

AN EXPLORATION OF HOW ASPECTS OF A NOTICING INTERVENTION SUPPORTED PROSPECTIVE MATHEMATICS TEACHER NOTICING

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Numerous studies, including our own, have documented that teacher noticing interventions can be effective in developing teachers' abilities to notice salient aspects of the mathematics classroom. In this study, we explore how specific aspects of one such intervention may have supported three prospective teachers in learning to notice high-potential instances of student mathematical thinking. The findings provide evidence that it was not one particular aspect of the intervention that was effective in supporting their noticing, but a combination of factors that include the use of a noticing framework, interactions with their peers and a facilitator, and targeted learning-to-notice activities.

Keywords: Classroom Discourse, Instructional Activities and Practices, Teacher Education-Preservice

The ability of a teacher to attend to and make sense of important events or aspects of the classroom – *teacher noticing* – is recognized as an important component of teaching expertise (Sherin, Jacobs, & Philipp, 2011). However, “the noticing required in teaching is specialized, it is not a natural extension of being observant in everyday life” (Ball, 2011, p. xx) and thus is a skill that must be learned. Fortunately, research has shown that noticing interventions, in a variety of forms, can be successful in helping teachers notice salient aspects of the classroom. For instance, interventions have been found to help prospective mathematics teachers become more focused on students' mathematical thinking (e.g., Mitchell & Marin, 2015), more discriminating about what is important to attend to in a classroom (e.g., Sherin & van Es, 2005), and better able to make connections between teacher actions and student learning (e.g., Roth McDuffie et al., 2014). In general, research suggests that teachers can become more attentive to whatever aspect of instruction is the focus of an intervention.

While it is clear that targeted interventions can be effective in scaffolding noticing, it is less clear why particular interventions work. Researchers have hypothesized a range of explanations, including the use of specific frameworks or targeted prompts (e.g., Roth McDuffie et al., 2014), discourse among participants (e.g., Mitchell & Marin, 2015), and multiple opportunities to engage in noticing activities (e.g., Santagata, Zannoni, & Stigler, 2007). Facilitation has been found to play a critical role in video analysis, a common feature of noticing interventions, since the facilitator must support teachers to “not only see what is worthwhile but how to dissect the details of the interactions represented in this video...to draw informed interpretations of teaching and learning” (e.g., van Es, Tunney, Goldsmith, & Seago, 2014, p. 352).

Our own work with prospective teachers (PTs) has documented that our noticing intervention has helped PTs become more focused on individual students' thinking, better able to articulate the specific mathematics underlying that thinking, and more capable of identifying instances of student thinking that have significant potential to be used to support students' learning (Stockero, Rupnow, & Pascoe, 2017). Like others, we have hypothesized aspects of the intervention that supported PT learning: using a framework, interacting with peers and a facilitator, and requiring a response template to structure PTs' reflections. We also suspect that some of the learning took place as a result of many opportunities to engage in noticing activities over time. The purpose of this study is to begin to explore how specific aspects of a noticing intervention may have supported the changes in noticing we have documented, and is thus at the crossroads of past teacher noticing research that focused on

Galindo, E., & Newton, J., (Eds.). (2017). *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Indianapolis, IN: Hoosier Association of Mathematics Teacher Educators.

whether interventions can work and future research that is necessary to understand why such interventions work. To do so, we examine the cases of three PTs who formed one cohort that engaged in the noticing intervention. Specifically, this exploratory study focuses on the question: How do particular features of a noticing intervention support PTs' ability to notice high-potential instances of student mathematical thinking?

Theoretical Framework

Although teachers need to attend to a variety of classroom features while enacting a lesson, we focus our work on the noticing of students' mathematical thinking. This choice is grounded in our goal of helping teachers learn to enact *ambitious teaching* (Lampert, Beasley, Ghouseini, Kazemi, & Franke, 2010), teaching that is intentionally responsive to students' current thinking as a means of helping all students develop a deep mathematical understanding. We adopt Jacobs, Lamb and Philipp's (2010) definition of *professional noticing of [students'] mathematical thinking* to include the skills of attending, interpreting, and deciding how to respond. In this study, we focus specifically on the first two components. We hold the perspective that not all instances of student thinking should be given equal attention, however, since they do not all have the same potential to enhance student learning. We focus specifically on noticing instances of student thinking that have significant potential to be used during a lesson to support mathematical learning. We use the MOST Analytic Framework (Leatham, Peterson, Stockero, & Van Zoest, 2015) as a tool to identify such instances—those that occur at the intersection of student mathematical thinking, significant mathematics, and pedagogical opportunity. In the framework, each of these three characteristics has two criteria. Student mathematical thinking requires inferable *student mathematics* and an associated *mathematical point*; significant mathematics requires that the mathematical point is *appropriate* and *central* to student learning goals; and pedagogical opportunity requires that the instance of student thinking creates an *opening* to build on student thinking and that the *timing* is right to take advantage of the opening at the moment it occurs (for more detail about the framework, see Leatham et al., 2015). We prioritize the noticing of MOSTs because they are instances that have significant potential to advance students' mathematical understanding if built upon by a teacher – that is if made “the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea” (Van Zoest et al., 2017, p. 36).

Methodology

This study is part of a larger research project focused on supporting PTs' ability to notice MOSTs that surface during a classroom lesson. In this study, we focus on the last of five iterations of the intervention; this iteration was selected because it was found to be the most successful in supporting PT noticing (Stockero et al., 2017).

Intervention

The intervention took place during a one-semester early field experience course at a Midwestern US university. The participants were three PTs who comprised the fall 2014 cohort. Each PT completed weekly observations in a local, secondary school mathematics classroom, with the PTs taking turns recording a lesson in their classroom each week. The common full-length classroom video was analyzed individually by the PTs and by the research team each week. The research team used the PTs' and their own analyses to strategically select video instances to discuss at a weekly group meeting among the members of the cohort, facilitated by the first author (see Stockero et al., 2017, for more detail about the instance selection processes). The participants analyzed 9 different videos and attended 11 weekly meetings.

At the start, all of the PTs' video analyses focused on identifying “mathematically important moments that the teacher should notice”; they tagged such instances on a video timeline and

annotated each instance with their reason for its selection. The PTs were introduced to the MOST Analytic Framework (Leatham et al., 2015) at the end of the Week 2 meeting to define important mathematical instances and provide focus for their video analysis. In Week 3, the PTs used the framework to re-analyze the two videos they had analyzed in the prior weeks. Subsequently, they used it to analyze each new video for MOSTs. In the Week 9 of the intervention, the PTs completed an activity focused on identifying in a set of statements those that represented a *mathematical point* that a given instance of student thinking could be used to work towards and rewriting those that did not. When analyzing the last two videos (Weeks 10 and 11), the PTs were provided a template to structure their video annotation by requiring them to address all six MOST criteria in their reasoning about instances.

Data Collection and Analysis

The data for the study include the PTs' individual video analysis timelines, the research team's weekly meeting plans, and video of each group meeting. In all of the data analysis, the data were first coded individually by two or three members of the research team. The team then met to compare coding and discuss coding differences until consensus was reached.

The unit of analysis for the PTs' video timelines was each instance marked by a PT, including their annotation. These instances were analyzed in two ways. First, each was coded according to three characteristics: *agent* (who was noticed), *specificity* (the level of detail with which the mathematics was discussed), and, for instances where the agent included student(s), *focus* (what about the student(s) was noticed) (adapted from van Es & Sherin, 2008; for more detail see Stockero et al., 2017). This coding was used to analyze changes in specific characteristics of the PTs' individual noticing in relation to our noticing goals: individual student agent, specific mathematics, and a focus on noting or analyzing student mathematics. Second, each instance was coded according to whether it aligned in the video with a MOST identified by the research team and whether the PTs' reasoning was consistent with what made it a MOST, an indication of whether the PTs were noticing high-leverage instances of student thinking.

The meeting videos were also analyzed in two ways. The first analysis focused on identifying instances of what we call *analytic discussion*. Here the unit of analysis was a segment of the meeting discussion that focused on a single topic; for example, making sense of the student mathematics in an instance. Informed by Lohwasser's (2013) concept of *accountable talk* in teacher professional learning communities, we focused on identifying segments of dialogue that were likely to advance the PTs' learning—those that went beyond sharing their thinking or agreeing with one another. Instead, analytic discussion included making sense of ideas, critiquing the thinking of others, and providing alternative perspectives. In short, it is discussion that has the potential for “developing and creating usable...knowledge for teaching” (Lohwasser, 2013, p. 141-142). In the second analysis, each individual facilitator move was coded according to its purpose, using a coding framework that was informed by the facilitation moves described by van Es et al. (2014). In the analysis reported here, we focus specifically on *probing* and *challenging* moves—the moves that were most likely to directly influence PT learning. In a probing move, the facilitator pushes for more detail or specificity about a PTs' thinking. In a challenging move, the facilitator may point out a discrepancy in reasoning, or push the PTs to consider an alternative explanation or point of view, critique another PTs' explanation, or make a firm decision about the value of an instance.

The data analysis involved analyzing the coding to compare changes in the PT's individual noticing to key features of the intervention as documented in the meeting notes and group meetings. This analysis focused on determining whether changes in the PTs' noticing could be explained by particular aspects of the intervention.

Results

In the following, we first briefly describe the cases of the three participants – Claire, Aaron, and Ruth – in terms of the overall trajectory of changes in their noticing. We then use these cases as background for considering the extent to which particular aspects of the intervention appeared to support the PTs' noticing.

Cases of Learning to Notice

Claire's baseline noticing data showed that she was focused on the important mathematical ideas in the lesson, but in isolation from the students. For example, her annotation of one instance said, "Opposites added together always equal zero." None of her instances in the baseline data were coded as being consistent with MOSTs. After the introduction of the MOST framework, Claire quickly changed her focus and was able to maintain a productive focus throughout the remainder of the intervention. Beginning from the first week she used the framework, Claire consistently focused on the students' mathematics, discussed this mathematics in a specific and often analytical way, and noticed instances that satisfied at least some of the MOST criteria. Claire is a case of a PT who was quickly able to make sense of the framework she was provided and use it to identify high-potential instances of student thinking.

Aaron's initial noticing was focused on the teacher and non-mathematical issues, such as "getting everyone involved". It was thus inconsistent with MOSTs. After the introduction of the MOST framework, Aaron began to focus more on students, but moved back and forth between a teacher and a student focus for several weeks. Aaron was similar to Claire in that he began displaying analytical behavior early on. His noticing generally focused on instances that satisfied some of the MOST criteria beginning in Week 5, and his noticing became entirely aligned with MOSTs by the last two weeks. Aaron is a case of a student who took some time to make sense of the framework, but at the end of the intervention was displaying productive noticing skills.

Ruth's baseline noticing had a mixed focus on the students as a collective and on the teacher. Over half of her noticing was non-mathematical in nature and all of it was inconsistent with MOSTs. The introduction of the framework allowed Ruth to shift her focus to the students in the video and thus allowed her noticing to become more consistent with MOSTs. She also began to discuss some of the mathematics in a specific way, although much of such discussion was still at a very general level. She was slower than Claire in developing the ability to identify MOSTs; it was not until Week 8 that the majority of Ruth's noticing consistently focused on MOSTs. Ruth had the most difficulty with the interpreting aspect of noticing, as she only had instances coded as analyzing student mathematics in the last two weeks of the intervention, and even then only a single instance in each video reached this level. Ruth is a case of slow growth over time and of a student who may have benefitted from a longer intervention.

Supports for Noticing

Noticing framework. We first considered how the use of a framework supported the PTs' noticing. To understand its immediate impact, we analyzed the PTs' noticing in Week 3, when the MOST framework was used to reanalyze the videos from the first two weeks, and Week 4 when the framework was used to analyze a new classroom video.

The data suggests that the MOST framework immediately and effectively supported Claire's noticing. During her first use of the framework, her noticing shifted from teacher to students, and to instances that were MOSTs. Impressively, Claire's annotations were coded as analyzing student mathematics in over two-thirds of the instances she identified in Weeks 3 and 4, and nine of her ten identified instances were MOSTs. To give a sense of the type of noticing Claire engaged in during

her early use of the MOST framework, consider her annotation of an instance that occurred when students were being introduced to Pythagorean Triples:

Student: ‘Times 13^2 ’. One student thought that $5^2 + 12^2$ should be multiplied by 13^2 to find out the hypotenuse length. [T]his concept is not especially difficult, that it should [be] equal to 13^2 , but when this is just being introduced, it might be difficult for a student to understand how to know if a 5, 12, 13 triangle is a Pythagorean triple. At this point it is important to understand that they just need to plug the values into the equation $a^2 + b^2 = c^2$.

In this instance, Claire not only noticed an important error made by a student, she also hypothesized why the concept might be difficult (Pythagoreans triples have just been introduced) and explained what mathematical idea she would want the student to understand.

Aaron and Ruth, on the other hand, seemed to take more time and need more support to make effective use of the framework. Although about half of their noticing was focused on MOSTs in Weeks 3 and 4, each displayed key inhibitors to their noticing. Aaron’s was his continued focus on the teacher, despite the fact that the first MOST criteria is student mathematics (e.g., “The question is how to compute the hypotenuse given two legs. The goal is to be able to use the Pythagorean Theorem to do this. [The teacher] explains the central goal in detail so the students will understand this concept”). For Ruth, it was her vague explanations that lacked evidence that she was engaging in analysis and interpretation of the student’s thinking (e.g., “The students are all getting the problem wrong, and you can tell what they are thinking mathematically by their misspeaking or wrong answers.”). Thus, although the framework provided some focus to Aaron and Ruth’s noticing, it was not sufficient to focus them on noticing and interpreting MOSTs.

Group discussions. The data indicated that, on average, the PTs engaged in 14 episodes of analytical discussion in each weekly meeting, with a range from 8 to 19 episodes. The PTs’ participation in these discussions was found to be evenly distributed, so they all had equal opportunity to engage in discussions that were likely to promote their growth in noticing.

The Week 1 and 2 analytical discussions focused largely on distinguishing between teachers’ noticing and their use of prior knowledge to make instructional decisions, as well as on making sense of what it means for something to be mathematically important (versus important for some other reason). Inferring the student mathematics (SM) in the video was a primary focus of analytical discussions nearly every week, as was articulating a mathematical point (MP) that the student mathematics could be used to work towards, after this concept was introduced with the MOST framework. Other topics that were the object of analytical discussion were definitions of specific MOST criteria and considering these criteria within specific contexts (such as what it means for an idea to be central to student learning if it is not the focus of the current lesson, or what the SM is if two competing student solutions have been shared). During the second half of the intervention, proposing building moves was also a significant topic of discussion, although not the focus of our current analysis.

The SM and MP criteria are the most mathematical of the MOST criteria and are typically the components that require the deepest level of analysis to identify whether an instance is a MOST. They appear to be the components that were most challenging to Ruth and Aaron and thus affected the advancement of their noticing skill. There is evidence that a sustained focus on these topics during the meetings was effective, however, since Ruth and Aaron both continually increased in their focus on noting and analyzing the student mathematics and their ability to identify MOSTs. Aaron did so more quickly than Ruth as his noticing was coded as either noting or analyzing the student mathematics in nearly all instances beginning in Week 5, and by Week 6 he noticed mostly MOSTs. Ruth gradually increased in the percentage of instances coded as noting student mathematics and that were MOSTs, but only showed evidence of analyzing student mathematics in the last two weeks of the intervention. To give a sense of how the analytical discussion may have supported the PTs in

learning to infer the SM, consider an excerpt from the Week 5 meeting in which the PTs were discussing a video instance in which a student said that the coordinates of the x- and y-intercepts of a graph “both have a zero”:

Facilitator: What do you think the student is saying there?

Claire: I think she is noticing that the x-intercept is when y is zero and the y-intercept is when x is zero. That’s not what she says. She says they both have a zero, but I think that’s what she’s getting at.

Aaron: She just didn’t find the right way to explain it, a way that everyone else would understand what she meant by saying it, which is why [the teacher] later went into explaining when looking for the x-intercept, it’s when $y=0$, when it crosses the x-axis. She goes over a few ways to explain it. Actually explaining it in a way that everyone else would understand makes sense. It makes sense for her to interject there.

Facilitator: I heard Claire say that she didn’t really say that. Is there enough there to infer the student math?

Ruth: I feel like she wasn’t entirely clear on what the correct answer was. She just made the observation that each coordinate has a zero. That’s what I thought.

Facilitator: So that’s all you’re willing to infer, then? That each one has a zero in it.

Ruth: Yeah, she was just making that observation and the teacher then elaborated on it.

In this excerpt, we see Claire and Aaron make an inference about the SM that went beyond what should be reasonably inferred based on the student’s brief statement. Ruth contradicted their inference, however, providing the opportunity for them to reconsider their assertion about the SM. In fact, later in the discussion Claire noted that, “We’ve kind of thought that she made an observation that might or might not be correct, so [the teacher] needs to elaborate or figure out what she really means before it can be opened up to the class to discuss,” indicating that Ruth’s comments had caused Claire to reconsider her original inference.

The meeting data showed that Ruth struggled to articulate MPs related to the students’ ideas through much of the intervention. For example, a Week 7 meeting discussion focused on an instance in which a student had suggested that to convert the fraction $1/10$ to a percent, you could just use the denominator, so it would be 10%. When the facilitator asked, “Is there a mathematical point associated with that? In other words, what would you want him to understand?”, Ruth replied “That’s not how you do it,”—a response that typified her articulation of MPs and lacked the level of interpretation that was our goal. Despite participating in numerous discussions where the other PTs had articulated MPs, Ruth struggled to do so. This was a primary reason that she was unable to reach what we considered the most advanced level of noticing, where she was able to identify and also make sense of students’ mathematics.

Facilitation. Related to the *analytic discussion* findings, an examination of the role of the facilitator suggests that the meeting facilitation was also important to the PTs’ learning. In the ten meetings that focused on analyzing video instances (all except Week 9), 84% (119/155) of all analytic discussions were supported by either *probing* (79) or *challenging* (6) facilitator moves, or by both (34). This was relatively consistent across meetings, with between 69% and 92% of analytic discussion supported by such moves. Additionally, 77% (40/52) of all *challenging* facilitator moves during the intervention meetings coincided with analytic discussion, indicating that such moves were effective in causing the PTs to grapple with ideas. Together, these results suggest that the meeting facilitation played a key role in supporting changes in the PTs’ noticing during the intervention; in particular, the facilitator’s moves appeared to support discussion among the PTs that was likely to advance their learning.

Targeted activity and template. Prompted largely by the observation that Ruth was having difficulty advancing her noticing due to her inability to articulate MPs—an important part of the identification of MOSTs—in Week 9 we engaged the PTs in an activity where they worked on articulating MPs associated with instances of student mathematical thinking that they were provided. Following this activity, we also provided a template that prompted the PTs to address all six of the MOST criteria in their instance annotation beginning with the Week 10 analysis. Because these activities occurred simultaneously, it is difficult to separate their impact on Ruth’s noticing. There is evidence, however, that together they had a positive effect on her noticing.

In Week 8, a typical annotation by Ruth addressed the SM and the MP as follows: “The student math is that she discovered what the pattern is for getting the inverse [of a matrix]. Her point was closely related to what they're learning because it was the teacher’s next part of the lesson.” Note that this response alludes to the SM and to a MP, but does not precisely articulate either. In Week 10, however, her response addressed the same two criteria as follows:

Student Math: She said the absolute value of -5 was 5 because it's the opposite of 5.

Mathematical Point: The absolute value of a number is always the distance that number is away from 0. This is because absolute value is a measure of distance and distance is always positive.

Here, the level of detail and precision are both much improved from the prior example. Thus, it seems that the Week 9 activities and template did advance Ruth’s noticing. Even at the end of the intervention, however, she did not display the same level of analysis of student mathematics as her peers, with only a total of two coded instances reaching this level. In this case, a longer intervention may have allowed Ruth to continue to develop her noticing skill.

Conclusions

This exploratory study—at the crossroads of past teacher noticing research that focused on whether teacher noticing interventions could work to future research focused on understanding why they work—examined how various aspects of a noticing intervention that have been hypothesized to support teacher noticing appeared to help three PTs learn to recognize and interpret MOSTs in classroom video. As hypothesized, a provided framework did in fact support changes in their noticing, although the changes for two of the PTs were neither immediate nor drastic; in other words, the framework did not serve as an ‘answer key’ and was not by itself sufficient to support noticing. Rather, the PTs’ noticing appeared to develop over time though participation in regular group discussions that allowed them to grapple with components of the framework. Moreover, the evidence suggests that facilitation that probed or challenged the PTs’ thinking was important in supporting their engagement in analytic discussions that pushed their thinking and thus promoted learning. In the case of one PT, a direct intervention to support her in becoming more analytical of the mathematics underlying the student thinking was necessary to improve her noticing, but even then there was room for improvement. The results of this study highlight the significant effort required to develop skills in a practice as complex as noticing student mathematical thinking. Consistent with research on teacher professional development (e.g., Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2010), this study suggests that brief or minimally supported interventions are unlikely to fully develop such a practice.

Acknowledgments

This material is based upon work supported by the U.S. National Science Foundation (No. 1052958). Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the NSF.

References

- Ball, D. L. (2011). Forward. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. xx-xxiv). New York: Routledge.
- Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, 41(2), 169-202.
- Lampert, M., Beasley, H., Ghouseini, H., Kazemi, E., & Franke, M. (2010). Using designed instructional activities to enable novices to manage ambitious mathematics teaching. In M. K. Stein & L. Kucan (Eds.) *Instructional Explanations in the Disciplines* (pp.129-141). New York: Springer.
- Leatham, K. R., Peterson, B. E., Stockero, S. L., & Van Zoest, L. R. (2015). Conceptualizing mathematically significant pedagogical opportunities to build on student thinking. *Journal for Research in Mathematics Education*, 46, 88-124.
- Lohwasser, K. (2013). *Science-for-teaching discourse in science teachers' professional learning communities* (Unpublished dissertation). University of Washington, Seattle, WA.
- Loucks-Horsley, S., Stiles, K. E., Mundry, S., Love, N., & Hewson, P. W. (2010). *Designing professional development for teachers of science and mathematics*. Thousand Oaks, CA: Corwin.
- Mitchell, R. N., & Marin, K. A. (2015). Examining the use of a structured analysis framework to support prospective teacher noticing. *Journal of Mathematics Teacher Education*, 18(6), 551-575.
- Roth McDuffie, A., Foote, M. Q., Bolson, C., Turner, E. E., Aguirre, J. M., Bartell, T. G., Drake, C., & Land, T. (2014). Using video analysis to support prospective K-8 teachers' noticing of students' multiple mathematical knowledge bases. *Journal of Mathematics Teacher Education*, 17, 245-270.
- Santagata, R., Zannoni, C., & Stigler, J. W. (2007). The role of lesson analysis in pre-service teacher education: an empirical investigation of teacher learning from a virtual video-based field experience. *Journal of Mathematics Teacher Education*, 10, 123-140.
- Sherin, M. G., Jacobs, V. R., & Philipp, R. A. (2011). Situating the study of teacher noticing. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 3-13). New York: Routledge.
- Sherin, M. G., and van Es, E. A. (2005). Using video to support teachers' ability to notice classroom interactions. *Journal of Technology and Teacher Education*, 13(3), 475-491.
- Stockero, S. L., Rupnow, R. L., & Pascoe, A. E. (2017). Learning to notice important student mathematical thinking in complex classroom interactions. *Teaching and Teacher Education*, 63, 384-395.
- van Es, E. A., & Sherin, M. G. (2008). Mathematics teachers' "learning to notice" in the context of a video club. *Teaching and Teacher Education*, 24(2), 244-276.
- van Es, E. A., Tunney, J., Goldsmith, L. T., & Seago, N. (2014). A framework for the facilitation of teachers' analysis of video. *Journal of Teacher Education*, 65(4), 340-356.
- Van Zoest, L. R., Stockero, S. L., Leatham, K. R., Peterson, B. E., Atanga, N. A., & Ochieng, M. A. (2017). Attributes of instances of student mathematical thinking that are worth building on in whole-class discussion. *Mathematical Thinking and Learning*, 19, 33-54.