

Virtual Mathematics Education: Using Second Life to Model and Reflect upon the Teaching of Mathematics

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Two of the challenges faced by mathematics teacher educators involve the issue of pre-service teachers' mathematical content knowledge and the impact this has on their PCK, and the perceived gap between the theoretical knowledge received in their teaching course and the practical knowledge gained in the classroom while on professional experience. This paper examines the use of what is essentially a teacher educator tool or strategy, designed to give pre-service teachers a realistic environment in which to address these two challenges. The study uses data collected from pre-service teachers who participated in virtual lessons using Second Life, post-lesson interviews and a teacher educator survey to document the advantages and disadvantages of using such an approach. The results indicate that the use of Second Life has practical implications for teacher educators as an additional tool for modelling and reflecting upon the teaching of mathematics.

Concerns about pre-service teachers' content knowledge and the impact this has on pedagogical content knowledge (PCK) is widespread in the literature (e.g., Chick, Pham & Baker, 2006; Hill, Ball & Schilling, 2008; Maher & Muir, 2011). Similarly, concerns have been raised about teacher preparation in general, particularly in terms of meeting the challenges associated with bridging the perceived gap between theory and practice (e.g., Lin, 2005). Strategies to overcome these challenges have included the development of frameworks for studying pre-service teachers' content knowledge and PCK, (Ball, Thames, & Phelps, 2008; Chick et al., 2006) and the evaluation of teaching tools and strategies, such as teaching videos (e.g., Star & Strickland, 2008). Second Life offers a virtual environment which simulates 'real-life' situations, and has been used in educational contexts to recreate classrooms, particularly with an emphasis on behaviour management (e.g., Gregory et al., 2011). An extensive literature search, however, has revealed that other than being used as a vehicle for teaching mathematical concepts (e.g., Sweeney, 2008), Second Life has not been utilised to explicitly teach mathematics pedagogy. This paper adds to the limited research in this area through describing a project whereby pre-service teachers participated in two mathematics lessons designed to teach fraction concepts, and were then interviewed and surveyed about their experiences afterwards. Data were also collected from teacher educators to determine their perceptions of the practical uses and implications of the Second Life classroom.

Theoretical Framework

Teacher Content Knowledge and Pedagogical Content Knowledge

There is general consensus in the literature that effective teachers require both well-developed content knowledge and pedagogical content knowledge. Many elementary teachers, however, lack conceptual understanding of mathematics (Ma, 1999; Mewborn, 2001), and pre-service teachers' mathematical knowledge and confidence is of particular concern (e.g., Ball, 1990; Lange & Meaney, 2011). Recent graduate standards have

identified the need for graduates to “know the content and how to teach it” (Educational Services Australia, 2011, p.6) and excellent teachers of mathematics should demonstrate a sound, coherent knowledge of mathematics, appropriate to the students they teach (Australian Association of Mathematics Teachers, 2006). This has direct implications for the teacher educator who is primarily responsible for the preparation of these teachers. Shulman’s (1987) conceptualisation of PCK contributed greatly to current mathematics education research into what teachers need to know about students’ learning with a key aspect of PCK being the identification of students’ misconceptions and the strategies that teachers use to overcome misconceptions (Chick, et al., 2006). There is evidence to suggest that many pre-service teachers exhibit similar misconceptions to their student counterparts (Ryan & McCrae, 2005), and that even practicing teachers tend to overgeneralise their knowledge of whole numbers when working in the domain of rational numbers. In the domain of fractions, which was the topic of the lessons discussed further on in this paper, common misconceptions include misapplication of whole number thinking (e.g., $\frac{1}{8}$ is larger than $\frac{1}{4}$ because 8 is larger than 4), with other difficulties including the understanding of equivalence when using multiple representations and the overuse of an additive strategy when a multiplicative use is required (Ryan & Williams, 2007).

Linking Theory with Practice

It is generally agreed that pre-service teachers view classroom experience as the best teacher (Richardson, 1996), with evidence suggesting that pre-service teachers are not being adequately prepared for the challenges of teaching (Ferry, et al., 2004). Practice-based strategies, such as videos, have been found to be effective in bridging the perceived gap between theory and practice (Lin, 2005; Taylor, 2002) through challenging pre-service teachers’ images of mathematics and mathematics teaching (Taylor, 2002), and in developing their skills in observing teaching (Santagata, Zannoni, & Stigler, 2007). The use of video excerpts in pre-service teacher education is well documented in the literature (e.g., Star & Strickland, 2008), with the main benefit being the opportunity for classes to view the same teaching and learning episode and to share in a discussion of what has taken place. It is not always possible or practical however, to obtain or produce video excerpts that capture specific examples of instruction or situations that teacher educators can use to analyse classroom episodes. Similarly, it is seldom possible to capture video footage of pre-service teachers engaged in the teaching process and then view this footage in a social context. Second Life provides an alternative to locally or commercially produced video excerpts in that the teacher educator can identify, and to some extent influence, the objectives and outcomes of the lesson, it does not require ethical approval or sophisticated recording equipment, it has the advantage of involving the pre-service teachers in the action and is available for immediate viewing after the recording has occurred.

Second Life

Second Life (www.secondlife.com) is a popular, online virtual world where players represent themselves as avatars and interact with each other through conversations and gestures (Gao, Noh & Koehler, 2009). Virtual worlds have been designed to create simulations of real world phenomena to provide environments for experiential learning and training (Eschenbrenner, Fui-Hoon, & Siau, 2008). The benefits noted by educators include the opportunity to conduct activities and tasks in a risk-free environment, enhanced collaboration and communication between participants, including those who are geographically dispersed, and increased engagement (Gao, et al., 2009). Teacher educators

have utilised Second Life in particular to both supplement pre-service teachers' learning and to undertake preparation for professional experience (e.g., Gregory, et al., 2011). In order to address the inconsistencies between pre-service teachers' practical experiences, Gregory and her colleagues designed VirtualPREX in order to provide pre-service teachers with experiences that reflected the complex and diverse nature of a teacher's role in the classroom. VirtualPREX allowed the pre-service teachers to anonymously engage freely in practices, test their strategies, with no long-term effects on 'real' students, with the aim being to better understand themselves as teachers. While an extensive literature search failed to identify any examples of Second Life being used explicitly to teach mathematics pedagogy, it has been used to teach mathematical concepts (e.g., Sweeney, 2008) and for learning about mathematics. Valcke, Van Steenbrugge, and Veeragoudar (2008), for example, utilised Second Life as a mechanism to address their student teachers' attitudes and beliefs about learning mathematics. They created a Second Life environment, called the Fractal Village, in which groups of students worked on weekly tasks centred on the active manipulation of 3D objects. The researchers found that the Second Life environment had the potential to positively influence the student teachers' mathematical dispositions but conceded that further research was necessary in order to more accurately assess the impact.

Methodology

The main aims of the project were: to create a Multi-User Virtual Environment (MUVE or virtual classroom) using Second Life and teacher and student avatars; to provide pre-service teachers with the opportunity to take on the role of teacher and student avatars in a fictional, safe on-line environment; to better prepare pre-service teachers for the challenges of undertaking professional experience placements; to provide an avenue for pre-service teachers to engage in critical discussion and reflective practice and to explore whether or not Second Life could be used to model and reflect upon mathematics teaching and mathematical behaviour.

Virtual Classroom Design

The classroom was designed by an educational developer and consisted of a three stage process involving conceptualisation, finding and purchasing objects, and assembling the objects into a finished product. The researcher met regularly with the developer to discuss the design of the classroom and avatars, and to select objects for inclusion. Objects such as chairs and table, notebooks and pens were purchased using in-world Linden Dollars while other parts of the room such as walls, the roof and windows were constructed using the shape creation tools. Other infrastructure put in place to facilitate and record the sessions were signage and instructions, a specialised 3D navigator control to smoothly pan camera viewpoints, video capture software to record the sessions and video-editing software to produce the final movies. A screen shot of the classroom is shown in Figure 1.

The Avatars

One teacher and seven avatars were created. Initially each of the student avatars had particular behavioural characteristics as the focus was originally on inclusivity and behaviour management, but these were later adapted to describe mathematical behaviours, incorporating common misconceptions as identified in the literature. Each avatar also had a list of actions that he/she was programmed to use. Figure 2 shows a visual representation of one of the student avatars, Matak, who was characterised as being of 'above average' mathematical ability and able to, for example, compare and order fractions, place a variety

of fractions correctly on a number line and explain why. In contrast, Willow and Monty were characterised as being of ‘below average’ mathematical ability and described as showing:

Confusion with ordering fractions – might think, for example, that $1/9$ is larger than $1/4$ because 9 is bigger than 4; would have difficulty with putting fractions in order on a number line; could probably say that $2/4$ is the same as $1/2$ but not consistently; no real understanding of the relative size of a fraction or that a fraction is also a number; if asked to add fractions, would say that $1/2$ and $3/4$ would be $2/6$.



Figure 1. Snapshot of virtual classroom



Figure 2. Matak

Participants and Procedure

Eight pre-service teachers volunteered to take part in the study to trial the virtual classroom. All but one were enrolled as fully on-line students, six were mature-aged and all were in their third or final year of a Bachelor of Education (Primary) course. As we wanted to utilise the voice mechanism, the number of participants for any session was limited to eight to avoid noise congestion and feedback. Prior to each session, each participant was assigned an avatar, given a unique log-in name and password and a description of their avatar in terms of their mathematical ability. An overview of the lesson was emailed to participants, along with instructions for any activities and resources if required. At the beginning of the study, participants were given tutorials in Second Life and provided with guides to assist them with movements, gestures and speaking. Although not all participants were involved in all sessions, for most of the pre-service teachers, the mathematics lessons would have been the seventh and eighth lessons conducted in Second Life. Each lesson ran for about 20-30 minutes, was recorded using ‘machinema’, and was immediately followed up with a debriefing session, which was also recorded.

Data Collection and Analysis

Recordings of the lessons were fully transcribed and manually analysed using codes to identify instances of mathematical misconceptions. The debriefing sessions were also fully transcribed, with the codes relating to evidence of reflection and experiences as a learner. Further evaluation occurred through a follow-up questionnaire for all eight participants (including those who did not participate in the session/s). In addition, seven teacher educators were asked to view the machinema footage and respond to an emailed open-ended questionnaire designed to evaluate its use as a teaching tool. Common themes were identified from the respondents’ answers and are qualitatively described in the next section.

Results and Discussion

The Mathematics Lessons

The focus for the lessons was fractions, with experiences including counting by fractions, using a fraction wall to assist with identifying equivalent fractions and fraction sizes. The role of the teacher avatar in the first lesson was played by the researcher and there were four student avatars of varying mathematical ability. An analysis of the transcript of the first lesson showed that the pre-service teachers were willing to take on the mathematical behaviours of their allocated students as the following excerpts demonstrate (Willow was described as ‘below average’ and Jake as ‘average’):

Miss Carpenter: Can you see the fraction wall there Willow? Do you see the green bits at the top? We can say that’s one whole thing, and if you have a look at your quarters – how many of your quarters equal one of those half green bars?

Willow: Two eighths

Miss Carpenter: Are we looking at eighths or quarters?

Willow: I don’t know, but if I add them together ...

Miss Carpenter: Oh, but if you’re looking at a quarter, what does the bottom number say? There is a one on top, but what number is on the bottom?

Willow: Four?

Miss Carpenter: Good. So see the blue quarters. They are all quarters. How many of them do we need to make one whole thing?

Willow: One, two, three, four

Tr. Excellent So if four will make one whole thing, how many do we need to make half? To make the same as the green half?

Willow: Um, two?

Miss Carpenter: Very good.

Miss Carpenter: This time I’m going to ask you a question and I want you to tell me which fraction is bigger – OK – so Jake, if I said to you which would be bigger – one fifth or one eighth, which would you say is bigger?

Jake: Ah, one eighth?

Miss Carpenter: One eighth? OK, so have a look at the fraction wall, what colour are the fifths?

Jake: Yellow

Miss Carpenter: And what colour are the eighths?

Jake: Blue

Miss Carpenter: Blue, or dark blue so we don’t get confused. So is the yellow bit bigger or smaller than one of the dark blue bits?

Jake: Bigger

Miss Carpenter: OK, very good. So how many of those yellow bits does it take to make a whole one?

Jake: Five

Miss Carpenter: Five, good. And how many of those dark blue eighths does it take to make a whole one?

Jake: Eight

Miss Carpenter: OK, so can you see by looking at that, that we have cut up, like a chocolate bar, into eight little bits and had to share it among eight people that would be the size of the piece they would get. Whereas with the yellow one, you only had to share it among five people so they get a little bigger bit than the people who had to share with eight.

The transcripts show evidence that the pre-service teachers could simulate the behaviour of students who were incorrectly applying whole number thinking to compare the size of fractions and enabled the teacher to model how this would be addressed with the aid of a fraction wall. In the debriefing session afterwards, the pre-service teachers were directly asked to comment on the lesson and in particular how useful they found the fraction wall as a teaching tool. While they generally agreed that it was important to have access to the

concrete aids, one of the pre-service teachers suggested that it could have been more explicitly explained (“that each line is a whole part”), and another suggested that additional models, such as pizza or chocolate, may also have been useful. Responses from the pre-service teachers also indicated that they appreciated the modelling that occurred when addressing children’s incorrect answers:

I think that’s the most important thing because you can really get to see what responses you’re going to get and it’s going to put you on the spot in the real world because you’ve got to think on your feet and it’s a perfect way to sort of do that I think

In the second lesson, one of the pre-service teachers took on the role of Miss Carpenter and some of the student avatars were played by different participants to those who took part in the first lesson. The lesson focus moved on towards adding fractions with different denominators and the transcript provided examples of how the pre-service teacher would teach this concept and address any misconceptions that arose. There were a number of occasions when Miss Carpenter referred to the fraction wall to help make links between equivalent fractions, with the following excerpt showing an example of this:

Look at the fraction wall up there – so if you’ve got one of the green portions and you’re converting it into the blue ones, how many blue portions would you need to make one of the green ones?

There was also evidence, however, to suggest a primarily procedural approach was used when converting fractions to similar denominators. For example:

So all we’ve done is doubled – we’ve multiplied ... and all we’ve simply done is doubled both of those numbers, one and three, to get a two and a six ...so ... whatever we do to the numerator – the top number – we have to do to the denominator – the bottom number – as well.

This was also noted by one of the pre-service teachers who through the questionnaire completed after viewing the footage, questioned whether or not students would actually fully understand equivalence or just “simply find the same fraction size”. Having one of the pre-service teachers take on the role of Miss Carpenter did provide an added opportunity for reflecting upon one’s own performance as a teacher, which was not available in the first lesson. ‘Angela’, who played the role, commented that the “hardest part was that you can’t point to things ... it’s really hard to explain things if you can’t actually demonstrate them, you have to do it all via voice ... and you can’t write anything down on a whiteboard or anything. But it was good.” The follow-up questionnaire produced some more reflective comments, and although most of them related to the restrictions due to the virtual classroom environment, Angela did note that:

I shouldn’t have said ‘no’ to a student’s incorrect answer but rather asked her to explain her answer further, or asked other students to help, or failing all that, explicitly taught the concept.

Evaluation of Second Life as a Teaching Tool

Feedback received from the pre-service teachers through anecdotal, interview and survey data indicated that the main advantages included the opportunity to interact with peers in an on-line environment that was relatively anonymous, “the chance to experiment on computer students, as opposed to real people” and practice with “thinking on your feet”. Most participants agreed that it was possible to recreate an authentic classroom experience through using Second Life, with the following comment being illustrative of the feedback received:

It certainly creates an experience that is closer to authentic than text book scenarios. Practising how you respond to various situations is good practice – whether it is maths or any topic. I like the idea that to a degree you are placed out of your comfort zone as you are not completely sure what will occur, while at the same time you are within the safety of a learning environment. Often the first

practice we get are with real students and you don't have the opportunity to reflect and try again the next session (Rachel, 15/11/11).

The limitations identified included the time involved to master operating the avatar, technical issues associated with slow internet connections, restriction on the use of interactive 'props', and the inability to use spontaneous gestures.

The responses received from the teacher educators indicated that they perceived Second Life to have potential as a teaching tool within their courses. Some of the benefits identified included "it brings out the deficiencies and strengths in mathematical content knowledge as well as pedagogical content knowledge", opportunities for pre-service teachers to reflect upon their use of language and mathematical explanations, increased collaboration and social interaction for on-line students and the provision of a experience closer to reality than is usually provided in a university setting. Some suggestions for use included focusing on demonstrating a particular aspect of teaching and even modelling inappropriate teaching approaches. Like the pre-service teachers, they also perceived the main limitations as the time taken to become comfortable with operating the avatars and restrictions on interacting with the environment.

Conclusions and Implications

Conducting mathematics lessons in Second Life provided pre-service teachers with the opportunity to focus on the teaching of mathematics in a realistic and immersive environment, without having an impact on the learning of real students. It allowed the teacher educator to both role model the teacher's role and act as a facilitator following the sessions to encourage reflection and discussion about the pedagogy involved. Mathematics content knowledge and confidence with teaching mathematics continues to be a concern in teacher education, and Second Life provides an alternative to more traditional approaches, including the use of classroom video footage. The virtual environment can be 'controlled' by the instructor, providing the opportunity for focusing on particular topics, misconceptions, learning difficulties and pedagogical approaches. Feedback from participants indicated that despite some limitations, it had potential for enhancing the practical experience of pre-service teachers.

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