IMPROVING KNOWLEDGE OF ALGEBRAIC LEARNING PROGRESSIONS THROUGH PROFESSIONAL LEARNING IN COLLABORATIVE VERTICAL TEAMS

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The following research report documents a professional development that focused on promoting understanding of algebraic learning progressions across vertical teams of elementary and secondary teachers. Analyzing quantitative data from the pre and post MKT assessment and qualitative data from lesson reflections and final course reflections, revealed multiple outcomes for teachers. More specifically, elementary teachers had lower MKT than the secondary teachers in both pre and post assessment, and teachers exhibited a significant increase in their MKT as a result of the program. Further, all teachers gained more in depth knowledge about the learning progression of algebra through collaboration in vertical teams and revealed a focus on improving pedagogy through the use of high leverage teaching practices.

Keywords: Algebra and Algebraic Thinking, Learning Trajectories (or Progressions), Mathematical Knowledge for Teaching, Teacher Education-Inservice/Professional Development

States and districts often have to realign their curriculum when standards are revamped. In fact, forty-two states, the District of Columbia, four territories, and the Department of Defense Education Activity have adopted the Common Core State Standards and had to undergo realignment with their curricular standards (NGACBP & CCSSI, 2010). Although our state did not adopt the Common Core State Standards, the state standards were revised and realigned with some standards expanded to bring more rigor, depth, and breadth to the learning objectives. As district leaders, mathematics educators, and teacher leaders worked with classroom teachers to unpack the standards, we designed professional development that focused on helping teachers learn the mathematics concepts more deeply by mapping out the learning progressions that will guide the sequence of concepts crucial in building mathematical understanding. The importance of this knowledge for teachers leads to a critical, practical question: what professional development experiences are necessary for teachers to develop an understanding of the learning progression in algebraic thinking?

This study is aimed at examining the ways in which the designed professional development can engage teachers in deepening their understanding of algebra. Further, we wished to explore how engaging in vertical teams for collaborative planning of lessons can help teachers at different grade levels better understand students' learning progression of algebraic thinking.

Theoretical Framework

In the math community, the term "learning progression" has been used to describe the vertical articulation of standards with an emphasis on conceptual understanding (Confrey, 2012; Confrey, Maloney, & Corley, 2014; Wilson, Mojica, & Confrey, 2013). Understanding of learning progressions is important for teachers; they serve as the guidepost for analyzing student learning and tailoring their teaching sequence (Wilson et al., 2013). A research-based learning progression also informs how mathematical content knowledge and conceptual understanding for students develop over time.

Galindo, E., & Newton, J., (Eds.). (2017). *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Indianapolis, IN: Hoosier Association of Mathematics Teacher Educators.

Understanding How Vertical Knowledge of the Curriculum Contributes to Teachers' Mathematical Knowledge for Teaching

Mathematics knowledge for teaching (Hill, Sleep, Lewis & Ball, 2007) includes understanding of general content but also having domain specific knowledge of students. Teachers with mathematical knowledge for teaching must have an extensive and complex set of knowledge and skills that facilitates student learning across the learning progressions so that they learn the structure and relationship of students' understandings about a particular mathematical concept, teach specific strategies to elicit student thinking, strategically evaluate students' responses, and move the mathematical agenda forward (Wilson et al., 2013).

An example of the importance of learning progressions is found in the research on developing algebraic reasoning in the earlier grades through problem solving which requires depth of understanding. Blanton and Kaput (2008) reported that teachers become better at teaching algebraic reasoning when the teachers' own mathematical knowledge and understanding was increased and their algebra "eyes and ears" allowed them to bring out the algebraic reasoning while looking at student work and listening to their discussions and questions. To know what to look and listen for in the classroom, teachers must have a deep and profound understanding of the foundational concepts for algebraic reasoning through patterning, relations, functions, and representations using algebraic symbols and utilizing mathematical models to represent relationships (NCTM, 2000).

The knowledge of learning progressions is vertical knowledge. Vertical knowledge includes "familiarity with the topics and issues that have been and will be taught in the same subject area during the preceding and later years in school, and the materials that embody them" (Shulman, 1986, p. 10). This vertical knowledge can be supported through knowledge of learning trajectories and vertical teaming by teachers. Confrey states that using learning trajectories, teacher can "plan their instruction based on how student learning progresses. An added strength of a learning trajectories approach is that it emphasizes why each teacher, at each grade level along the way, has a critical role to play in each student's mathematical development" (Confrey, 2012, p. 3). However, teachers do not have frequent opportunities to work with teachers from different grade bands. Understanding the learning progression across grade levels requires the collaboration of teachers through meaningful vertical articulation and PD.

Context: Detailing Our PD Design and Activities

The designers and instructors of this project included a university mathematics educator, a mathematician, teacher leaders from the districts, and doctoral students in mathematics education. This PD was conducted as year one of a three-year Mathematics Science Partnership grant focused on algebraic thinking and proportional reasoning during the transition years from fifth grade into high school. We based this project's design on the current research and needs in mathematics education with a specific attention to creating opportunities for vertical articulation focused on algebraic learning progressions as students move into high school algebra. We considered all the core features of effective mathematics PD which includes having content as the focus, being sustained over time, collective participation of teachers working together on issues central to instruction, and focus on instructional materials that teachers use in their classrooms (Desimone, 2009).

To focus teachers' work with content, we used NCTM's standards (2000, p. 39) and explored the algebraic learning progression beginning with recursive patterns, representing mathematics situations with quantitative relationships, multiple representations of functions, including numeric, graphic, and symbolic, since the representational fluency develops a deeper understanding of functions (Moschkovich, Schoenfeld, & Arcavi, 1993). Weekly activities in content-based class sessions included modeled lessons using rich tasks combined with in-depth conversation about both the

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algebra content and the pedagogical strategies employed by facilitators. In addition, a centerpiece of the PD experience was having teachers learn about the high leverage practices outlined in the *Principles to Action* (NCMT, 2014), and asking teacher teams to select a goal that they wanted to focus on for improving their practice. Many of the teacher teams selected goals of facilitating meaningful discourse and posing purposeful questions. In addition, we shared Smith and Stein's (2011) *5 Practices for Orchestrating Productive Mathematics Discussions* with the PD participants and guided their use of the five practices in the planning and implementation of their lessons.

We conducted two iterations, each with a different cohort of teachers, of the content-focused course which included either a follow up Lesson Study or participation in a video-based teacher study group in school based vertical teams. We focused on engaging teachers in active learning through algebraic problem solving tasks and exploring pedagogical strategies. Our goal was to deepen teachers' algebraic thinking, encourage vertical articulation, and develop a productive disposition towards teaching through problem solving. The follow-up sessions were designed to provide teachers with continued support in professional learning implementing algebraic content, as well as providing opportunities for vertical articulation between and among grades levels.

Methods

For our study, we used a mixed methods approach to examine the outcome of the PD focused on learning progressions and vertical teaming of teachers. The quantitative research focused on examining teachers' content knowledge and the qualitative research focused on professional growth as identified by teachers' reflections.

Participants

The data were collected from the teachers who participated in the two cohorts. A total of 54 teachers participated in the study, 23 of whom were in Cohort 1 and 31 in Cohort 2. Most teachers worked in public schools (N=51 from seven school districts). A total of 35 schools were represented with 21 schools being represented by one teacher, nine schools by two teachers, and five schools by three teachers. The teachers also held various positions at their schools: elementary school teachers (N=22), secondary teachers (N=25), special education teachers (N=3), ELL teachers (N=1), and coaches (N=1). Two teachers did not report their positions. Specifically, the participants taught Grade 2 (N=1), Grade 3 (N=2), Grade 4 (N=3), Grade 5 (N=11), Grade 6 (N=11), Grade 7 (N=9), Grade 8 (N=3), and Grade 9 (N=6). Eight teachers did not indicate which grade they taught. On average, the teachers (N=52) had 9.42 years of teaching experience (SD=7.42) with a minimum of 0 and a maximum of 30 years.

Data Sources

Quantitative data. Prior to the beginning of the PD program, the Mathematics Knowledge for Teaching (MKT) of the participating teachers was assessed (Learning Mathematics for Teaching, 2007). Specifically, they completed the 2007 Middle School Patterns Functions and Algebra – Content Knowledge instrument, administered in two forms (N=24 for Form A; N=30 for Form B). At the end of the program, the same assessment was administered again (N=30 for Form A; N=24 for Form B). All teachers completed different forms for the pre and posttest. The forms were assigned to the teachers randomly. Due to non-equivalence of the test forms, teachers' raw test scores were converted into IRT scores, which were used in the analysis.

Qualitative data. The qualitative data sources included final course reflections, teacher reflections from lessons, video clips from the research lessons. To delve in deeper into the nature of the collaborative exchange, we collected a final course reflection, that the researchers created for the second PD cohort, which asked teachers to reflect on the nature of the vertical professional learning and how the focus on an instructional practice by the team contributed to their professional learning.

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We chose one of the video-based teacher study groups as a case that provided us the best opportunity to learn about how team members exchanged professional knowledge within the video lesson analysis.

Research Questions

1. What differences did we see in content knowledge of teachers at different grade levels?
   - Do the two cohorts of teachers differ in their MKT over the PD program?
   - Do elementary and secondary teachers differ in their MKT over the PD program?
2. What did teachers identify as areas of professional growth from their vertical teacher study groups focused on algebraic thinking and enhancing their instructional practices?
   - How did vertical teaming enhance teacher's planning and instruction?
   - What is the nature of the collaborative learning that teachers identified through their work with peer study groups?

Data Analysis

The quantitative data were statistically analyzed using IBM SPSS Statistics 22. To begin analyzing the themes in the qualitative data, we systematically analyzed the data by developing initial codes and used the method of axial coding to find categories in such a way that drew emerging themes (Miles, Huberman, & Saldaña, 2013). To verify and compare recurring themes and categories, the research team worked individually on coding the documents before comparing preliminary codes in order to agree upon recurring themes from the reflections. Dedoose, an internet-based data management tool (Dedoose Version 6.2.7) was used to code and analyze the data. Initial codes reflected specific teacher practices and actions (e.g., posing purposeful questions, learning content from peers, anticipating student responses). As codes were categorized, several main themes emerged: improved pedagogical practice due to instructional strategies, improved content knowledge; supporting student thinking in algebra; improved pedagogical practice resulting from collaboration, and gaining vertical knowledge of content from peer collaboration.

Results

Quantitative Results

The descriptive statistics for the pre and posttest IRT scores are presented in Table 1. In addition to the total, descriptive statistics by Cohort and Position at school are also presented. Due to the sample size restrictions, only elementary and secondary positions are considered (secondary teachers include both middle and high school teachers).

To further explore teachers' MKT, we conducted a comparative analysis for the cohorts and teachers' positions (a Bonferroni correction was employed). First, to determine if there was a difference between the two cohorts on the pre and posttest, we conducted independent-samples t-tests. The results showed that the cohorts did not differ in MKT either on the pretest $(t(52)=0.741, p=0.462)$ or on the posttest $(t(52)=0.428, p=0.670)$. Second, to determine if there was a difference between the pre and posttest for the two cohorts, we conducted paired-samples t-tests. The results indicated that teachers in Cohort 2 scored higher on the posttest than on the pretest $(t(30)=-2.546, p=0.016)$; however, no difference was observed between the pre and posttest scores for Cohort 1 $(t(22)=-1.219, p=0.236)$.

Next, to determine if there was a difference between elementary and secondary teachers on the pre and posttest, we conducted independent-samples t-tests. The results showed that the elementary teachers scored lower than secondary teachers on the both pre $(t(45)=-3.948, p=0.000)$ and posttest $(t(41.811)=-3.785, p=0.001)$. Finally, to determine if there was a difference between the pre and posttest for the elementary and secondary teachers, we conducted paired-samples t-tests. The results indicated no difference between the pre and posttest scores of either elementary *(t(21)=-2.248, p=0.035)* or secondary *(t(24)=-1.060, p=0.300)* teachers.

Table 1. Descriptive Statistics for the Pre and Posttest of MKT

Measure	Mean (SD)				
	Total (N=54)	By Cohort		By Position	
		Cohort 1 (N=23)	Cohort 2 (N=31)	Elementary (N=22)	Secondary (N=25)
Pretest	-0.049 (1.000)	0.068 (0.918)	-0.137 (1.063)	-0.532 (0.749)	0.497 (1.000)
Posttest	0.185 (0.917)	0.248 (0.936)	0.139 (0.915)	-0.244 (0.632)	0.645 (0.962)

Research question #1a. First, we aimed to determine whether the two cohorts differed in MKT over time. To answer this research question, we conducted the two-way mixed design ANOVA with Cohort (Cohort 1 and Cohort 2) as a between subject factor and Time (pre and post) as a within subject factor. The results indicated the main effect of Time *(F(1, 52)=6.502, p=0.014)*, i.e., teachers' scores, averaged across cohorts, were higher on the posttest than on the pretest. However, there was no main effect of Cohort *(F(1,52)=0.393, p=0.534)*, i.e., teachers' scores, averaged across time, did not differ between the cohorts. Additionally, no interaction effect of Time and Cohort was observed *(F(1, 52)=0.289, p=0.593)*. Thus, considering these results and comparisons above, we decided to combine the two cohorts for the further analysis.

Research question #1b. Next, we aimed to determine whether elementary and secondary teachers differed in MKT over time. To answer this research question, we conducted the two-way mixed design ANOVA with Position (elementary and secondary) as a between subject factor and Time (pre and post) as a within subject factor. The results indicated the main effect of Position *(F(1, 45)=17.084, p=0.000)*, i.e., elementary teachers had lower scores on MKT, averaged across time, than secondary teachers. The comparisons above also suggest that elementary teachers had lower MKT on both pre and posttest. Additionally, there was the main effect of Time *(F(1, 45)=5.187, p=0.028)*, indicating that teachers' scores, averaged across positions, were higher on the posttest than on the pretest. However, according to the comparisons above, when the teachers are split into elementary and secondary levels, the effect of time does not hold for either level. This finding is also consistent with the absence of the interaction effect of Time and Cohort *(F(1, 45)=0.532, p=0.470)*. Larger sample sizes may be needed to determine differences within the levels (i.e., elementary vs. secondary).

Qualitative Results

While the MKT results showed that teachers overall made gains in their content knowledge as a result of the PD, they also reported, universally, through the qualitative data that the PD led to improvement in their understanding of algebraic content and pedagogy for math instruction. We will next examine reflections of the teachers from the second cohort to identify which particular benefits the teachers gained from the PD.

Vertical teaming and its impact on teacher's planning and instruction of algebra across the learning progression. All of the second cohort teachers who completed the final reflection indicated that they learned from their peers and found added value in that learning because the teams were made up of teachers from multiple levels (i.e., elementary and secondary). Barbara, an algebra teacher, described the benefit of working in vertical teams:

> There really should be more collaboration between elementary, middle, and high school. I loved hearing from the elementary teachers because of all their use of manipulatives and their different approaches can also work for high school and be helpful, especially for low level students.

By working together in vertical teams, the teachers built a broader foundation of both personal understanding of content and knowledge of how students at different levels (both in age and readiness) approach algebraic thinking. Ralph, a 5th grade teacher stated:

> The wide-range of grade levels that are taught within our group served as a catalyst for a deeper understanding of the levels of reasoning and problem-solving students are at and what can be achieved during those levels and what can be and needs to be done for those that may be behind developmentally.

Finally, teachers indicated that they had a firmer grasp of the learning progression necessary for students to succeed in algebraic thinking at all levels. Rebecca, a 4th grade teacher shared:

> I really was able to open my eyes to how the ideas that start in early elementary are so foundational to how students think when they get to their algebra class. The connections that can be built through patterns and being able to recognize them is so valuable.

Collaborative professional learning through teacher study groups. Consistent with the quantitative results, almost all participants (30/32) specifically indicated that working with their peers improved their knowledge of math content. More than half (18/32 total teachers) identified that they learned math content from their colleagues: elementary teachers learned to see equations and formal algebra from their secondary colleagues (10/18 elementary teachers) and secondary teachers learned to utilize manipulatives and visual models from their elementary colleagues (8/14 secondary teachers). In fact, most teachers indicated that working with peers expanded their knowledge of diverse strategies and the use of varied representations when solving problems (29/32). Jan, an 8th grade math teacher elucidates this idea:

> Being able to see how others approach problem-solving was very enlightening. It was so interesting to me to see that there are so many different ways of solving problems. I was never taught and haven't felt comfortable using manipulatives, etc., but I really appreciated seeing this strategy being used. Now when I look at a problem, I am able to see different ways of approaching it.

Teachers also learned to improve their pedagogy through collaboration with their peers. Half the teachers made specific comments that their pedagogical practice improved due specifically to their interactions with their peers (16/32). Elizabeth, a 5th grade teacher, stated, "I liked brainstorming with my group members and thinking outside of my box. I realized that anticipating the questions that students might ask prior to the lesson is very helpful!" Further, Jocelyn, a middle school math teacher reflected that, "Working with the group helped frame the plan of what ideas to share and how to make those mathematical connections between the students' strategies and the 'Big Ideas'."

While teachers' reflections offered insights into their thinking, their video discussions allowed us to view their practice. We selected one case study that we felt demonstrated notable exchanges among team members in regards to the benefits of collaboration. The team was composed of five teachers, ranging from 2nd grade to 8th grade algebra. The study group chose a growing toothpick pattern task which they modified for implementation at each grade level. We coded the commentaries from the collaborating teachers on each video. These exchanges revealed ways in which peers exchanged their knowledge for teaching and assisted one another. One of the major themes was how peers validated each teacher's instructional practices.

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Ann, a 4th grade teacher presented the Hexagon Pattern. A peer-teacher, Karen, commented that Ann did not jump in to tell a student she had the wrong number of toothpicks for two hexagons. Instead, Ann allowed the student to discover the error when building the pattern.

Karen: I love how you didn't originally tell her that 12 was wrong, she noticed it was wrong when she got up to place the toothpicks!

Another peer-teacher, Holly, viewing Ann's lesson commented on Ann's use of posing purposeful question to follow up on the student's discovery:

Holly: Purposeful questions - "If we know that hexagons have 6 sides and we are building 2 then why do we have 11 toothpicks instead of 12?" Great job!

In another notable exchange, two teachers share the challenge of knowing and gauging the right balance between allowing students to experience "productive struggle" and knowing when to ask a probing question while commenting on a lesson taught in Micki's classroom.

Micki: Another moment where a purposeful question would have been helpful. This student got stuck and I gave her time to think, believing I was supporting productive struggle, but I should have left her with a question to help move her thinking along.

Sara: Sometimes it is really hard to find that balance of when to let them keep going with the productive struggle and when would it be better to give them a purposeful question to help them along. Remember hindsight is 20/20 and it is why we should always reflect on our lessons. If you do another activity with your class before the end of the year, you will know this student needs a little more support.

Conclusions and Implications

For several decades, the mathematics education community has focused their attention on the importance of the *Mathematical Knowledge for Teaching* (Hill et al, 2007) that defines the deep, broad, and well connected knowledge that is needed to decompose and unpack content to make it comprehensible by students. In addition to conceptual understanding of the knowledge needed for teaching, teachers must have command of high leverage teaching practices. Our PD design focused on the task of exposing teachers to the complexity of ambitious teaching. To examine the benefits of participation in our PD, we conducted a mixed-methods study.

With our quantitative analysis, we found that teachers were able to gain significant knowledge during the PD course. Our qualitative results supported the increase in teachers' content knowledge and also indicated learning benefits beyond the content knowledge; the teachers content scores did not tell the whole story. In particular, perhaps the most encouraging result from our study is the notion that PD utilizing collaborative, vertical teams of teachers can contribute to teachers' professional learning as they examine their practice and work to improve their pedagogical skills. The evidence also suggested that validation from their peers supports teachers' continued growth. As we consider Synergy at the Crossroads, mathematics educators and researchers may consider providing more opportunities for teachers to work in vertical professional learning communities focused on understanding the mathematics learning progressions in future PD offerings to increase teachers' MKT and improve their practice.

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