MATHEMATICS TEACHERS' TAKE-AWAYS FROM MORNING MATH PROBLEMS IN A LONG-TERM PROFESSIONAL DEVELOPMENT PROJECT

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Considering the role of mathematics-focused professional development programs in improving teachers' content knowledge and quality of teaching, we provided teachers opportunities for dealing with mathematics problems and positioning themselves as students in a large-scale long-term professional development (PD) project. In this proposal, we aimed to understand the impact of engaging in morning math problems on teachers in terms of their mathematical understanding and teaching practices. Both written work and interviews showed that solving open-ended problems helped teachers better understand the mathematics content and students' challenges as they solve problems; thus, suggested an effective means of PD for teachers.

Keywords: Teacher Education- Inservice/Professional Development, Problem Solving

As many teacher education researchers highlighted, mathematics-focused professional developments play a central role in efforts to improve teachers' knowledge base (Ball, 1990; Hill, 2007; Moss, 2006). Through encountering mathematics problems and positioning participating teachers as students, we sought to improve not only the teachers' mathematical content knowledge but also pedagogical knowledge, pedagogical content knowledge, and beliefs about what it means to 'do' mathematics. Considering this role, we aimed to provide teachers opportunities of dealing with mathematics problems and positioning themselves as students in a large-scale long-term professional development (PD) project. Each professional development session started by asking teachers to work on authentic and challenging mathematics problems. We also interviewed teachers at the end of the project to learn their thoughts about morning math sessions, the nature of the math problems they worked on, and what they learned about mathematics and mathematics teachers. We particularly focused on the following research questions:

1. What do teachers think about the role of morning math sessions on their improvement as mathematics teachers?
   1.1. What did teachers gain in terms of mathematical content from morning math sessions?
   1.2. What did teachers gain in terms of mathematics teaching from morning math sessions?

We found these questions significant to investigate to better understand the role of teachers' solving math problems and experiencing student position as a means of professional development. Thus, this study links to the conference theme, *Synergy at the Crossroads: Future Directions for Theory, Research, and Practice*, in that it introduces a promising component of professional development for mathematics teachers, discusses the role of morning math sessions in improving teachers' mathematics teaching practices, and makes suggestions for future directions to develop more effective professional development sessions for mathematics teachers.

Theoretical Framework

Teacher Knowledge and Role of PD in Teachers' Knowledge Development

Over the last 40 years, understanding what teachers need to know has become one of the most important concerns in the field of education (Cochran-Smith & Lytle, 1999). While some studies

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have focused on the knowledge that teachers need to know as professionals (Grossman & Richert, 1988; Shulman, 1987), others have aimed to understand the knowledge that teachers need to know for the practice of teaching (Hiebert, Gallimore, & Stigler, 2002). Cochran-Smith and Lytle (1999) took this distinction between professional and practitioner knowledge further and suggested three conceptions of knowledge: (1) knowledge-*for*-practice, (2), knowledge-*in*-practice & (3) knowledge-*of*-practice. Among these three conceptions, knowledge-*for*-practice referred to the formal knowledge that teachers gained in teacher education and professional development programs. In this vein, in the mid-1980s, Shulman (1987) had proposed seven categories of teacher knowledge: (i) content knowledge; (ii) general pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organization that appear to transcend subject matter; (iii) curriculum knowledge, with particular grasp of the materials and programs that serve as "tools of the trade" for teachers; (iv) pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding; (v) knowledge of learners and their characteristics; (vi) knowledge of educational contexts, ranging from the workings of the group or classroom, the governance and financing of school districts, to the character of communities and cultures; and (vii) knowledge of educational ends, purposes, and values, and their philosophical and historical grounds. (p. 8)

Shulman argued that these categories constituted a teacher knowledge base which was supported by both theoretical and empirical sources of knowledge. Shulman's perspective viewed the teacher as a trained professional who could learn about subject matter, curriculum, educational philosophy and history and as an active member of a scholarly community, who could pursue and help others pursue intellectual development.

Understanding those categories of teachers' content knowledge provides a strong basis for designing effective teacher education and professional development opportunities. Especially, mathematics-focused professional developments play a central role in efforts to improve teachers' knowledge base (Ball, 1990). As argued by Moss (2006, p.97), "In order to encourage their students' mathematical thinking, teachers must be able to appreciate and evaluate the reasonableness of their thinking. However, to be able to do this, they must have for themselves a deeper understanding of mathematics." Thus, providing teachers opportunities of evaluating their understanding of mathematics is important for teacher development.

Five Practices for Orchestrating Productive Mathematics Discussions

The PD sessions at the focus of this study were designed using the five practices for orchestrating productive mathematics discussions (Stein & Smith, 2011); therefore, PD trainers demonstrated these five practices during morning math sessions. Stein and Smith (2011) developed these five practices to help teachers design and implement lessons involving mathematically rich discussions and enhancing students' mathematical understanding. Stein and Smith (2011) summarized these five practices as follows:

1. anticipating likely student responses to challenging mathematical tasks;
2. monitoring students' actual responses to the tasks (while students work on the tasks in pairs or small groups);
3. selecting particular students to present their mathematical work during the whole-class discussion;
4. sequencing the student responses that will be displayed in a specific order; and
5. connecting different students' responses and connecting the responses to key mathematical ideas (p. 8).

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In addition to those, Stein and Smith (2011) proposed that setting instructional goals and selecting appropriate tasks could be viewed as the practice zero, which had to be ensured before five practices. It is important to note that these practices do not serve as a manual but suggest an effective way for teachers to characterize their work of orchestrating student-centered discussion by ensuring they make sense of students' work and connect students' thinking to the big mathematical ideas.

Mode of Inquiry

Professional Development Project and Participants

Approximately twenty-two elementary teachers from across three school districts participated in the two-year professional development program focused on preparing mathematics teacher leaders. The professional development program consisted of monthly sessions and intensive summer PD workshops. All sessions were focused on engaging teachers' in activities to develop their teachers' mathematical knowledge for teaching and leadership skills. They were involved in 16 monthly 8-hour workshops and 80 hours (10 days) of PD over two summers for a total of 26 workshops days. The sessions involved in a range of activities (e.g., video discussions, math content sessions, rehearsals etc.) and teachers were engaged in coaching sessions (McGatha, 2009) with the professional developers. In this study, we focus on the activities involved in one regular PD session we called *Morning Math*. During each Morning Math session, teachers were given a problem (or two) to solve. These problems were selected to not resemble typical textbook tasks; rather, they represented true problems for the teachers.

Data

In this proposal, we focused only on teachers' work on morning math problems during two summer PD sessions; a five-day PD in June 2015 and a five-day PD in July 2015. In particular, we examined teachers' work on four mathematics problems in June PD and five mathematics problems. The names of the problems that teachers worked during each PD session are given in the table below.

Table 1: Morning Math Problems

Name of the Math Problem	PD Session
Darts Remainder 4 Milk Chocolates Painted Cubes Rocket Science	June PD
Marbles Gum Drops The Sheep Activity Coin Sums	July PD

Although teachers worked on these problems in groups, they recorded their thinking on provided worksheets, and regularly recorded their work on large flipchart paper to share with peers. Furthermore, their group discussions were video recorded. In addition, teachers were individually interviewed. Thus, data set also consisted of transcriptions of teachers' interviews.

Data Analysis Process

In the initial round of data analysis, teachers' written work on math problems were analyzed based on content analysis for understanding the range of solutions (Neuendorf, 2016). Then, the video records of group working were analyzed using more focused coding about the ways in which

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teachers approached these problems. In the final round of data analysis, the teachers' interviews were examined using thematic analysis to understand whether teachers thought the morning math sessions impacted their practice as mathematics teachers (Clarke & Braun, 2013).

Findings

In this proposal, we only present teachers' work on the Milk Chocolates problem. To introduce the Milk Chocolates problem, we showed a box of chocolates as shown in Figure 1. We also provided a model of this chocolate box and asked whether they could find the number of milk chocolates in the box without counting. Moreover, we explained to the teachers that we were not only interested in the correct answer to the problem but also the ways in which they got the answer. Therefore, we asked them to find the answer in as many different ways as they could and record a numerical expression that modeled their thinking.

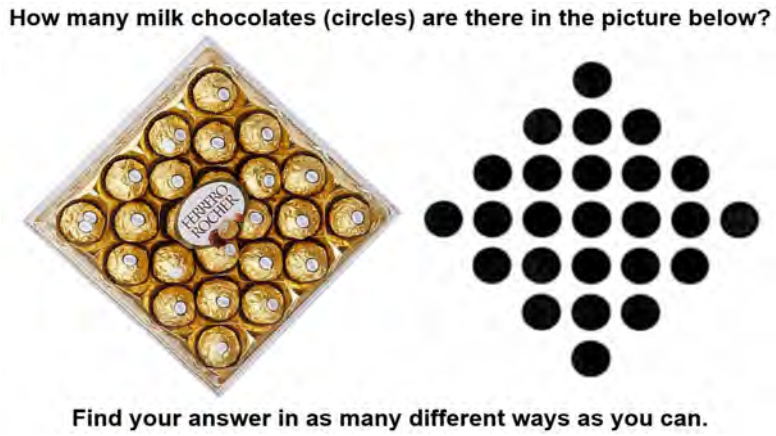

Figure 1. Milk Chocolates Problem (adapted from Balka & Hull, 2012).

Teachers' Content-Related Take-Aways

Teachers worked in groups of 2-3 and produced fifteen different solution methods as shown in Figure 2 below. As demonstrated by these solutions, teachers developed different ways of counting the chocolates in the box. When they started to work on this problem, they had not anticipated that there would be fifteen different solutions. As they solved the problem, we observed that they were changing their perspectives: "Ohh, okay. There might be a couple other ways to get the answer." As the discussion ensued and additional solutions were shared, they were very excited and engaged as they saw many different ways that their colleagues shared on the board.

This is also evident in their interviews that most of the teachers mentioned about more than one solution method when they were asked about morning math problems. The following excerpts illustrate this issue:

> Teacher A: I just learned different strategies during morning math and I learned, you know, not to give up and continue to keep trying.
>
> Teacher B: Well, there's definitely more than one strategy. One way to think about something that everybody takes a different way, and, um, it seems that when you work in groups, um, with the teacher walking around and kind of, like, monitoring.
>
> Teacher C: Just how many different ways people approach things. Because I try to think-- well, I seem to think kind of 1, 2, 3 stepwise. And then people are pulling out all this other stuff that I've never even dreamed of thinking of. So it's good to see all the different ways that people approach things.

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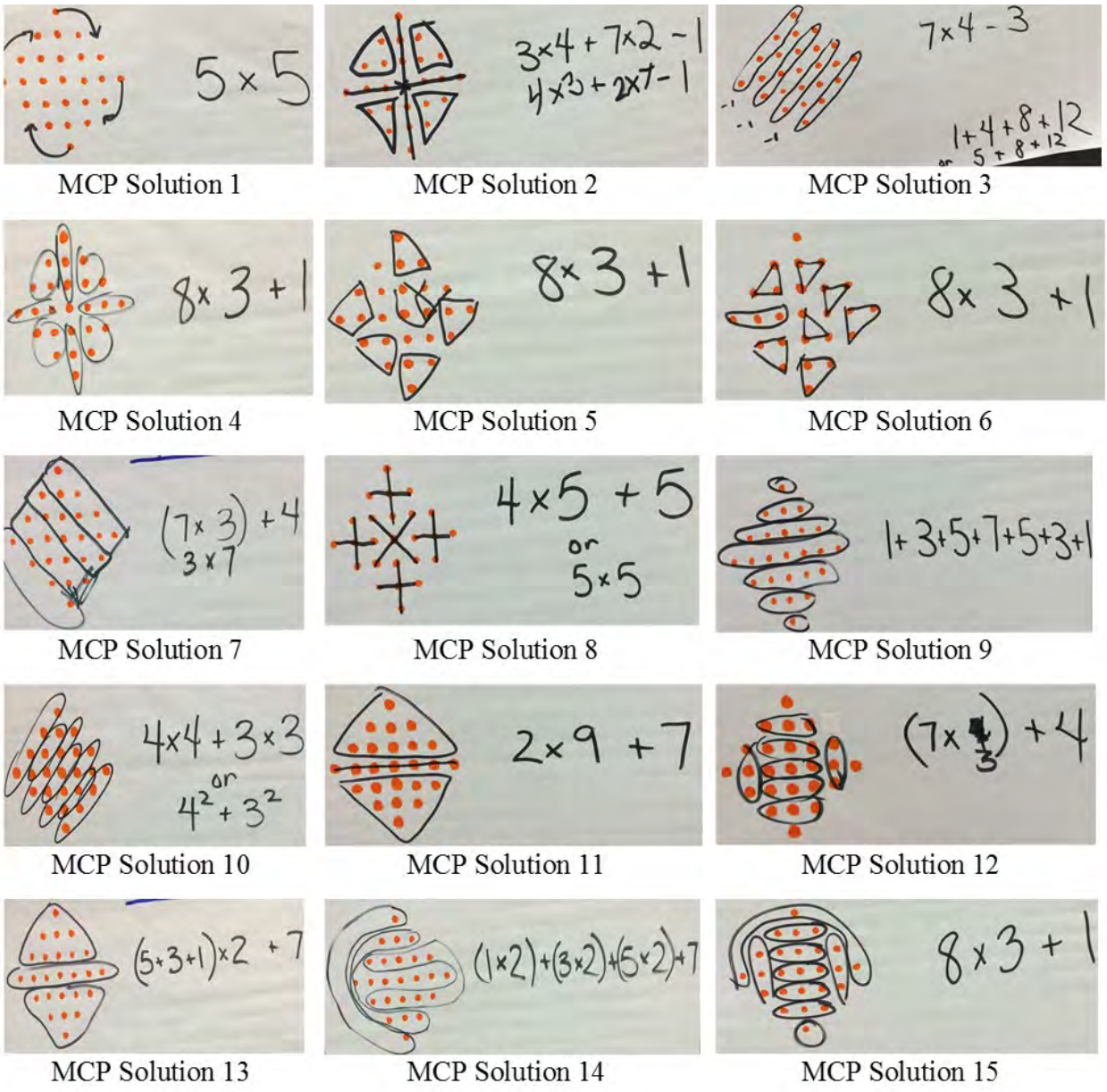

Figure 2. Teachers' solutions on Milk Chocolates Problem (MCP).

Like many other teachers, these three teachers highlighted different solution methods that were developed by other teachers and that they would not have anticipated otherwise. After sharing fifteen solutions to the milk chocolates problem, we asked teachers an extension question:

- What if the size of the box changes?
- Which method would you use to find the number of chocolates in these boxes (see Figure 3 below)?

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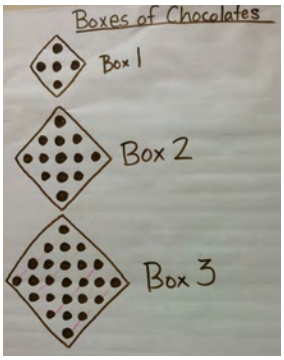

Figure 3. Milk Chocolates Problem Extension Part 1 (adapted from Goldenberg et al., 2015).

- If the pattern (shown in figure 3) continued, how could you find the number of chocolates in the box of any size? (i.e., Milk Chocolates Problem Extension Part 2)

In this extension part, teachers picked the method they thought to be more efficient to find the number of chocolates in Box n. It was challenging because not all of the solutions shown in Figure 2 led to a more general solution. Figure 4 presents four of the fifteen methods that teachers utilized to develop algebraic expressions.

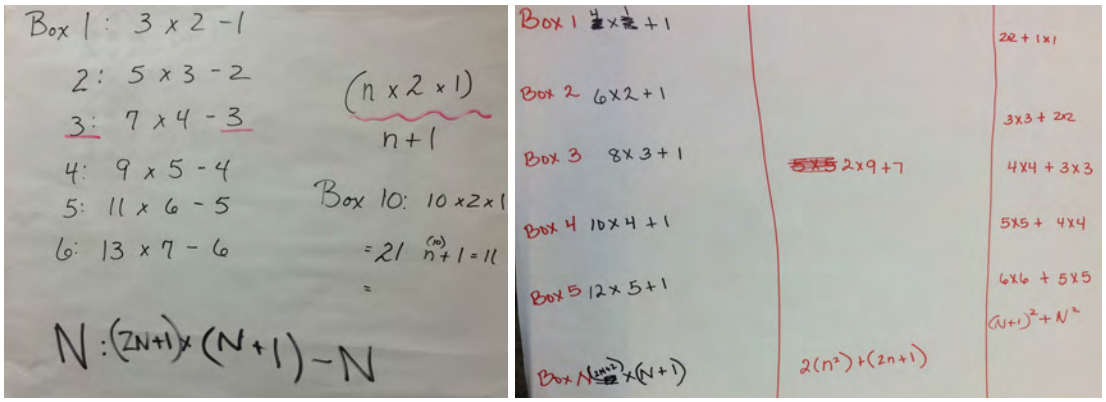

Figure 4. Teachers' Solution to the Milk Chocolates Extension Problem

After this work was shared by teachers, they engaged in a discussion about how they found different algebraic expressions and whether each of these expressions was a different one. During this discussion, teachers reached a conclusion that each of the methods showed them a pattern in counting, but these patterns resulted with the same simplified algebraic expression (i.e., $2n^2+2n+1$). Similarly, teachers had not thought that they would reach an algebraic expression with one unknown at the end of this problem, and they did, in fact, find the same expression even though they developed different solution methods.

Teachers' Teaching Practices-Related Take-Aways

During the interviews, teachers mentioned about morning math problems in relation to the 'selecting' and 'sequencing' practices:

> Teacher D: Well, we worked on--there's usually--definitely there's more than one way to answer the problems and all of them and the way that he [*PD trainer*] just placed them is all--generally, the concept--the overall concept that he's trying to get everybody to see, the real wow moment, he saves for the end, like when there's a formula, he saves them for the end, so

he's working simplest to most complex is what it looks like in, uh, showing the students work and that is a good strategy that I feel I could benefit from.

Teacher E: Well, it really has helped me to, um, well, learn that, uh, there's different ways that people think about a problem and that, um, you have to help, uh, persevere through some of the problems because they really help you to really think and make connections to other, um, other math concepts that you weren't even thinking at first.

Those teachers described that the morning math problem sessions went beyond only showing that multiple solutions existed. The multiple solutions were discussed, compared with one another and through engaging in the 'connecting' practice, wisely connected to a mathematical big idea. In fact, the open-ended nature of the problem (i.e. involving more than one answer), the potential for multiple solutions, and the opportunity to build understanding of an underlying math idea were three characteristics named by teachers as they describe the nature of the morning math problems and the quality of math problems in general.

Another important role of the morning math was that it required teachers to engage in productive struggle and that perseverance is important in the problem-solving process. The excerpt of a teacher below illustrates the sentiments of several teachers' in terms of the impact of morning math sessions in that the struggle and perseverance were necessary for learning new mathematical ideas.

Teacher F: Uh, I've learned--actually one, it's taking me back to, like, what children experience and how they have to persevere and just struggle through and go back--and go back to all those ideas and teachings that you learn from the past. So, it brought me back to that and understanding what they have to experience. … And then also, with the right coaching, with the right assistance through those tests, I think that that's been a really big eye-opener, um, made me think past first-grade math.

Interviewer: (laughs) and that's good or bad or…

Teacher F: Oh, that's good because, I mean, I like Math so I try to refresh and keep myself as, I want to say up-to-date as I can. Because not too long ago, I was looking at some Algebra II books and just, you know, just for the sake of time, just messing around and just refreshing it. … knowing how to persevere and understanding what children feel. … I think it makes anyone a better teacher because the more you see it, the more you understand that person's experiences, the better you're able to actually help them through their experiences.

Having first-hand experience of what students were experiencing while learning new mathematical knowledge helped teachers think more about how they could support students differently; thus, this experience helped them to improve their understandings and skills for "monitoring" practice. As many teachers also pointed out, one of the ways of enhancing student learning is by selecting good problems which are challenging but attainable and which will allow making real-life connections. In this vein, we could observe that teachers understood the importance of the practice zero, "setting goals and selecting tasks." To sum up, morning math sessions of the PD not only provided teachers direct experience with mathematical ideas but also demonstrated the ways in which such problems enhance students' learning.

Conclusions

Our aim in this particular study was to understand the role of morning math PD sessions on teachers' understanding of mathematics content and mathematics teaching. As illustrated with the data shared in this proposal, we argue that providing teachers direct experiences with mathematics problems is important for them to learn mathematical ideas conceptually, to understand the challenges that students experience as they learn, to appreciate the effectiveness of challenging, open-ended, and sense-making problems in learning mathematics, and to understand the role of

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cooperative learning in teaching mathematics. These findings are important for teacher educators and teacher education researchers since they indicate morning math problem-solving sessions as an effective means of professional development for mathematics teachers. However, the role of morning math sessions did not solely result from teachers' solving *any* mathematics problems. In fact, the problems which connected with real life situations, which had multiple solutions and sometimes multiple answers, and which were challenging but attainable, help teachers improve themselves in terms of mathematical content and mathematics teaching skills, particularly around five practices for orchestrating productive mathematics discussions because teachers also observed PD trainers as they monitored their work, selected and sequenced the solution methods, and connected with the big mathematical idea (Stein & Smith, 2011). By presenting teachers' take-aways from morning math sessions of this PD project, we provide insight to teacher educators and teacher education researchers for improving professional development sessions designed for mathematics teachers, which we expect to result in improvement in teachers' mathematics teaching practices.

References

Balka, D., & Hull, T. (2012). *Visible thinking activities: 23 lessons for problem solving*. Rowley, MA: Didax

Ball, D. L. (1990). Breaking with experience in learning to teach mathematics: The role of a preservice methods course. *For the Learning of Mathematics*, *10*(2), 10-16.

Clarke, V. and Braun, V. (2013) Teaching thematic analysis: Over-coming challenges and developing strategies for effective learning. *The Psychologist, 26*(2), 120-123.

Cochran-Smith, M., & Lytle, S. (1999). Relationships of knowledge and practice: Teacher learning in communities. *Review of Research in Education*, *24*, 249–305.

Goldenberg, E.P., Mark, J., Kang, J. M., Fries, M., Carter, C. J., & Cordner, T. (2015). *Making sense of algebra: Developing students' mathematical habits of mind.* Portsmouth, NH: Heinemann.

Grossman, P. L., & Richert, A. E. (1988). Unacknowledged knowledge growth: A re-examination of the effects of teacher education. *Teaching and Teacher Education*, *4*(1), 53-62.

Hiebert, J., Gallimore, R., & Stigler, J. W. (2002). A knowledge base for the teaching profession: What would it look like and how can we get one? *Educational Researcher*, *31*(5), 3–15.

Hill, H. C. (2007). Mathematical knowledge of middle school teachers: Implications for the No Child Left Behind policy initiative. *Educational Evaluation and Policy Analysis*, *29*(2), 95-114.

McGatha, M. (2009). Mathematics specialists and mathematics coaches: What does the research say? *National Council of Teachers of Mathematics Research Briefs*.

Moss, M. V. (2006). *Specialized understanding of mathematics: A study of prospective elementary teachers*. Retrieved from ProQuest Digital Dissertations (UMI No: 3235503).

Neuendorf, K. A. (2016). *The content analysis guidebook*. Los Angeles: Sage.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, *57*(1), 1-23.

Smith, M. S., & Stein, M. K. (2011). *Five practices for orchestrating productive mathematics discussions*. Reston, VA: NCTM.

Galindo, E., & Newton, J., (Eds.). (2017). *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Indianapolis, IN: Hoosier Association of Mathematics Teacher Educators.