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Cognitive Profiles Associated With Responsiveness to Fraction Intervention

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This study examined differences in cognitive processing between 4th-grade students who respond adequately, as opposed to inadequately, to intervention on 3 fraction outcomes: number-line estimation, calculation, and word problems. Students were assessed on 7 cognitive processes and on the 3 fraction outcomes. Students were grouped as adequate or inadequate responders, using as the cut-point the control-group mean on pre-to-post improvement on the relevant measure. Between-group differences identified reasoning, concept formation, and listening comprehension related to all 3 fraction outcomes. On the number-line outcome, within-group profile analysis indicated that inadequate responders experienced low reasoning ability relative to their other forms of cognitive processing.

Fraction knowledge is central to students' mathematical development (National Mathematics Advisory Panel [NMAP], 2008; Siegler et al., 2012; Siegler, Fazio, Bailey, & Zhou, 2013). Competence with fractions is a pivotal achievement in students' mathematics learning because it plays a foundational role in advanced mathematics learning - most notably algebra learning, the gatekeeper for higher learning in mathematics and science (Bailey, Hoard, Nugent, & Geary, 2012; Booth & Newton, 2012; NMAP, 2008). Yet fractions are consistently one of the most difficult mathematics topics for students to master (e.g., Bright, Behr, Post, & Wachsmuth, 1988; Hiebert, 1985; Perle, Moran, & Lutkus, 2005; Stigler, Givvin, & Thompson, 2010). This difficulty is often attributed to *whole-number bias*—the interference of students' whole-number knowledge in their development of the concept of fractions (Cramer, Post, & delMas, 2002; Cramer & Wyberg, 2009; Lamon, 1999; Post, Cramer, Behr, Lesh, & Harel, 1993; Siegler, Thompson, & Schneider, 2011). For example, students may consider fraction numerators and denominators as separate whole numbers, or may believe that the value of a fraction is dependent on the value of the numerator or denominator, rather than understanding that the numerator and denominator work together, as one number, to determine value (Mack, 1995; Stafylidou & Vosniadou, 2004). Although there does not yet appear to be consensus on the nature of the effects of whole-number bias, the prior knowledge and experience that students bring to number acquisition tasks does appear to play a significant role, at least in early fraction understanding (Ni & Zhou, 2005).

Although difficulty in fraction learning is not unique to at-risk students, it is especially common in students with mathematics learning difficulties (Algozzine, O'Shea, Crews, & Stoddard, 1987; Berch, 2016; Tian & Siegler, 2016), who comprise an estimated 5–7 percent of school-age children in the U.S. (Berch & Mazzocco, 2007; Geary, 2004, 2011). Failure to achieve competence with fractions can limit access to educational, economic, and employment opportunities (Adelman, 2006; NMAP, 2008).

With effective fraction intervention, however, at-risk students' fraction knowledge improves (Fuchs, Schumacher et al., 2016; Fuchs, Schumacher et al., 2013; Fuchs, Sterba, Fuchs, & Malone, 2016), and the gap between the level of fraction performance of at-risk and not-at-risk students narrows (Fuchs, Sterba et al., 2016). Yet an estimated 3–6 percent of students respond inadequately to mathematics interventions demonstrated to be generally effective (Fuchs & Fuchs, 2005; Fuchs, Fuchs, & Compton, 2013). The response deficit of this population of at-risk students highlights a critical need to increase understanding of the factors that influence students' response to fraction intervention.

The purpose of this study was to describe the cognitive processes of students who respond inadequately to generally effective fraction intervention. To provide context for this study, we begin this introduction by identifying and providing a rationale for the three strands of fraction outcomes investigated. Next, we summarize prior work on cognitive predictors of these strands, and provide an overview of studies examining cognitive moderators of responsiveness to fraction intervention. Finally, we specify the approach taken in this study.

Three Strands of Fraction Performance

We identified three fraction strands frequently investigated within the fraction literature: number-line estimation, calculation, and word-problem solving (e.g., Booth & Newton, 2012; Hecht, Close, & Santisi, 2003; Jordan et al., 2013). These strands parallel those for whole-number competence frequently used in prediction studies.

Number-line estimation with fractions indexes a student's ability to represent relative fraction magnitudes through

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the accuracy of placement of fractions on a number line. Understanding of fraction magnitudes is a critical component of fraction knowledge between grades three and five (Jordan et al., 2013; Vukovic et al., 2014) and a predictor of advanced mathematics achievement in areas such as algebra (Bailey et al., 2012; Fazio, Bailey, Thompson, & Siegler, 2014; Fuchs, Schumacher et al., 2013; Fuchs et al., 2014; Siegler et al., 2011). For example, Siegler et al. reported correlations of .54 to .86 between fraction number-line estimation and mathematics achievement at sixth and eighth grade.

Calculating with fractions is also a strong predictor of general mathematics achievement. Bailey et al. (2012) demonstrated that measures of fluency with fraction calculations significantly predict seventh-grade mathematics achievement. Siegler and Pyke (2013), who examined developmental and individual differences in sixth and eighth graders' fraction calculation skills, found significant relations with overall math achievement test scores, which increased with age. In this study, we examined addition and subtraction calculation with fractions, because this study focused on fourth grade (National Governors Association Center for Best Practices, 2010).

Less is known specifically about the role of word-problem solving involving fractions. Yet solving word problems is generally challenging for students who are at risk for mathematics difficulties (e.g., Jitendra, Hoff, & Beck, 1999; Montague, Enders, & Dietz, 1996). Moreover, word-problem solving is a strong school-age predictor of occupational success in adulthood (e.g., Hudson, Price, & Gross, 2009; Murnane, Willett, Braatz, & Duhaldeborde, 2001). For these reasons, in this study, we included fraction word problems as one of the three strands of fraction performance. Specifically, we included multiplicative fraction word problems that require students to make fractions from units, or to make units from fractions, using multiplicative reasoning. We were interested in multiplicative word problems because (a) multiplicative thinking is central to fraction knowledge, as reflected in the fact that finding equivalent fractions requires multiplying or dividing the numerator and denominator, and (b) multiplicative thinking with fractions can be counterintuitive, in part because multiplying two proper fractions results in smaller quantities, while dividing a proper fraction by a proper fraction produces larger amounts.

Prior Work on Cognitive Predictors of Fraction Performance

Across the three strands of fraction performance, studies focusing on cognitive predictors of fraction development suggest that aspects of language comprehension, attentive behavior, processing speed, reasoning, and working memory play a role in students' responsiveness to fraction intervention. The literature examining cognitive predictors of number-line estimation, calculation, and word problems is more extensive for whole numbers than for fractions. Yet evidence suggests that whole-number and fraction competence follow similar paths of development for this population and often rely on similar cognitive processes (Namkung & Fuchs, 2016). Some pathways to competence are, however, distinctive.

We identified three prior studies that explored underlying cognitive processes associated with students' whole-number number-line estimation (Bailey, Siegler, & Geary, 2014; Geary, Hoard, Nugent, & Byrd-Craven, 2008; Namkung & Fuchs, 2016) and fraction number-line estimation (Bailey et al., 2014; Namkung & Fuchs, 2016). Bailey et al. found that first-graders' working memory span significantly predicted eighth-grade fraction number-line estimation competence. Geary and colleagues administered a number line task to first- and second-grade students and found working memory span to predict various aspects of number line performance. Namkung and Fuchs, who examined shared and distinct predictors of whole-number versus fraction number-line estimation, found that the working memory span uniquely predicts whole-number number-line estimation, but not fraction estimation. Conversely, language comprehension uniquely predicted fraction number-line estimation, but not whole-number number-line estimation.

Prior work on cognitive predictors in the calculation strand is more extensive. Findings point to different aspects of working memory as important predictors for whole-number (Fuchs et al., 2010; Hecht & Vagi, 2011; Jordan et al., 2013; Seethaler, Fuchs, Star, & Bryant, 2011) and fraction calculations (Hecht et al., 2003; Siegler & Pyke, 2013). For example, Seethaler and colleagues identified working memory span as a unique predictor of third-grade students' whole-number calculation, while Siegler and Pyke found that eighth graders' updating and inhibitory working memory skills correlated with their fraction calculation accuracy. Attentive behavior as measured by teacher ratings (e.g., SWAN; Swanson et al., 2004) has also been identified as a predictor of whole-number calculations (Fuchs & Fuchs, 2005; Fuchs et al., 2006, 2008, 2010; Namkung & Fuchs, 2016) and fraction calculations (Fuchs, Fuchs et al., 2013; Hecht et al., 2003; Hecht & Vagi, 2011; Namkung & Fuchs, 2016). Indeed, working memory and attentive behavior are sometimes viewed as overlapping constructs (e.g., Awh & Jonides, 1998; Corbetta, Kincaid, & Shulman, 2002). Moreover, risk for math difficulties and poor attentive behavior often co-occur (Fletcher, Shaywitz, & Shaywitz, 1999; Gross-Tsur, Manor, & Shalev, 1996). Raghubar et al. (2009) found that students rated as less attentive made more calculation errors than students rated as more attentive. This finding makes sense because performing calculations accurately requires students to maintain attentive behavior in order to keep track of procedural steps. Further, inattentive students may have difficulty with inhibitory control (e.g., Bull & Scerif, 2001; Cepeda, Cepeda, & Kramer, 2000), making it difficult to avoid interference of incoming information or switch to a new set of procedures. Processing speed (i.e., speed of processing perceptual stimuli for similarities or differences) (Kaufman, Raiford, & Coalson, 2016), has also been explored as a predictor of whole-number and fraction calculations, with inconsistent results. While Seethaler and colleagues did not identify a significant association, other studies demonstrated a relation (Fuchs, Fuchs et al., 2013; Namkung & Fuchs, 2016). Automatic execution of tasks, reflected in processing speed, appears transparently involved in executing calculation procedures, and may be relevant to both whole-number and fraction calculations. Finally, prior studies of fraction competence suggest that listening comprehension is associated specifically with fraction calculations. For example, Seethaler and colleagues found that listening comprehension was a significant predictor of later fraction calculation performance.

In terms of word problems, we identified only one study that has explored the role of cognitive predictors specifically for fractions. Hecht et al. (2003) found that attentiveness in the classroom uniquely contributed to the fraction word-problem performance of fifth-grade students. A larger literature does exist for whole-number word problems. Fuchs et al. (2006), who focused on third grade, found listening comprehension and vocabulary to be uniquely predictive. Fuchs et al. (2006, 2010) also identified inductive reasoning, in the form of concept formation, as a predictor of whole-number word-problem solving. Jordan, Levine, and Huttenlocher, (1995) found that kindergarten and first-grade students with low language skills performed significantly worse than their typical peers on whole-number word problems.

Prior Work on Cognitive Moderators of Fraction Performance

Although limited, some studies have also identified cognitive processes as potential moderators of students' fraction performance. These cognitive moderators are baseline characteristics that interact with the fraction intervention to affect the outcome. For example, Fuchs, Geary et al. (2013) found that attentive behavior and language comprehension moderated first-grade students' responsiveness to arithmetic intervention on whole-number word-problem outcomes. Fuchs et al. (2014) and Fuchs, Schumacher et al. (2016) also found that working memory moderated the effect between two forms of fraction intervention for at-risk fourth-grade students on the number-line outcome, and in the 2016 study, reasoning ability moderated the effect of two forms of fraction intervention on word-problem outcomes. Even so, across the three strands of fraction performance, few studies have considered the child-level cognitive processes associated with responsiveness to fraction intervention. It is important to explore whether certain cognitive profiles have the potential to moderate the effects of fraction intervention, because the cognitive processes associated with inadequate response to intervention may provide insight into instructional techniques for addressing the needs of students with the most challenging learning problems. Additional research is clearly warranted.

This Study

In this study, we explored the cognitive processes involved in responsiveness to fraction intervention on fraction number-line estimation, fraction calculation (addition and subtraction), and fraction word problems. Although the literature on overall predictors of success with fractions, along with a smaller literature on moderators of responsiveness to fraction intervention, suggest that cognitive processes may differ for adequate responders versus inadequate responders, only a handful of studies have directly examined cognitive profiles in mathematics intervention generally and fraction intervention specifically. The key research question is, on which cognitive processes do inadequate responders demonstrate substantially low performance relative to their own performance on the other cognitive processes?

We extended the available literature by contrasting the cognitive processes of adequate versus inadequate responders in the context of fourth-grade fraction intervention by exploring whether conclusions vary as a function of analytic method for considering the cognitive processes associated with responsiveness. The first analytic procedure was within-group profile analysis, in which the shape of the cognitive profile for the inadequate responder group is examined for cognitive processes that differ from this group's other cognitive processes. The second analytic procedure was the more common approach: between-group differences that test mean-level differences between adequate and inadequate responder groups. We also extended the literature by exploring whether the cognitive processes associated with responsiveness differ as a function of fraction outcome. Our overall goal was to provide insight into directions for developing intervention methods that expand responsiveness for a greater proportion of students.

METHOD

Overview

The data described in this analysis were collected in a series of field-based randomized control trials examining the effects of an intervention designed to improve at-risk fourth-grade students' understanding of and procedural skill with fractions (Fuchs et al., 2014; Fuchs, Schumacher et al., 2016; Fuchs, Malone et al., 2016). In this section, we refer to these three investigations as Study A, Study B, and Study C. During these larger parent studies, participants were identified as at-risk based on low mathematics performance at the start of the school year, and then were randomly assigned to a business-as-usual control condition or one of two variants of a multi-component fraction intervention. Both before and after the study, students were assessed on fraction measures that included number-line estimation, calculations, and word problems. For the purposes of this study, due to the participants' grade level, the calculation strand of fraction performance addressed addition and subtraction. Additionally, at the start of the study, students were assessed on a set of cognitive processes.

For the purpose of the present analysis, three analytic samples were created from the students randomly assigned to one of the two variants of the multi-component intervention conditions. The *Number-Line Sample* and the *Calculation Sample* comprised students who received intervention on either variant (both focused on fraction magnitude understanding) in Studies A, B, and C. The *Word-Problem Sample* comprised students who received intervention on fraction magnitude understanding but also received intervention on multiplicative word problems in Studies B and C (see Figure 1).

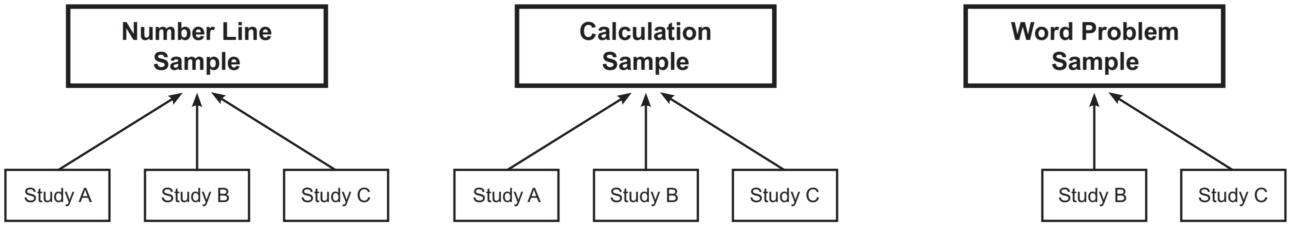

FIGURE 1 Origins of participants by analytic sample.

Next, we identified the responsiveness status of each student in each analytic sample as adequate or inadequate. The cut-point for responsiveness was the control-group mean on pre-to-post improvement on the relevant measure. We then analyzed each of the three samples to assess the cognitive variables associated with responsiveness for each of the fraction outcomes, using each of the two analytic methods: profile analysis and between-group differences.

Participants

Participants were drawn from a southeastern metropolitan school district. In the larger studies, they had been identified as being at risk for poor learning in fractions based on performance below the 35th percentile on a broad-based mathematics computation assessment (Wide Range Achievement Test–4 [WRAT-4]; Wilkinson & Robertson, 2006). Furthermore, because the larger studies were not focused on intellectual disability, we excluded students earning T-scores below the 9th percentile on both subtests of the Wechsler Abbreviated Scales of Intelligence (WASI; Wechsler, 1999). To ensure strong representation across the range of scores below the 35th percentile, we sampled approximately half the students from below the 15th percentile and half from between the 15th and 34th percentiles. For the present analysis, control group participants were excluded, because the interest was responsiveness to intervention; this exclusion left 448, 448, and 144 students in the Number-Line, Calculation, and Word-Problem analytic samples, respectively. See Tables 2 and 3 for demographic information and pre- and post-test performance on the cognitive processes and fraction measures.

Procedure

Data used for the present analysis were collected using a multi-step process. First, during August and September, examiners screened potential participants for whom parental consent was obtained, by administering the *WRAT-4-Math Computation* (Wilkinson & Robertson, 2006) in whole-class settings. Students who met the criterion for at-risk status were individually administered *WASI* and the cognitive process measures. Second, during September and October, examiners pre-tested students on the three fraction measures. *Fraction Addition* and *Fraction Subtraction* were administered in whole-class settings; *Fraction Number Line* was administered individually. During Studies B and C, examiners also administered *Multiplicative Word Problems* in whole-class settings. Third, students were post-tested on the three fraction measures in March, after intervention ended, using a parallel form of the pre-test. *Fraction Addition*, *Fraction Subtraction,* and *Multiplicative Word Problems* were administered in whole-class format; *Number Line* was administered individually. (*Multiplicative Word Problems* was not administered in Study A.)

All assessments were administered by trained examiners, blind to conditions, each of whom demonstrated acceptable fidelity (±90 percent) during mock assessment administrations. All test administration sessions were audio-recorded, and 20 percent of sessions, stratified by examiner, were randomly sampled and checked for fidelity by an independent scorer using a checklist. Agreement on test administration and scoring exceeded 97 percent. Testers were blind to conditions when administering and scoring tests.

Screening Measures

The mathematics screening measure was *WRAT-4-Math Calculations* (Wilkinson & Robertson, 2006), in which students completed calculation problems of increasing difficulty. Alpha on the primary studies' samples ranged from .74 to .87. The IQ screening measure was the *WASI* (Weschler, 1999). With *Vocabulary,* students identified pictures and defined words. With *Matrix Reasoning*, students selected the option that best completed a visual pattern. Reliability exceeded .92 on both measures (Sattler, 2008).

Fraction Measures

With the *Fraction Number-Line Task* (Hamlett, Schumacher, & Fuchs, 2011, adapted from Siegler et al., 2011), which was administered via computer, students placed common fractions and mixed numbers on a number line labeled with endpoints of 0 and 2. A fraction was presented below the number line, and students used a computer mouse to place the fraction on the number line. Students first practiced with two fractions and then estimated the location of 20 fraction items presented in random order. The score for each item was the absolute difference between the placement and the correct position. Scores were averaged across items, divided by two (the numerical range of the number line), and multiplied by 100 to derive the percent of absolute error. Lower scores indicated stronger performance (in some analyses, we multiplied scores by -1). Test-retest reliability, on 63 students across two weeks, was .80.

The calculation measure was from the *Fraction Battery* (Schumacher, Namkung, Malone, & Fuchs, 2013). *Fraction Addition* included five problems with like denominators and seven with unlike denominators; *Fraction Subtraction* included six problems with like denominators and six with unlike denominators. In each subtest, half the problems were presented vertically and half horizontally. One point was awarded for the correct numerical answer; 2 points if simplified (i.e., reduced) one time (7 addition items; 8 subtraction items; e.g., $\frac{1}{8} + \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$; $\frac{3}{4} + \frac{2}{4} = \frac{5}{4} = 1\frac{1}{4}$); 3 points if simplified two times (1 subtraction item: $\frac{10}{6} - \frac{2}{6} = \frac{8}{6} = 1\frac{2}{6} = 1\frac{1}{3}$). We used the total score across subtests ($r = .83$), with a maximum score of 41. Alpha on the parent studies' samples ranged from .90 to .94.

Multiplicative Word Problems, from the *Fraction Battery* (Schumacher et al., 2013), included six problems requiring students to make fractions from units (the "splitting" problem type), six problems requiring students to make units from fractions (the "grouping" problem type), and two distractor problems requiring students to compare fraction quantities (e.g., "Ruby ate $\frac{1}{4}$ of the pizza, and Bob ate $\frac{1}{8}$ of the pizza. Who ate less pizza?"). None of the tested problems was used for instruction. Two near-transfer splitting problems relied on the vocabulary and question structure used in instruction (e.g., "Lauren has 3 yards of ribbon. She cuts each yard of ribbon into sixths. How many pieces of ribbon does Lauren have now?"). Four far-transfer splitting problems included novel vocabulary or questions (e.g., "Jamie has 5 cups of batter to make cupcakes. Each cupcake needs $\frac{1}{2}$ cup of batter. How many cupcakes can Jamie make?" [novel vocabulary and question because *5 cups of batter* is the unit and *cupcakes* are the "pieces" in this problem, when students generally think of *cupcakes* as a unit]). Four near-transfer grouping problems incorporated unit fractions (e.g., "Dante is making 8 peanut butter bars. Each peanut butter bar needs $\frac{1}{4}$ cup of peanut butter. How many cups of peanut butter does Dante need?"); two far-transfer grouping problems included non-unit fractions (e.g., "Gabby needs to read 3 chapters in her book. Each chapter takes $\frac{2}{3}$ of an hour to read. How many hours does Gabby need to spend reading?"). The tester read each item aloud while students followed along and then completed their work on individual paper copies. Students were able to ask for one re-reading of each item. For each problem, students earned 1 point for the correct numerical answer and 1 point for the correct label (e.g., pieces of ribbon). The maximum score was 26 (for distractor problems, only 1 point can be earned, which is for finding the correct numerical answer). Alpha on the parent studies' samples ranged from .80 to .90.

Cognitive-Processing Measures

Reasoning

WASI Matrix Reasoning (Wechsler, 1999) measured reasoning with pattern completion, classification, analogy, and serial reasoning tasks. Students selected among five response options to complete a matrix with a missing section. At age 9, internal consistency reliability was .94. *Concept Formation* from the Woodcock-Johnson III (WJ-III; Woodcock, McGrew, & Mather, 2001) measures fluid inductive reasoning by requiring students to employ rule application and frequent switching from one rule to another. Median test reliability was .94.

Processing Speed

Cross Out from the WJ-III (Woodcock et al., 2001) measured processing speed by asking students to locate and draw a line through five pictures that match a target picture in that row. Students had 3 minutes to complete 30 rows. Reliability was .91.

Working Memory

To assess the working memory span, we used two subtests from the central executive scale of the *Working Memory Test Battery for Children* (WMTB-C; Pickering & Gathercole, 2001). Both included six dual-task items at span levels from 1–6 to 1–9. Passing four items within a level moved the child to the next level. At each span level, the number of items to be remembered increased by one. Failing three items terminated the subtest. We used the trials-correct scores. With *Listening Recall*, the child determined whether each sentence in a series is true, and then recalled the last word of each sentence. Test-retest reliability ranged from .84 to .93. With *Counting Recall*, the child determined how many objects are in an array and then recalled the series of counts in the trial. Test-retest reliability ranged from .82 to .91.

Listening Comprehension

With the *Woodcock Diagnostic Reading Battery - Listening Comprehension* (WDRB; Woodcock et al., 2001), students supplied the word missing at the end of sentences or passages that progress from simple verbal analogies and associations to discerning implications. At age 9, internal consistency reliability was .81.

Attentive Behavior

The *Strengths and Weaknesses of ADHD-Symptoms and Normal-Behavior* (SWAN; Swanson et al., 2004) samples items from the Diagnostic and Statistical Manual of Mental Disorders-IV-TR (4th ed., text rev.; DSM–IV–TR; American Psychiatric Association, 2000) criteria for Attention Deficit Hyperactivity Disorder for inattention (9 items) and hyperactivity-impulsivity (9 items), but scores are normally distributed. Teachers rated items on a 1–7 scale. We reported data only for the inattentive subscale as the average rating across the nine items. The SWAN correlates well with other dimensional assessments of behavior related to attention. Alpha for the inattentive subscale on the parent studies' samples was .96.

Intervention

In all three larger studies, students who were assigned to the intervention conditions received a base fraction intervention in small groups of two-three students for 30–35 min per day, three days per week, for 12 weeks (three lessons per week) from late October to early February. Each of 36 lessons (35 minutes each) was delivered in groups of two students. Students in Studies B and C also received instruction on multiplicative word problems.

Base intervention

The base intervention provided students with explicit instruction using a multi-component fraction intervention referred to as *Fraction Face-Off!* (Fuchs, Schumacher, Malone, & Fuchs, 2015). *Fraction Face-Off!* was organized into a manual of lesson guides providing models of each lesson and the language of explanations. Each 35-minute lesson comprised six activities. Activity names reflected a sports theme. During "Word-Problem Warm-Up" (7 min; introduced in lesson 7), students received instruction on word problems. During "Training" and "Relay" (20 min combined), students received instruction on fraction magnitude understanding. "Sprint" (2 min; introduced in lesson 10) provided strategic, speeded practice on four fraction magnitude topics: identifying whether fractions are equivalent to $\frac{1}{2}$; comparing the values of proper fractions; comparing the values of a proper and an improper fraction; and identifying whether numbers are proper fractions, improper fractions, or mixed numbers. During the "Individual Contest" (5 min) and "Scoreboard" (1 min), students independently completed cumulative review in the form of paper-pencil problems based on that day's Training topics. Tutors then scored students' work and provided corrective feedback. In the first 3 weeks, the Training and Relay were extended to account for the full 35 minutes. In the last 2 weeks, the Training and Relay were replaced with the "Fraction Championship," in which students competed by solving fraction problems of varying difficulty, with differing predetermined point values. Number lines, fraction tiles, and fraction circles were used to explain concepts throughout the lessons.

Fraction Magnitude Instruction

The primary focus of the base intervention was fraction magnitude understanding, which was provided during the Training and Relay activities. During Training, tutors introduced fraction magnitude concepts, skills, problem-solving strategies, and procedures, while relying on manipulatives (e.g., fraction tiles and fraction circles) and visual representations. Initial instruction provided introductory concepts (e.g., fraction vocabulary definitions, naming fractions), and relied on a combination of part/whole relations (e.g., objects with shaded regions) and equal sharing examples to build on prior knowledge and classroom instruction; the focus then emphasized the roles of the numerator and denominator. Instruction then centered around representing, comparing, ordering, equivalencies, and placing fractions on number lines in relation to benchmark fractions. For example, tutors introduced strategies for comparing two fractions with unlike numerators and denominators, and taught students to convert between improper fractions and mixed numbers. See Table 1 for the sequence in which topics were introduced. Relay involved group work on concepts and strategies taught during that day's Training. Students took turns completing problems while explaining their work to the group. All students simultaneously showed work for each problem on their own papers. Note that after a topic was introduced, it was cumulatively reviewed, and that approximately 85 percent of content was allocated to understanding fractions and word problems (rather than calculations).

TABLE 1
Topics Introduced by Week

Week(s)	*Topic*
1–2	Fraction foundations Key vocabulary: numerator, denominator, unit, equivalent, equal parts Meaning of fractions (equal sharing, part-whole, quotients) Role of numerators vs. denominators Naming fractions Comparing fractions with like N's, like D's, and fractions equivalent to one whole Proper and improper fractions equal to one
3–5	Magnitude reasoning when comparing 2 fractions and ordering 3 fractions $\frac{1}{2}$ and equivalencies ($\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$, $\frac{6}{12}$) as benchmarks to compare fractions with unlike numerators and denominators Placing 2 fractions on 0–1 number lines marked with 0, $\frac{1}{2}$, and 1 Word problems
6	Improper fractions and mixed numbers >1 and <2 0–2 number lines Converting between and equivalent properties of improper fractions and mixed
7–8	Comparing, ordering, and number-line activities integrating proper fractions, improper fractions, and mixed numbers
9	Adding/subtracting proper and improper fractions, first with like D's, then with unlike D's
10	Adding mixed numbers Adding/subtracting mixed numbers Removal of $\frac{1}{2}$ from 0–1 and 0–2 number lines Removal of 1 from 0–2 number lines
11–12	Equivalencies for $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ via multiplication Review

Note. After a topic was introduced, cumulative review occurred thereafter. Note that none of the number-line activities relied on computers, as done in the pre/posttest assessment task. Adapted from Fuchs, Schumacher et al. (2016).

Multiplicative Word-Problem Intervention

Multiplicative word-problem instruction was introduced in lesson 7 and comprised approximately 7 minutes of each tutoring session. Instruction relied on schema theory, through which students categorize word problems as belonging to one of two word-problem types based on their underlying mathematical structure and then apply a word problem-solving strategy specific to that type (e.g., Fuchs et al., 2010; Jitendra & Star, 2012). With schema-based instruction, students are taught to represent the underlying structure of the word-problem type with a number sentence (e.g., Fuchs et al., 2009; Jitendra, DiPipi, & Perron-Jones, 2002) or visual display. Students conducted their work on individual worksheets by first labeling the word problem with the determined problem-type category and then carrying out the appropriate strategy in the space below the word problem. The two multiplicative word-problem types were "Splitting" and "Grouping."

Splitting word problems describe a unit being cut, divided, or split into equal parts (e.g., "Melissa had 2 lemons. She cut them in half. How many pieces of lemon did she have?"). Tutors introduced Splitting word problems by first presenting an intact story (no missing value or questions) and using fraction circles (units and halves) to illustrate the meaning of the narrative. Tutors then demonstrated a worked example, providing a rationale for each step of the word problem-solving strategy. Tutors then taught students a series of strategic steps to help students organize their work, synthesize information in the word problem, and solve the problem. First, students underlined the unknown amount; in this example, students underlined "pieces of lemon." Second, they identified and labeled the units and the size of each piece. In this example, students wrote "U" above "2 lemons" and "S" above "half." Third, students created an array to represent the underlying numeric structure of the units and sizes within the word problem in order to show how each unit divided into fractional pieces (e.g., for each unit divided into fifths, $\frac{1}{5}$ was written five times to represent each piece for each unit). Finally, students solved the word problem and wrote their numerical answer and word label.

During lesson 16, tutors introduced Grouping word problems (e.g., "Keisha wants to make 8 necklaces for her friends. For each necklace, she needs $\frac{1}{2}$ of a yard of string. How many yards of string does Keisha need?"). In Grouping problems, students were required to make units from fractions by determining the number of fractional pieces needed to comprise the given group. First, students identified the "items" (in this example, necklaces) that represent the fractional pieces needed to comprise the group. Students then solved the problems using parallel methods to Splitting problems, but with an array representing the underlying structure distinct to Grouping problems.

Once both word problem types were taught, practice included *distractor* word problems with the aim of increasing students' ability to recognize non-examples of the taught word-problem types, thus decreasing the tendency to overgeneralize strategies. See Fuchs, Schumacher et al. (2016) and Fuchs, Malone et al. (2016) for additional information on word-problem instruction, tutor training, materials structure, activities, and promotion of task-oriented behavior.

Control-Group Instruction

Students assigned to control received classroom fraction instruction relying on *enVisionMATH* (Scott Foresman-Addison Wesley, 2011), the district's mathematics curriculum. At fourth grade, the fraction units are "Understanding Fractions" and "Adding and Subtracting Fractions." The program relies mainly on part-whole understanding by using shaded regions and other area-model manipulatives.

Distinctions between Intervention and Control

In terms of content, there were three major distinctions between the control group and the two intervention conditions. First, the control group focused predominantly on part-whole understanding, whereas both intervention conditions emphasized the magnitude understanding of fractions. Second, the control group addressed some advanced skills not covered in the intervention conditions, such as estimation and word problems. Third, the control group did not restrict the range of fractions, whereas the intervention conditions limited the pool of denominators to 2, 3, 4, 5, 6, 8, 10, and 12, and the pool of equivalent fractions and reducing activities to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{1}$. The amount of mathematics instructional time was similar for intervention and control students.

DATA ANALYSIS AND RESULTS

To designate responsiveness, the cut-point was the control-group mean on pre- to-post improvement on the relevant measure. Although the control group was used to designate responsiveness status for students receiving the intervention, control group data were otherwise excluded from analyses (because those students did not receive intervention). The percentage of students designated as inadequate responders was 13.8 for the Number-Line Sample, 4.2 for the Calculation Sample, and 11.8 for the Word-Problem Sample.

Demographic Comparability of Adequate and Inadequate Responsiveness Groups

We examined frequencies for gender, ethnicity, subsidized lunch, special education status, and English-language learner status for each analytic sample (see Table 2). Chi-square tests indicated no significant differences between adequate and inadequate responders by gender, ethnicity, subsidized lunch, or English-language learner status. As might be anticipated, however, students designated as inadequate responders were significantly more likely to qualify for special education services than were students designated as adequate responders: Number-Line Sample, χ^2 (1, $N = 448$) = 22.52, $p = .002$; Calculation Sample, χ^2 (1, $N = 448$) = 23.24, $p = .002$; Word-Problem Sample, χ^2 (1, $N = 144$) = 37.56, $p < .001$.

Cognitive Processes Associated with Responsiveness

To examine the cognitive processes associated with responsiveness, we conducted two types of analyses, separately for each analytic sample. The first procedure was within-group

TABLE 2
Demographic Information for Study Participants by Responder Group by Analytic Sample

	Number Line Sample		*Calculation Sample*		*Word Problem Sample*	
	(n = 448)		*(n = 448)*		*(n = 144)*	
Characteristic	*Adequate Responders (n = 386) n (%)*	*Inadequate Responders (n = 62) n (%)*	*Adequate Responders (n = 429) n (%)*	*Inadequate Responders (n = 19) n (%)*	*Adequate Responders (n = 127) n (%)*	*Inadequate Responders (n = 17) n (%)*
Gender						
Female	221 (57%)	38 (61%)	248 (58%)	11 (58%)	72 (57%)	6 (35%)
Male	165 (43%)	24 (39%)	181 (42%)	8 (42%)	55 (43%)	11 (65%)
Race						
Black	219 (57%)	36 (58%)	243 (57%)	11 (57%)	72 (57%)	9 (56%)
Hispanic	63 (16%)	8 (13%)	67 (16%)	5 (26%)	21 (17%)	6 (19%)
White	90 (23%)	15 (24%)	102 (24%)	3 (16%)	30 (24%)	2 (22%)
Biracial	8 (2%)	0 (0%)	10 (2%)	0 (0%)	2 (1%)	0 (1%)
Other	6 (2%)	3 (5%)	7 (2%)	0 (0%)	2 (1%)	0 (1%)
Low Income	354 (92%)	57 (92%)	393 (92%)	19 (100%)	117 (92%)	15 (92%)
English Language Learner	60 (16%)	11 (18%)	67 (16%)	4 (21%)	18 (14%)	2 (5%)
Special Education	35 (9%)	15 (24%)	46 (11%)	5 (26%)	11 (9%)	10 (15%)

Note. Analytic samples did not differ significantly on demographic characteristics ($p > .05$). Low-income status denotes qualification for subsidized lunch. Special Education status denotes qualification for special education services.

profile analysis, in which the shape of the cognitive profile for the inadequate responder group is examined for cognitive processes that differ from this group's other cognitive processes. The second analytic procedure was the more common approach: between-group differences that test mean-level differences between adequate and inadequate responder groups (See Table 3).

Profile Analysis

For methods and applications of profile analysis, see Bernstein, Garbin, and Teng (1988) and Fletcher, Shaywitz, Shankweiler, Katz, and Al (1994). We conducted profile analysis in four steps. First, we transformed data from each cognitive process onto the same scale by calculating z-scores across the participants. This was done separately for each of the three fraction outcome samples. Second, for each responsiveness group within each analytic sample, we calculated the *elevation* of the group's cognitive performance by deriving the grand mean across the z-scores for the seven cognitive processes. As expected, for each type of fraction outcome (each analytic sample), the grand mean was higher for the adequate than for the inadequate responder group.

The main focus of profile analysis, however, is not elevation but rather *shape*, in which differences among the cognitive processes within the inadequate responder group are of primary interest. To isolate the shape effect, our third step was to subtract the grand mean from the mean performance level of each of the seven means. Isolating the shape effect was done for each of the two responder groups for each of the three fraction outcome samples. With elevation removed, the plotted cognitive profile for the inadequate responder group more clearly reveals greater variability in the cognitive profile for the adequate responder group. Finally, for the adequate and inadequate responder groups separately for each fraction outcome sample, we identified which cognitive processes were more than 1 standard error of measurement above or below the inadequate responder group's grand mean.

These profiles are plotted in Figure 2, which shows relatively flat cognitive process scores for adequate responders compared to the profiles of inadequate responders. The flatness, relative to the shape of the inadequate responder group, indicates that the responders performed more similarly across the various measures than did the less responsive group. Within the Number-Line Sample, inadequate responder performance on reasoning and counting recall was more than one standard error of measurement from the group's grand mean: reasoning was below the grand mean; counting recall, above the grand mean. By contrast, for adequate responders, none of the mean values was discrepant from that group's grand mean. For the Calculation Sample and the Word-Problem samples, none of the cognitive variables was more than 1 standard error away from the inadequate responder subgroup's grand mean, as was the case for the adequate responder group.

Between-Group Mean Differences

We next tested whether the performance of the adequate responder group was significantly different from the performance of the inadequate responder group. We did this for each cognitive process, separately for each fraction outcome sample. In Table 4, for each of the three fraction outcome samples and each of the seven cognitive processes, we present means and standard deviations as well as F-values and effect sizes, calculated on sample-based z scores for the adequate and inadequate responder groups.

TABLE 3
Means and Standard Deviations of Raw and Standard Cognitive Process Performance by Responder Group by Analytic Sample

	Number Line Sample				Calculation Sample				Word Problem Sample			
	Adequate Responders (n = 386)		*Inadequate Responders* (n = 62)		*Adequate Responders* (n = 429)		*Inadequate Responders* (n = 19)		*Adequate Responders* (n = 127)		*Inadequate Responders* (n = 17)	
Cognitive Process	*Raw Score M (SD)*	*Standard Score M (SD)*	*Raw Score M (SD)*	*Standard Score M (SD)*	*Raw Score M (SD)*	*Standard Score M (SD)*	*Raw Score M (SD)*	*Standard Score M (SD)*	*Raw Score M (SD)*	*Standard Score M (SD)*	*Raw Score M (SD)*	*Standard Score M (SD)*
Reasoning	17.4 (5.9)	46.9 (9.7)	12.3 (5.0)	39.0 (8.1)	16.8 (6.0)	46.0 (9.9)	13.8 (6.4)	41.6 (9.7)	17.4 (6.1)	N/A	13.8 (5.8)	N/A
Processing Speed	14.7 (3.1)	91.2 (12.7)	13.4 (3.0)	86.2 (13.9)	14.6 (3.0)	90.8 (12.4)	13.5 (5.0)	85.6 (22.7)	14.8 (3.2)	91.9 (13.8)	12.2 (3.2)	80.9 (13.1)
Concept Formation	15.3 (16.0)	86.8 (11.4)	11.8 (5.8)	80.3 (11.5)	14.9 (6.0)	86.1 (11.6)	11.9 (6.1)	79.3 (11.2)	15.5 (6.1)	86.7 (11.6)	11.2 (5.0)	77.3 (9.9)
Listening Comprehension	21.1 (3.6)	90.4 (14.4)	18.8 (4.0)	82.2 (12.3)	20.9 (3.7)	89.5 (14.3)	18.9 (4.6)	83.4 (14.1)	21.6 (3.4)	92.3 (14.7)	19.6 (3.6)	84.4 (10.9)
Listening Recall	9.8 (3.2)	89.0 (17.1)	8.5 (3.2)	82.7 (15.9)	9.7 (3.2)	88.4 (17.1)	8.0 (3.0)	80.3 (14.2)	10.0 (3.4)	90.2 (17.3)	9.1 (2.9)	85.0 (15.0)
Counting Recall	16.6 (4.4)	77.3 (15.1)	15.5 (3.8)	72.8 (13.3)	16.5 (4.4)	76.9 (14.9)	15.1 (4.5)	71.0 (15.6)	16.8 (4.6)	78.0 (15.5)	15.5 (4.4)	73.0 (14.9)
Attentive Behavior	35.1 (11.1)	N/A	30.5 (10.2)	N/A	34.6 (11.1)	N/A	30.6 (10.5)	N/A	4.3 (11.6)	N/A	27.2 (9.3)	N/A

Note. Reasoning is-WASI Matrix Reasoning. Processing Speed and Concept Formation are from the Woodcock-Johnson III. Listening Comprehension is from the Woodcock Diagnostic Reading Battery. Listening Recall and Counting Recall are from the Working Memory Test Battery for Children. Attentive Behavior is from the Strength and Weaknesses of ADHD-Symptoms and Normal-Behavior. Standard scores are mean = 100 (SD = 15) except reasoning (mean = 50, SD = 10).

As shown, for the Number-Line Sample, significant differences were found between adequate and inadequate responders on reasoning, processing speed, concept formation, listening comprehension, listening recall, and attentive behavior. For the Calculation Sample, significant differences were found between adequate and inadequate responders on reasoning, concept formation, listening comprehension, and listening recall. For the Word-Problem Sample, significant differences were found between adequate and inadequate responders on reasoning, processing speed, concept formation, listening comprehension, and attentive behavior.

DISCUSSION

The purpose of this study was to explore the cognitive processes associated with at-risk students' responsiveness to generally effective fraction intervention. This study extends this literature by focusing on three key outcomes (number-line estimation, calculations, and word problems), while examining a large battery of cognitive processes and contrasting two analytic procedures: within-group profile analysis and between-group differences. Here we discuss findings, directions for future research, and implications for practice and remediation – first for within-group profile analysis, and then for between-group differences.

On the fraction number-line outcome, within-group profile analysis revealed distinctively low reasoning ability for the inadequate responders relative to this group's own performance on the other six cognitive processes. In other words, reasoning was a key limitation for students who responded inadequately to the generally effective fraction intervention on the number-line outcome. This finding corroborates previous studies demonstrating a role for reasoning ability in responsiveness to mathematics intervention (Fuchs, Geary et al., 2013; Fuchs, Malone et al., 2016).

In this study, reasoning was operationalized with the WASI Matrix Reasoning task, which requires students to identify visual patterns and relationships while inferring and adhering to analytical rules (Nutley et al., 2011). Estimating the placement of fractions on number lines that are marked only with endpoints (in this study, 0 and 2) transparently engages this form of visually demanding reasoning. It requires students to interpret visual stimuli and to understand and compare the magnitude of the target fraction against strategically selected *benchmark* fractions along the number line (e.g., $\frac{1}{2}$ and $\frac{3}{4}$ serve as benchmarks for the target fraction $\frac{5}{8}$), while engaging relational thinking to determine where the target fraction is situated in relation to the benchmark fractions.

Accordingly, to support the at-risk sample's success with this challenging task, the generally effective fraction intervention not only incorporates a strong emphasis on understanding how numerators and denominators operate together to determine fraction magnitude. It also explicitly teaches strategies, and the conceptual basis for those strategies, for placing fractions on number lines marked only with endpoints. These strategies, which involve marking the paper number lines with the benchmark fractions and other

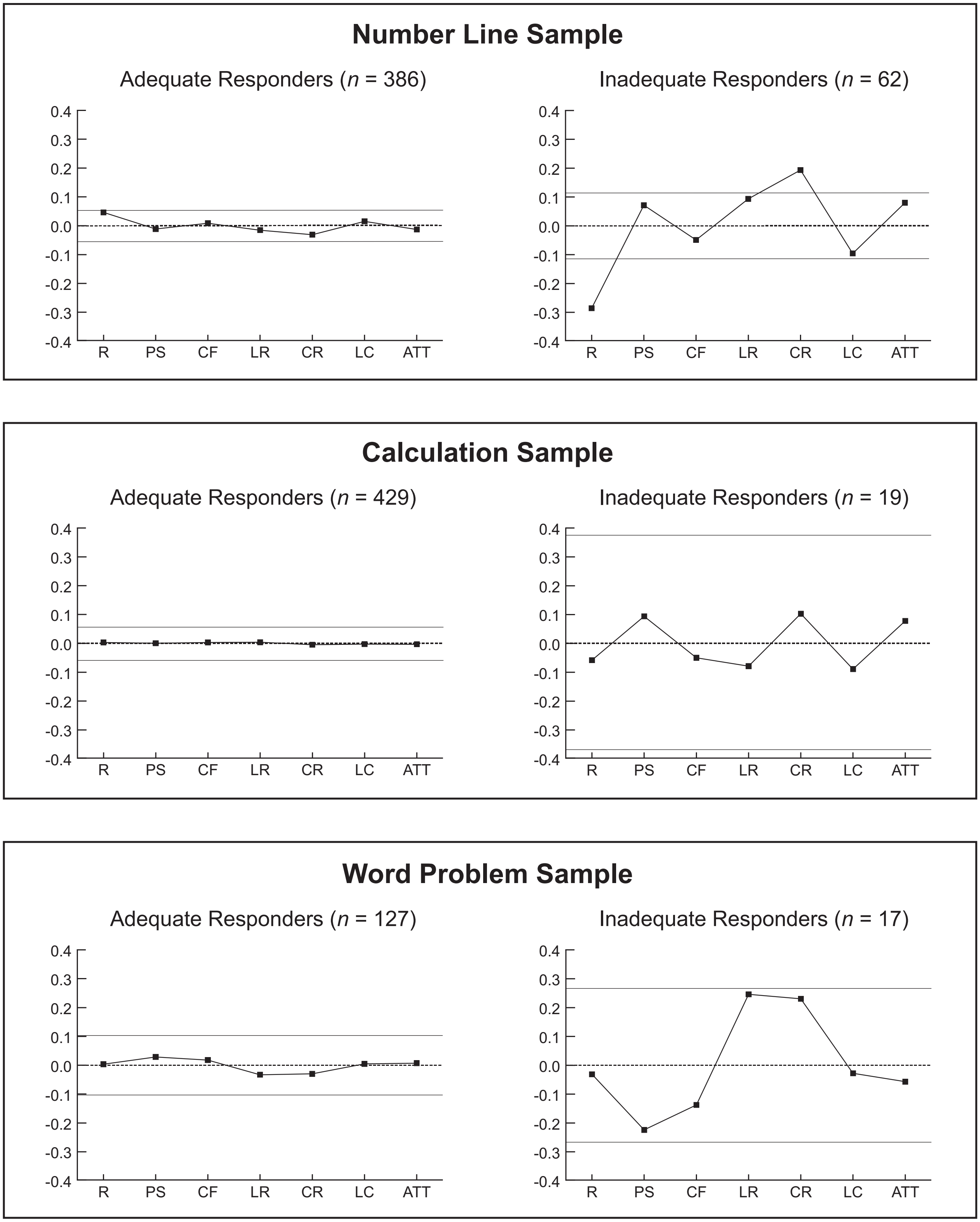

FIGURE 2 Cognitive profiles by responsiveness to intervention.

TABLE 4
Means and Standard Deviations on Sample-Based z-Scores With F-Values and Effect Sizes by Responder Group by Analytic Sample

	Number Line Sample						Calculation Sample						Word Problem Sample					
	Adequate Responders ($n = 386$)		Inadequate Responders ($n = 62$)				Adequate Responders ($n = 429$)		Inadequate Responders ($n = 19$)				Adequate Responders ($n = 127$)		Inadequate Responders ($n = 17$)			
Cognitive Dimension	*M*	*SD*	*M*	*SD*	*F*	*ES*	*M*	*SD*	*M*	*SD*	*F*	*ES*	*M*	*SD*	*M*	*SD*	*F*	*ES*
Reasoning	0.12	0.98	−0.73	0.83	41.73	−0.88*	0.02	0.99	−0.49	1.06	4.72	−0.51*	0.07	1.00	−0.51	0.95	5.18	−0.58*
Processing Speed	0.06	1.00	−0.36	0.96	9.54	−0.42*	0.01	0.97	−0.34	1.62	2.26	−0.35	0.09	0.97	−0.71	0.97	10.29	−0.82*
Concept Formation	0.08	0.99	−0.49	0.95	17.77	−0.58*	0.02	0.99	−0.48	1.00	4.56	−0.50*	0.08	1.00	−0.62	0.81	7.77	−0.71*
Listening Comprehension	0.09	0.96	−0.54	1.08	22.24	−0.64*	0.01	0.98	−0.52	1.23	5.21	−0.53*	0.07	1.02	−0.51	1.04	5.22	−0.57*
Listening Recall	0.06	0.99	−0.35	0.99	8.81	−0.41*	0.03	1.00	−0.51	0.93	5.14	−0.54*	0.03	1.00	−0.24	0.85	1.09	−0.27
Counting Recall	0.04	1.02	−0.23	0.87	3.80	−0.27	0.02	1.00	−0.32	1.02	2.09	−0.34	0.04	0.98	−0.25	0.96	1.24	−0.30
Attentive Behavior	0.06	1.00	−0.36	0.92	9.70	−0.42*	0.02	1.00	−0.35	0.95	2.41	−0.37	0.07	1.00	−0.54	0.80	5.81	−0.62*

Note. Scores are reported as *z* scores. Effect sizes are reported as Cohen's *d*. Significant effects are denoted with an asterisk ($p < .035$).

notations (e.g., marking the left side with the letter "L" for "less" and the right side with the letter "G" for greater) to narrow down the estimate of where the fraction belongs, were systematically faded over time and eventually invoked on an as-needed basis for instructional purposes. On the computer number-line outcome measure, however, struggling students could not mark the number line with supportive notations in any way, due to the administration mode of the task (i.e., computer instead of paper and pencil). For all these reasons, it is not surprising that students who failed to respond adequately to this intervention were characterized, via the profile analysis, as experiencing very low visual-reasoning ability.

The percentage of students who met our benchmark for *in*adequate response on the number-line outcome was not inconsequential: 62 of the 448 students. Given that the fraction number-line task is a key indicator of future success with more advanced mathematics (Siegler et al., 2012), identifying ways to extend intervention to address these students' limitations in reasoning ability is critical. For example, inadequately responsive students may require an expanded or more innovative instructional approach, perhaps with additional reasoning-based activities and with activities that deliberately exercise visually oriented proportional reasoning in connection with number lines. There are, however, processing abilities that underpin the visual reasoning needed to accurately place a fraction on a number line. For example, the deficit may stem from poor spatial skills, as spatial skills have been shown to predict linear number-line knowledge (e.g., Gunderson, Ramirez, Beilock, & Levine, 2012; LeFevre et al., 2013). Further research is needed to explore the association between spatial skills and fractional number-line placement to determine whether training of spatial skills may be advisable, as suggested in other forms of mathematics performance (Cheng & Mix, 2011; Uttal et al., 2013).

At the same time, it is interesting to consider that the cognitive profile analysis also revealed that inadequate response on the number-line task was associated with strong performance on the counting recall task (a form of working memory span) relative to these inadequate responders' own performance on other cognitive processes. Working memory span seems transparently involved in supporting students' placement of fractions on the number line. For example, making a considered placement of $\frac{5}{12}$ on a 0–2 number line involves identifying $\frac{1}{2}$ as a strategic benchmark and holding this visual marker, while the student determines the denominator of that fraction equivalent ($\frac{6}{12}$) and comparing $\frac{5}{12}$ to $\frac{6}{12}$ to determine that $\frac{5}{12}$ is less than $\frac{1}{2}$. Yet the counting recall task, which involves whole numbers, may not be the best form of working memory to tap inadequate responsiveness on the fraction number-line placement, .

One might have expected relatively strong working memory span capacity to compensate for limitations in reasoning. Yet the present pattern of findings, in which students with marked limitations in reasoning accompanied with relative strength in working memory span suffered poor responsiveness, suggests otherwise. Perhaps the subset of students with this combination of relative strength and weakness relied to an inadvisable extent on working memory span to place fractions on the number line, without sufficient emphasis on the need to exercise the visually demanding reasoning strategies required to accurately place fractions on an unmarked number line. Speculation aside, this finding underscores the importance of reasoning ability for success with the challenging number-line task.

On the most procedural outcome, fraction calculations, the percentage of inadequate responders was substantially lower (4.2 vs. 13.8 for the number-line outcome). Given the small number of inadequate responders ($n = 19$) on the fraction calculations outcome, the absence of distinctive cognitive processes is not surprising. It may be due to inadequate power and unreliability in the shape of the profile analysis.

Even so, on the more conceptually demanding fraction word-problem outcome, which places similar reasoning demands, and where the rate of inadequate response was more similar (11.8 percent) to that of the Number-Line Sample (13.8 percent), no reliable pattern of cognitive processes was associated with inadequate response. Additional research to identify innovative directions for extending fraction word-problem intervention is clearly warranted, as word-problem solving is a strong predictor of later employment and wages (Parsons & Bynner, 1997).

Given that the within-group profile analysis revealed few distinctive cognitive processes for the inadequate responder groups, it is interesting to consider results of the second analytic procedure, in which differences *between* adequate and inadequate response groups were tested. This between-group analysis represents a more common approach for investigating such distinctions. It is, nevertheless, a less stringent method due to the elevated performance *across* cognitive processing variables for the adequate responder group. Profile analysis removes this elevation.

It is not surprising that the between-group-differences approach identified many more cognitive weaknesses compared to the profile analyses' focus on within-group shape. For all three fraction outcome samples, inadequate responders scored significantly lower than adequate responders on three cognitive processes: reasoning, concept formation, and listening comprehension. This pattern suggests that competence in the three fraction strands develops similarly and relies on common cognitive resources representing the kinds of higher-order cognitive processing expected for handling fractions. However, given that development of competence in the three fraction strands involves varied knowledge types (i.e., number-line task relies on number sense; fraction calculations rely on procedural knowledge; word-problem solving relies on understanding of schema), this pattern was unexpected.

At the same time, between-group differences indicate that three additional forms of cognitive processes, all lower-order forms, are involved in responsiveness to fraction intervention, each for two of the three fraction outcomes. Across the number line and the calculation outcome samples, inadequate responders scored significantly lower than adequate responders on the working memory span listening recall task. During the listening recall task, examiners read aloud a series of sentences; the student decides if each is true or false and at the end of the series, recalls the last word in each sentence. The ability to maintain information on line in this way while processing new information is transparently involved not only in comparing fraction magnitudes (as in the number-line task) but also while performing fraction calculations. Listening recall's role in responsiveness for the number line and calculation outcome samples (respective effect sizes of 0.41 and 0.54) was expected. Its lack of significance for the word-problem outcome sample (effect size of 0.27) was also expected, given that students listen to word problems being read aloud before solving them. Its lack of significance may be due to the fact that students also have the word problems available for reading along and rereading during test administration.

Across the number line and word-problem outcome samples, inadequate responders scored significantly lower than adequate responders on processing speed and attentive behavior. Attentive behavior has been identified in prior work as a robust predictor of learning in mathematics as well as reading. It is not clear why it failed to distinguish adequate from inadequate responders for the fraction calculations sample. This may be a power issue, as already discussed (the effect size difference between adequate responders and inadequate responders on attentive behavior for the calculations outcome sample was 0.37). It may also be due to an interaction between attentive behavior and working memory. Processing speed, the efficiency with which simple tasks are executed (Case, 1985), has also been identified as salient in prior work. However, in contrast to present findings, it has been identified as distinctively associated with calculation, not word-problem difficulty (Fuchs et al., 2008). Yet in this study, the effect size between adequate responders and inadequate responders on processing speed was 0.35 for the calculations outcome sample, 0.42 for the number-line outcome sample, and a surprisingly large 0.82 for the word-problem outcome sample.

Additional work is required to explore whether (and if so, how) processing speed is associated with fraction word-problem performance. This future work must also tease out the role of working memory span, given its association with processing speed and working memory (Fry & Hale, 1996). Perhaps inefficient processing impairs effective retrieval of word-problem type schemas. Alternatively, given that processing speed is positively linked to reading skill (Just & Carpenter, 1992; Kail & Hall, 1994), impaired processing speed may compromise access to text. Note, however, that for the word-problem measure used in the present analysis, testers read problems aloud while students followed along. Finally, a deficit in processing speed may increase susceptibility to interference or decay during word-problem solving. With faster processing, reasoning is more likely to reach resolution before the requisite information is lost or altered (Jensen, 1993; Miller & Vernon, 1996).

Before closing, we note several limitations that should be considered when interpreting this study's findings. First, we dichotomized participants' responsiveness to intervention to identify adequate and inadequate responder groups. Such bifurcation via an arbitrary point of demarcation may mask salient differences. Yet schools do need to designate adequately and inadequately responsive groups of students for the purpose of moving students between tiers of intervention. In this way, bifurcation mirrors the challenge schools face. The best strategies for formulating sound responsiveness decisions require further investigation. Future work may also consider alternative methods of analysis based on continuous dichotomous distributions of responsiveness, to contrast how they elucidate information on the salient child-level variables associated with responsiveness to fraction intervention. A second limitation is that we used a single measure, rather than latent constructs, to represent each cognitive process. Future work may use multiple and varied methods to represent each cognitive process. In this vein, future work may also consider conducting task analyses of the fraction performance tasks to explore closer links between the cognitive processes and the demands of the specific tasks. A third limitation is the potential issue of power, as already discussed. Given the small number of inadequate responders on the fraction calculation outcome ($n = 19$) and the word-problem outcome ($n = 17$), there is the possibility that nominal effects went undetected. Finally, although we considered the contribution of seven cognitive processes to the shape of students' cognitive profiles, other potential contributors such as metacognition and visual-spatial processing were not considered. Future work should include these constructs.

With these limitations in mind, findings offer insight into directions for expanding the efficacy of fraction interventions for a broader range of at-risk students. Results suggest that fraction intervention may embed activities designed to improve students' reasoning, concept formation, and listening comprehension. Alternatively, fraction intervention may be strengthened by more effectively compensating for these cognitive process limitations within the explicit, direct skills intervention approach taken within the intervention used in this study. The most compelling target for enhancing the design of fraction intervention, as revealed across the profile analysis and between-group mean differences analysis results, is reasoning, where additional strategies and interventions to compensate for such limitations are required.

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