more time and more effort. It is precisely because rewriting is *not* efficient that it indicates that this is truly a way in which Dylan is compensating for particular characteristics of her disability. Attending to instances in which Dylan uses rewriting, highlights both how she is compensating and suggests potential areas of difficulty.

## Endnotes

[1]We are not arguing that deriving arithmetic facts is not desirable or productive for many students, however when students use derived facts to solve a problem, the answer is often achieved in a couple seconds. Our point here is that the time and process that Dylan used to solve the problem "8x3=" is unusual for a statistics major.

## Conclusion

Dylan reported difficulties in remembering numbers, connecting symbols to their underlying meaning, and dealing with symbolic ambiguity. Mathematical symbols can be thought of as being at least somewhat inaccessible to Dylan. She reported several different ways in which she used the compensatory strategy of "rewriting" to accomplish the same goals. Although the three forms of rewriting presented here involved different features to accomplish different goals, in each case she rewrote something in a more accessible form. Similar to her strategy for solving "8x3=", Dylan's strategy produces the correct answer, but time consuming and more cognitively demanding.

In this paper we have attempted to push the boundaries of how MLDs have traditionally been conceptualized in two specific ways. First, we rejected the deficit-model of the learner and adopted a Vygotskian notion of disability focusing explicitly on a student who has developed sophisticated ways of compensating. Second, our collaborative effort represents break down the traditional researcher-participant hierarchy. Dylan was positioned as the expert and was a meaningful collaborator in the conception, design, analysis, and dissemination of this research. We believe that pushing on these boundaries enables possibilities for innovative research where new questions are posed and new avenues pursued because the participant has a voice and power over determining the direction of the research.

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