The Role of Domain-General Cognitive Abilities and Decimal Labels in At-Risk Fourth-Grade Students' Decimal Magnitude Understanding

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Abstract

The purpose of the study was to determine whether individual differences in at-risk 4th graders' language comprehension, nonverbal reasoning, concept formation, working memory, and use of decimal labels (i.e., place value, point, incorrect place value, incorrect fraction, or whole number) are related to their decimal magnitude understanding. Students (n = 127) completed 6 cognitive assessments, a decimal labeling assessment, and 3 measures of decimal magnitude understanding (i.e., comparing decimals to the fraction $\frac{1}{2}$ benchmark task, estimating where decimals belong on a 0-1 number line, and identifying fraction and decimal equivalencies). Each of the domain-general cognitive abilities predicted students' decimal magnitude understanding. Using place value labels was positively correlated with students' decimal magnitude understanding, whereas using whole-number labels was negatively correlated with students' decimal magnitude understanding. Language comprehension, nonverbal reasoning, and concept formation were positively correlated with students' use of place value labels. By contrast, language comprehension and nonverbal reasoning were negatively correlated with students' use of whole number labels. Implications for the development of decimal magnitude understanding and design of effective instruction for at-risk students are discussed.

Keywords: individual differences; mathematics; rational numbers

The Role of Domain-General Cognitive Abilities and Decimal Labels in At-Risk Fourth-Grade Students' Decimal Magnitude Understanding

Many students struggle when the curriculum shifts from whole numbers to rational numbers in the upper elementary grades. The ability to accurately assess magnitude is thought to be key for consolidating properties of whole numbers and rational numbers, since magnitude is a unifying property of *all* numbers (Siegler, Thompson, & Schneider, 2011) and rational number magnitude knowledge is related to future mathematics achievement (Bailey, Hoard, Nugent, & Geary, 2012; Booth & Siegler, 2008; DeWolf, Bassock, & Holyoack, 2015; Fazio, Bailey, Thompson, & Siegler, 2014; Rittle-Johnson, Siegler, & Alibali, 2001, 2012; Siegler & Pyke, 2013). Students at risk for mathematics difficulties demonstrate pervasive and systematic misconceptions related to estimating rational number magnitude (e.g., Jordan et al., 2016; Malone & Fuchs, 2016), but much of the research has centered on common fractions $\left(\text{i.e., } \frac{a}{b}\right)$. It is unclear whether the development of decimal magnitude understanding among at-risk students, the focus of the present study, parallels that of fraction magnitude understanding. Understanding individual differences in at-risk students' development of decimal magnitude understanding provides insight into the cognitive abilities required to develop competence with decimals, which in turn can guide the design of early screening tools and interventions.

Most college and career-ready state standards emphasize decimal magnitude understanding. By end of fourth grade, students should be able to compare decimal tenths and hundredths and reason about their size. However, 67% of fourth-grade students could not estimate the location of a decimal on a number line on the National Assessment of Education Progress (U.S. Department of Education, 2011). Many students incorrectly apply whole-number logic to decimals, e.g., assuming 0.274 is greater than 0.83 because 274 is greater than 83 (Rittle-

Johnson et al., 2001), and these misconceptions are difficult to correct (Kallai & Tzelgov, 2014; Resnick et al., 1989;; Stafylidou & Vosniadou, 2004; Vamvakoussi & Vosniadou, 2004).

In this paper, a *decimal* refers to a number written with digits to the right of the decimal point (e.g., 0.25). Decimal magnitude understanding refers to the ability to estimate and reason about the size of a decimal. For this study, we focused on determining the location of a decimal on a number line, rationalizing about the size of a decimal compared to the benchmark fraction $\frac{1}{2}$, and judging the validity of decimal and fraction equivalencies. The purpose of the study was to determine whether individual differences in cognitive abilities and use of decimal labels (i.e., place value, point, incorrect place value, incorrect fraction, or whole number) are related to at-risk fourth-graders' decimal magnitude understanding.

Potential Cognitive Predictors of Decimal Magnitude Understanding

We focus on individual differences in language comprehension, nonverbal reasoning, concept formation, and working memory, as these four cognitive resources are related to the development of fraction understanding (e.g., Hecht & Vagi, 2010; Jordan et al., 2013; Namkung & Fuchs, 2015; Seethaler, Fuchs, Star, & Bryant, 2011; Vukovik et al., 2014), and we located no prior studies examining cognitive predictors of decimal magnitude understanding. Although fractions and decimals have different symbolic notation (e.g., $\frac{1}{2}$ vs. 0.5) and labeling convention (e.g., "one-half" vs. "five-tenths"), they also have similar properties (i.e., both are rational numbers that can signify magnitudes less than one) and students tend to struggle with both fractions and decimals (e.g., Kallai & Tzelgov, 2009, 2014; Ni & Zhou, 2005).

In the present study, we operationalized language comprehension as the ability to accurately define printed words or use a word to describe a picture. To index nonverbal reasoning, we assessed the ability to solve logical puzzles and define relationships between pictures. For concept formation, we focused on the ability to apply a rule to a pattern of objects. For working memory, we focused on span tasks assessing the central executive component of working memory, or the ability to hold pieces of information in the mind while performing cognitive tasks. We focus on span tasks involving both sentences and numbers, as both forms have been found to be related to fraction understanding and mathematics achievement in general. The cognitive abilities incorporated in the studies described below are consistent with these methods for operationalizing these four cognitive processes.

Seethaler et al. (2011) found that language comprehension, nonverbal reasoning, concept formation, and working memory were unique predictors of fraction-calculation skill. Although the present study is not about calculations, research suggests that improved magnitude understanding improves calculation skill (e.g., Fuchs et al., 2014), as these processes likely develop iteratively (Rittle-Johnson & Siegler, 1998; Rittle-Johnson et al., 2001). Namkung and Fuchs's (2015) findings support this. They found that language comprehension, concept formation, and nonverbal reasoning play a role in fourth-grade students' development of accurate fraction number line estimation. Similarly, in a two-year longitudinal study, Jordan et al. (2013) found that third-grade students' language comprehension and nonverbal reasoning, (along with calculation fluency, reading fluency, and attentive behavior) predicted their development of conceptual understanding of fractions in fourth grade, including the ability to estimate fraction magnitude on the number line.

Vukovik et al. (2014) found a somewhat more nuanced set of relations among these domain-general abilities and the development of fraction magnitude understanding. First-grade students' language comprehension, nonverbal reasoning, working memory, and attentive behavior, were measured along with their whole-number knowledge. In second grade, students' whole-number knowledge was again indexed; then students' understanding of fractions was examined in fourth grade. Language comprehension, working memory, and attentive behavior predicted fraction understanding, including the ability to estimate fractions on the number line. Yet, although these domain-general abilities predicted fourth graders' understanding of fractions, these effects were completely mediated by students' second-grade whole-number skill.

Although their findings stand in contrast to the earlier studies, language comprehension, working memory, and attentive behavior have also been found to predict whole-number calculation skill (e.g., Seethaler & Fuchs, 2006; Seethaler et al., 2011). In the Vukovic et al. (2014) study, these domain-general abilities did not predict rational number knowledge beyond whole-number calculation skill (which is why we control for students' whole-number knowledge in the present study). It stands to reason that these abilities are important for developing competence with both whole numbers and rational numbers.

Despite some inconsistency in findings across these studies, language comprehension, nonverbal reasoning, concept formation, and working memory appear related to the development of fraction understanding, especially developing number line estimation skill. We therefore hypothesized a similar developmental pattern for decimals as fractions. At the same time, important distinctions may emerge, considering that decimals and fractions have different symbolic notation, which affects labeling conventions. For labeling fractions, the numerator and denominator have a special term (e.g., read $\frac{1}{2}$ as "one-half and $\frac{1}{3}$ as "one-third"). By contrast, labeling decimals reflects place value (e.g., read 0.2 as "2 tenths" and 0.35 as "35-hundredths"). Therefore, the bipartite $\left(\frac{a}{b}\right)$ structure of fractions may impose additional cognitive demands for estimating magnitude over what is involved for decimals (e.g., DeWolf, Grounds, Bassok, & Holyoak, 2014). Although our study did not compare and contrast fraction and decimal labeling conventions, we did investigate whether decimal labels are related to students' decimal magnitude understanding and cognitive abilities, which indirectly addresses these differences.

Labeling Decimals

We identified one study that suggests using decimal place value labels is related to increased magnitude understanding. Mazzocco and Delvin (2008) investigated whether low-achieving, typically-achieving, and learning disabled sixth graders' decimal magnitude understanding and knowledge of decimal place value labels (e.g., reading 0.49 as "forty-nine hundredths") predicted their ability to rank order fractions and decimals at eighth grade. Students with mathematics learning disabilities had the most difficulty labeling decimals with place value labels, and these difficulties persisted into eighth grade. By contrast, incorrectly labeling decimals in sixth grade for low-achieving and typically-achieving students were not predictive of students' ability to name decimals with place value labels in eighth grade. That is, some low-achieving and typically-achieving students failed the naming test in sixth grade, but passed in eighth grade. This is likely because 94% of students who mastered the ranking test in eighth grade used some place value labels for decimals in sixth grade. They concluded that the inability to correctly name a decimal with place value labels may represent a key deficit in rational number understanding among students with mathematics learning disabilities, which suggests that decimal labels may play an important role in students' development of rational number understanding. The authors did note, however, that using place value labels in sixth grade did not guarantee greater magnitude understanding in eighth grade, as 22% of students who used place value labels in sixth grade failed the ranking test in eighth grade.

There is also some evidence in the fraction literature to suggest a positive relation between the quality of fraction labels and developing fraction knowledge. Miura, Okamoto, Vlahovic-Stetic, Kim, and Han (1999) compared U.S., Croatian, and Korean first- and second-graders' initial fraction ideas. They hypothesized that Korean students would have the greatest foundational knowledge of fractions because the Korean naming system for fractions directly supports magnitude understanding. That is, the direct translation of a unit fraction such as $\frac{1}{3}$ is "of three parts, one," whereas English and Croatian refer to it as "one-third," which does not produce a direct mental image of magnitude. As expected, Korean students had greater foundational knowledge of fractions than students in the other two countries. So, explicit vocabulary linked to magnitude understanding (as reflected in the Korean language) positively influenced students' initial concepts of fractions.

Paik and Mix (2003) extended Miura et al. (1999) by examining differential fraction competency among Korean and U.S. first- and second-grade students. Paik and Mix found that when students were provided fraction labels mimicking Korean usage, U.S. students actually outperformed Korean students on measures of fraction competence. Both studies reveal the importance of explicitly defining relevant vocabulary when introducing novel concepts, such as rational numbers, to students.

Given the importance of magnitude language for enhancing initial rational number understanding, we were interested in assessing whether place value labels activate magnitude representations that children can use to guide problem solving. The present study extends prior work by identifying a range of decimal labels used by at-risk fourth-grade students (i.e., place value, point, incorrect place value, incorrect fraction, or whole number) and investigating the potential cognitive correlates of decimal label use. We focused on fourth grade, the first year of intensive focus on rational numbers. This study therefore has important theoretical and practical implications. If naming decimals with place value labels represents a key deficit among eighth-grade students with mathematics learning disabilities, as shown in Mazzocco and Delvin (2008), understanding potential cognitive correlates of such deficits can assist in the design of screening tools and interventions to enhance outcomes at the upper elementary grades.

Based on prior research, we hypothesized that naming decimals with place value labels is positively correlated with greater accuracy on each of the decimal magnitude tasks. Although Mazzocco and Delvin (2008) found that using place value labels did not guarantee greater decimal magnitude understanding, Miura et al.'s (1999) study on early fraction knowledge development indicated a positive relation between fraction labels and enhanced fraction understanding. Therefore, using place value labels may represent an important indicator of magnitude understanding. Further, as previously discussed, developing rational number understanding requires substantial background knowledge, including relevant vocabulary, using deductive and inductive reasoning, and holding information in the memory while solving complex problems. For these reasons, we hypothesized that students' ability to name a decimal with place value labels is positively correlated with language comprehension, nonverbal reasoning, concept formation, and working memory. This is parallel to our hypothesis about the relation between students' domain-general abilities and their decimal magnitude understanding. Note that naming a decimal with place value labels is a possible indicator of magnitude understanding, but naming a decimal with place value labels does not guarantee that a student has any knowledge about the decimal value (Mazzocco & Delvin, 2008). The remaining labels are not indicators of magnitude understanding. We therefore did not expect that students' use of point, incorrect place value, incorrect fraction, or whole number labels is related to magnitude understanding or domain-general cognitive abilities.

Method

Participants and Screening

Participants were 127 fourth-grade students from the southeastern region of the United States. All participants had whole-number mathematics difficulty, defined as scoring below the 35th percentile on the *Wide Range Achievement Test – 4th Edition* (WRAT-4; Wilkinson & Robertson, 2006) at the beginning of their fourth-grade year, and were therefore at-risk for difficulty in learning about decimals. The WRAT-4 includes 40 problems of increasing difficulty. Alpha on this sample was .76.

Qualified students participated in two 45-minute individual testing sessions, which included the cognitive batteries. We excluded students who performed below the 9th percentile on both subtests of the *Wechsler Abbreviated Scale of Intelligence* (WASI; Wechsler, 1999), because this study is not about intellectual disabilities. Some students in our sample were control students in a randomized-control trial testing the efficacy of a rational number intervention (Fuchs & Malone, 2015). Our sample did not receive intervention. Demographics of the sample was as follows: M_{age} = 9 years, 5 months (August of fourth-grade); 57.6% female; 48.0% African American, 22.8% Caucasian, 21.3% Hispanic, and 7.9% other; 92.1% of students received subsidized lunch; 17.3% were English-Language Learners; and 15.0% received special education services (10.2% for learning disabilities).

Cognitive Predictors

We assessed students’ language comprehension with *WASI-Vocabulary* (Wechsler, 1999). Students identify pictures and define words (42 items). The picture items score “1” (correct) or “0” (incorrect). The words score “0”, “1”, or “2”, depending on the sophistication of the answer. Testing discontinues after five consecutive scores of 0. Reliability is .88.

We assessed students' nonverbal reasoning with *WASI-Matrix Reasoning* (Wechsler, 1999), which requires students to complete a pattern on each page by selecting one of the five choices on the bottom of the page. Each puzzle is increasingly difficult. Testing discontinues after four consecutive scores of 0 or four scores of 0 out of five items. Reliability is .93.

We assessed students' concept formation using the subtest from Woodcock Johnson-III (Woodcock, McGrew, & Mather, 2007). Students see a series of pictures (40 items across six spans), and determine a categorization rule for why the pictures are in a box. Each span includes practice to cue the types of rules and patterns for a span. Each span has a threshold for moving on and a ceiling to discontinue testing. Reliability is .76.

We assessed students' central executive working memory using two subtests from the *Working Memory Test Battery for Children* (WMTB-C; Pickering & Gathercole, 2001): *Listening Recall* (sentences) and *Counting Recall* (numbers). For Listening Recall, the tester reads a sentence. The student states whether the sentence is true or false, and then recalls the last word of the sentence. There are six spans with six trials per span. With each span, the number of sentences the student must remember and recall increases by one. The tester discontinues testing after three incorrect answers within a span. Reliability is .82-.91.

For Counting Recall, students count dots on different pages, and must recall the number of dots in the order they were presented. There are six spans with six trials per span. With each span, the number of pages the student must count and recall increases by one. The tester discontinues testing after three incorrect answers within a span. Reliability is .82-.91.

Decimal Outcome Measures

For *Compare to* $\frac{1}{2}$ (Malone, Loehr, & Fuchs, 2015), students were presented with a 0-1 number line on an 8.5"x11" piece of cardstock with $\frac{1}{2}$ marked as a fraction (not as the decimal

0.5), to decrease the probability of using whole number rules to assess magnitude. Students labeled 10 decimals (0.6, 0.20, 0.03, 0.09, 0.682, 0.25, 0.5, 0.49, 0.12, and 0.70) and then determined whether the decimal was less than, greater than, or equal to $\frac{1}{2}$ (read as "one half"). They did not place the decimals on the number line. Each item received a label code and an accuracy score. Alpha was 0.76. See Table 1 for definitions and examples of the five labels identified.

With *Decimal Number Line* (Hamlett, Kelley, Malone, & Fuchs, 2014, adapted from Siegler et al., 2011) students placed 14 decimals (0.6, 0.95, 0.7, 0.58, 0.9, 0.38, 0.69, 0.4, 0.82, 0.5, 0.75, 0.47, 0.8, 0.3) on a computer number line with endpoints 0 and 1 (no tick marks). The score for each item is the absolute difference between the student's estimate and where the decimal actually goes, divided by the scale of the number line endpoints (i.e., 1). Scores are averaged to yield average absolute error. Since lower scores indicate stronger performance, we multiplied scores by -1 for data analyses so that a positive score indicates stronger estimation ability. Test-retest reliability on a similar fraction assessment was .85.

The *Decimal Equivalency* task (Malone et al., 2015) requires students to judge the accuracy of 10 equivalency sentences (i.e., a=b). Three items included only decimals (0.9=0.90; 0.20=0.201; 0.1=0.01), six items included one decimal and one fraction ($\frac{6}{10}$=0.60; 0.4=$\frac{4}{100}$; $\frac{8}{10}$=.08; 0.30=$\frac{30}{100}$; $\frac{50}{100}$=0.05; 0.2=$\frac{20}{100}$), and one item included a decimal and whole number (0.7=7). Instructions included a true statement and a false statement with whole numbers: The tester says "3+4=7 makes sense, so you would say 'true'." Then the tester says, "3+4=12 does *not* make sense, so you would say 'false'." Then, the tester shows the student the 10 flashcards, one-by-one, and the student says "true" or "false" for each item. Alpha on this sample was 0.71.

Procedure

Trained research assistants (graduate students pursuing a master's degree in the school of education) administered all tests. In the fall of fourth grade, students were individually tested on the cognitive assessments in two 45-min sessions. In the spring of fourth grade (after rational number instruction occurred in the classroom), students were individually tested on all decimal outcome measures din one 60-min session. We listened to 99% of individual testing sessions. Accuracy of administration and scoring was 96.43%.

Data Analysis

Before conducting regression analyses, we tested all assumptions of linear regression. We then ran a regression model for each of the three decimal magnitude outcomes using WASI Vocabulary, WASI Matrix Reasoning, Concept Formation, Counting Recall, and Listening Recall raw scores as the predictor variables and controlling for students' incoming calculation ability (i.e., WRAT-4). We calculated correlations between each of the decimal magnitude tasks, students' use of decimal labels, and cognitive abilities.

Results

Cognitive Predictors of Students' Decimal Magnitude Understanding

See Table 2 for means and standard deviations of all variables, along with correlations between decimal magnitude measures and cognitive predictors. See Table 3 for a summary of the regression results for all three decimal magnitude outcomes. For Compare to $\frac{1}{2}$, students' incoming calculation ability, language comprehension, nonverbal reasoning, concept formation, and working memory together accounted for 13.2% of the variance, $F(6, 120)=3.03$, $p=.009$. WASI Vocabulary and Counting Recall predicted students' ability to accurately compare a decimal to one-half. Matrix Reasoning, Concept Formation, and Counting Recall failed to predict students' ability to compare a decimal to one-half, controlling for the other variables.

For Decimal Number Line, students' incoming calculation ability, language comprehension, nonverbal reasoning, concept formation, and working memory together accounted for 12.9% of the variance, $F(6, 120)=2.97$, $p=.01$. WRAT-4 and Counting Recall predicted students' decimal number line estimation ability. WASI Vocabulary, Matrix Reasoning, Concept Formation, and Listening Recall failed to predict students' accuracy on the number line, controlling for the other variables.

For Decimal Equivalency, students' incoming calculation ability, language comprehension, nonverbal reasoning, concept formation, and working memory together accounted for 16.3% of the variance, $F(6, 120)=3.89$, $p=.001$. WRAT-4 and Concept Formation predicted students' accuracy in assessing whether magnitude statements were true or false. WASI Vocabulary, WASI Matrix Reasoning, Counting Recall, and Listening Recall failed to predict students' accuracy in assessing whether decimal equivalency statements were true or false, controlling for the other variables.

Decimal Labels

See Table 4 for frequency of label use across trials and students. See Table 5 for correlations among students' magnitude understanding, cognitive abilities, and use of decimal labels.

Discussion

We investigated whether individual differences in language comprehension, nonverbal reasoning, concept formation, working memory, and use of decimal labels was related to at-risk fourth-graders' decimal magnitude understanding. The following includes a discussion of our findings, followed by recommendations for designing decimal instruction.

Cognitive Predictors of Students' Decimal Magnitude Understanding

Developing decimal magnitude knowledge does not exactly parallel that of fractions (DeWolf et al., 2014), given that nonverbal reasoning did not predict any of the outcomes. It may be that the bipartite structure of fractions (i.e., $\left[\frac{a}{b}\right]$) increases the reasoning demands required to accurately assess magnitude. Or, in the case of decimals, the other three cognitive predictors (i.e., language comprehension, concept formation, and working memory) may be more salient predictors than our nonverbal reasoning measure. Each of the three decimal tasks tapped students' decimal magnitude knowledge, but results indicate each require a slightly different set of skills.

To successfully complete the Compare to $\frac{1}{2}$ task, students must understand what $\frac{1}{2}$ represents (i.e., written as a fraction versus a decimal). They must understand what the numerator and the denominator mean (thus involving language comprehension) and how they work together before determining how $\frac{1}{2}$ relates to decimal magnitude. While holding this information in their mind (i.e., working memory), they must see that $\frac{1}{2}$ can be converted to decimal format before determining whether the decimal is less than, greater than, or equal to $\frac{1}{2}$. Note that the fraction $\frac{1}{2}$ does not match decimal language nor does it allow students to use whole number knowledge to compare magnitudes (e.g., 0.6 is greater than 0.5 because the whole number 6 is greater than 5). The Decimal Number Line was a more explicit estimation task than the Compare to $\frac{1}{2}$ task, yet required similar skills.

Only numerical working memory (i.e., Counting Recall), a more domain-specific working memory task (Raghubar, Barnes, & Hecht, 2010), predicted accuracy on both number line tasks. Because both working memory with sentences (i.e., Listening Recall) and numbers have been found to be related to fraction understanding (e.g., Seethaler et al., 2011; Vukovick et al., 2014), including moderating the effects of fraction intervention on number line estimation (e.g., Fuchs et al., 2014), we included both tasks in our theory-driven model. This could signify a difference between learning fractions and learning decimals. That is, assessing fraction magnitude may tax working memory more than assessing decimal magnitude.

Unlike the number line tasks, only concept formation predicted accuracy on the Decimal Equivalency task. To score well on the concept formation assessment, students must generalize a common rule that applies to a set of objects (e.g., a series of different colored squares and circles). The equivalency statements in this task included a mix of fractions, decimals, and whole numbers, and students therefore had to assess the commonality between the two numbers before judging whether the equivalency statement was true. This task also relies on other cognitive resources, but switching back and forth between different number representations may have overshadowed the need for them. The ability to assess commonalities between numbers (i.e., concept formation) appears to be most important in assessing these difficult equivalency statements.

Decimal Labels

As expected, language comprehension, nonverbal reasoning, concept formation, and working memory were positively correlated with students' use of place value labels. This makes sense, since labeling the decimal 0.12 as "twelve hundredths", for example, requires all the of these cognitive resources. The student must name the numeral and recognize and select relevant place value positions (i.e., language comprehension), process this information (i.e., reasoning abilities), and hold this information in the mind (i.e., working memory) to construct an accurate label. Students must also attend to relevant and irrelevant information (i.e., concept formation), for example, how to attend to trailing and leading zeros.

In terms of magnitude understanding, using place value labels did not guarantee greater magnitude understanding, as revealed by the nonsignificant correlation between place value labels and accuracy on the Compare to $\frac{1}{2}$ task. This is consistent with Mazzocco and Delvin's (2008) findings. It could be that benchmarking to $\frac{1}{2}$ (rather than 0.5) served as a distractor, and students did not rely on their place value knowledge on this task. Students likely needed to convert $\frac{1}{2}$ to a decimal before assessing magnitude. Understanding place value may *assist* students in assessing decimal magnitude, especially on items with a fraction and a decimal like 6/10=0.6 on the Decimal Equivalency task, likely because these equivalency statements primed students to use their place value knowledge. To assess this, we examined correlations between students' composite place value score and accuracy on the six true/false items with a fraction and a decimal (i.e., 6/10=0.6, 8/10=0.08, 0.4=4/100, 0.30=30/100, 50/100=0.05, and 0.20=20/100). Place value label use was indeed positively correlated with accuracy on these items (r=.43, p<.001). Note that each of these items had fractions presented in base-10 format (which was not the case on the Compare to $\frac{1}{2}$ task).

Unlike place value, point labels were not correlated with students' decimal magnitude understanding and negatively correlated with concept formation. That is, the lower students' score on concept formation, the more likely they were to use point labels. This parallels our finding that concept formation was a significant predictor of the Decimal Equivalency task. It appears that concept formation may play a significant role in supporting students' place value understanding, given that students' use of *place value* labels were positively correlated with accuracy on the six decimal equivalency items that included both a fraction and a decimal. Naming decimals with point labels does not imply fundamental misunderstanding of decimals; rather, it reflects the common, informal language that many adults use. However, point labels do not reflect sophisticated understanding of *magnitude* language (like place value). That is, naming the decimal 0.12 as "point 1…2" does not invoke an image of magnitude. This is not to say that using point labels reflects unsophisticated mental representations of decimal magnitude. Rather, point labels do not reflect sophisticated understanding. Our findings parallel those of Miura, et al. (1999) and Paik & Mix (2003) that using labels for fractions that reflect magnitude understanding (like those used in the East Asian languages) enhance students' understanding of fractions. Thus, teaching decimal place value labels may be one way to support decimal magnitude understanding.

Interestingly, using *incorrect* place value labels was positively correlated with language comprehension and accuracy on the Compare to $\frac{1}{2}$ task. This result is difficult to interpret, because there is no advantage to incorrectly labeling a decimal and assessing whether it is less than, greater than, or equal to one-half. Students with greater magnitude understanding may be more likely to attempt to use a place value label (even if it is incorrect). (However, the correlation between students' use of place value labels and accuracy on this task was not significant). Attempting to label a decimal with place value language may represent a transitional phase in their learning. So, students with greater language comprehension may have been more likely to attempt to use place value labels, even if they were inaccurate.

Using whole number labels demonstrates pervasive misunderstanding of decimals, which helps to explain why using whole number labels was associated with lower accuracy on the

Decimal Number Line and Decimal Equivalency. To glean a better understanding of students' whole-number misconceptions, we assessed how children generally think about decimal magnitude comparison problems using two versions of a "hidden decimal task" (Resnick et al., 1989). (Note that this was for descriptive purposes only; it was not the focus of the study.) One item asked students to identify whether 0.X or 0.XXXX was greater (Xs represent numbers covered by pieces of paper), or if it was impossible to know. Only 14% of children answered correctly. Instead, 86% of children identified 0.XXXX as the greater decimal, which is consistent with whole number logic. The second item provided the value of the tenths place (0.8 or 0.2XX) to reveal how competing strategies (length of digits versus comparing tenths) influence responses. The decimal with more digits is the smaller decimal. Increased accuracy (45% of children answered correctly) on this item is difficult to interpret, since students can successfully use whole number logic to compare the numbers in the tenths place without having any understanding of what the decimals represent (Loehr & Rittle-Johnson, 2015).

These whole number misconceptions appear to be related to deficits in cognitive abilities. This unexpected finding suggests that whole number bias may be pervasive among students with weaker cognitive abilities and that these students have deep misconceptions of decimals. Students with deficits in these areas may be more likely to rely on less sophisticated strategies (e.g., relying on whole number logic) to approach problems with which they are unfamiliar (e.g., assessing decimal magnitude).

Instruction Designed to Compensate for Incoming Cognitive Strengths and Weaknesses

Based on our results, and in conjunction with findings from previous research, we offer three recommendations for designing instruction to anticipate and avoid misconceptions among at-risk fourth grade students by designing instruction that compensates for incoming cognitive deficits to improve students' decimal magnitude understanding. First, we recommend using magnitude language to define decimals (i.e., always using place value labels) to reduce potential misconceptions that decimals have the same properties as whole numbers (Cramer, Monson, Wyberg, Leavitt, & Whitney, 2009; Fyfe, McNeil, & Rittle-Johnson, 2015; Gelman & Markman, 1986; Graham, Kilbreath, & Welder, 2004; Loehr & Rittle-Johnson, in press; Waxman & Gelman, 1986). Second, we suggest providing students with strategies for explicitly comparing and contrasting decimals, whole numbers, and fractions, with the goal of improving students' ability to reason about the size of numbers and fluidly transitioning from one rational number notation to another. This includes the inclusion of correct and incorrect examples to minimize potential whole number misconceptions (Durkin & Rittle-Johnson, 2012). Finally, instruction should be designed to incorporate extensive supports, scaffolds, and explicit strategies to minimize the amount of information students have to hold in memory to successfully assess decimal magnitude (Fuchs et al, 2014). However, more work is needed to assess the tenability of these recommendations as there is very little research on improving at-risk students' decimal magnitude understanding.

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Table 1

Decimal Label Definitions

Label	Definition	Example (0.25)
Place Value	Correct use of place value labels	“twenty five hundredths”
Point	Correct use of informal label	“point twenty five” or “point 2…5”
Incorrect Place Value	Incorrect use of place value labels	“twenty five tenths”
Incorrect Fraction	Incorrect misapplication of fraction terms for decimals	“two fifths”
Whole Number	Incorrect use of whole number labels for decimals	“twenty five” or “two…five”
Other	Incorrect answers that could not be classified	“twenty five hundred”, “two hundred twenty five”, or other

Note: Of these five labels, we consider place value labels the most sophisticated, as they are the only labeling convention that reflect magnitude language. The other four labels do not reflect decimal magnitude language (note that point labels are correct, whereas the remaining three labeling conventions are incorrect). If the student’s response did not fit into any of these categories, the code *other* was used. Note that naming the benchmark fraction (i.e., “one-half) did not clue students to how to name a decimal. Therefore, the frequency of decimal labels students can be considered a proxy of their knowledge of decimal labels.

Table 2

Means, Standard Deviations, and Correlations for Decimal Magnitude Tasks and Cognitive Predictors

Measures	Standard Scores		Raw Scores									
	M	*(SD)*	*M*	*(SD)*	1	2	3	4	5	6	7	8
Math Achievement												
1. WRAT-4			24.39	(2.13)								
Magnitude Understanding												
2. Compare to ½			4.61	(1.70)	.11							
3. Number Line			0.31	(0.09)	.22*	.25**						
4. Decimal Equivalency			6.00	(1.83)	.27**	.23**	.20*					
Cognitive Predictors												
5. WASI Vocabulary	47.08	(9.64)	31.81	(6.89)	.17	.18*	.18*	.21*				
6. WASI Matrix Reasoning	45.78	(9.00)	16.57	(5.47)	.07	.12	.12	.19*	.11			
7. Concept Formation	84.13	(14.03)	13.83	(7.36)	.13	.11	.10	.31***	.49***	.26**		
8. Listening Recall	87.73	(19.48)	9.57	(4.04)	.16	.00	.00	.15	.51***	.12	.42***	
9. Counting Recall	77.62	(15.44)	16.71	(4.43)	.16	.23**	.23**	.13	.39***	.09	.35***	.39***

Note: ***$p < .001$, **$p < .01$, *$p < .05$.

Table 3

Summary of Regression Results

Outcome	Predictors	B	SE	*t*	*p*
Compare to ½	(intercept)	4.610	0.144	31.95	< .001
	WRAT-4	0.045	0.070	0.64	.497
	WASI Vocabulary	**0.062**	**0.027**	**2.32**	**.009**
	WASI Matrix Reasoning	0.036	0.027	1.34	.183
	Concept Formation	0.007	0.024	0.27	.787
	Listening Recall	–0.078	0.045	–1.73	.087
	Counting Recall	**0.078**	**0.037**	**2.09**	**.038**
Number Line	(intercept)	0.309	0.008	39.28	< .001
	WRAT-4	**0.008**	**0.001**	**2.10**	**.038**
	WASI Vocabulary	0.002	0.001	1.64	.103
	WASI Matrix Reasoning	0.002	0.001	1.08	.284
	Concept Formation	–0.000	0.001	–0.21	.838
	Listening Recall	–0.005	0.002	–1.94	.054
	Counting Recall	**0.005**	**0.002**	**2.29**	**.024**
Decimal Equivalency	(intercept)	6.000	0.153	39.32	< .001
	WRAT-4	**0.200**	**0.074**	**2.67**	**.009**
	WASI Vocabulary	0.014	0.028	0.51	.608
	WASI Matrix Reasoning	0.037	0.029	1.27	.206
	Concept Formation	**0.059**	**0.026**	**2.32**	**.022**
	Listening Recall	–0.011	0.048	–0.24	.809
	Counting Recall	–0.003	0.039	–0.07	.945

Note: Bold indicates significant predictor.

Table 4

Frequency of Decimal Label Use

Label	% Used Across Trials[c]	% Used ≥ 1[d]	*M* (*SD*) if Used ≥ 1[e]
Place Value[a]	39.3%	58.3%	6.74 (2.47)
Point[a]	13.5%	20.5%	6.42 (3.68)
Incorrect Place Value[b]	14.1%	49.6%	7.38 (3.65)
Incorrect Fraction[b]	2.8%	10.2%	2.69 (2.59)
Whole Number[b]	24.4%	33.1%	7.38 (3.65)
Other[b]	6.3%	28.3%	2.22 (2.46)

Note: [a]Place value and point labels are *correct* labels. [b]Incorrect place value, incorrect fraction, whole number label, and other labels are *incorrect* labels. [c]To compute the percent used across trials, we counted the frequency with which students used a specific label across trials and then divided it by the total number of trials, multiplied by 100. For example, there were 1,270 trials (127 children each answered 10 items). The place value label was used on 499 out of 1,270 trials (i.e., 39.3%). [d] To calculate percent used at least one (i.e., % Used ≥ 1), we calculated the proportion of students who used each error code at least once across the 10 trials. To compute this, we summed the number of students who used each label at least once and divided it by the total number of students, multiplied by 100. For example, 74 students used the place value code at least once across the 10 trials (59.3%). [e]We calculated the mean and standard deviation of label use among the proportion of students who used the label *at least once* (*M* (*SD*) if Used ≥ 1; see column d for the percentage of students who used the label at least once). For example, among the 58.3% of students who used a place value label at least once, they used a place value label an average of 6.74 times across the test.

Table 5

Correlations between Students' Magnitude Understanding, Cognitive Abilities, and Use of Decimal Labels

Measures	Decimal Labels			
	Place Value[a]	Point[a]	Incorrect Place Value[b]	Whole Number[b]
Magnitude Understanding				
Compare to ½	.13	–.05	.18*	–.14
Number Line	.32**	–.08	.14	–.32**
Decimal Equivalency	.45**	–.16	.05	–.34**
Cognitive Predictors				
WASI Vocabulary	.27**	.02	.23**	–.36***
WASI Matrix Reasoning	.24**	–.07	–.08	–.08
Concept Formation	.37***	–.19*	.11	–.18*
Listening Recall	.20*	–.02	.10	–.17
Counting Recall	.18*	–.06	.05	–.13

Note: ***$p < .001$, **$p < .01$, *$p < .05$. We do not report correlations for incorrect fraction or other labels, because frequencies were below 10%. [a]Place value and point labels are *correct* labels. [b]Incorrect place value and whole number labels are *incorrect* labels.