

Does Knowing More Advanced Mathematics Ensure Effectiveness of Working Towards Demonstrating Specialised Mathematical Content Knowledge of Second-Year Pre-Service Teachers?

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The theoretical understanding that underpins a teacher's foundation knowledge draws on their common content knowledge (CCK) and influences their mathematics' teaching (Rowland, Turner, Thwaites, & Huckstep, 2009). Teachers who have specialised content knowledge (SCK) demonstrate a unique kind of content knowledge which is more than knowing the content (Ball, Thames, & Phelps, 2008). This study reports on a comparison of two second-year pre-service teachers who had varied mathematical content knowledge (MCK) at the beginning of their Bachelor of Education course. It investigated whether knowing more advanced mathematics or foundation knowledge (Rowland et al., 2009) facilitates working towards demonstrating SCK. The results draw on a qualitative analysis, categorising lesson observation and interview responses using foundation knowledge and connections constructs of the 'Knowledge Quartet' framework (Rowland et al., 2009). Both pre-service teachers experienced course opportunities that consolidated foundation knowledge and demonstrated connections. During their lesson observation they relied on procedural explanations and neither especially demonstrated working towards SCK.

Within a large lecture theatre of pre-service teachers it can be difficult to get to know what is going on in the minds of beginning teachers. All possess a different knowledge of mathematics, mostly gained from their own educational background. At the university where this study took place, many pre-service teachers will gain employment in the Western Metropolitan Region. This region is experiencing a rapid growth in population.

This study endeavours to outline implications that will inform planning and delivery of teacher training for future pre-service teachers and the development of primary mathematics' teachers for local schools and/or beyond. Identifying course experiences that assist pre-service teachers to develop MCK can support their students' mathematical growth. Building on pre-service teachers' foundation knowledge for teaching mathematics is important. "It is fundamental because it underpins all the decisions about which examples or representations to use, connections to make, or how to respond to pupils' ideas" (Rowland et al., 2009, p. 152).

This paper focuses on the experiences and learning related to the developing MCK of two second-year pre-service teachers. The study aims to identify the opportunities and influences that enhanced their MCK during the first and second years of their course. Did knowing more mathematics at the start of their course make a differencing when comparing examples of foundation and connected MCK? These findings will contribute to a larger four-year longitudinal study of 17 pre-service teachers' MCK. The literature highlights a need for cross case analyses of different approaches, programs and settings that affect content knowledge over time, as well as longitudinal studies of learning to teach (Adler, Ball, Krainer, Lin, & Novotna, 2005; Mewborn, 2001).

Literature Review

Research on the study of teacher knowledge is recent when compared to the history of teaching and research on teaching. For the past two decades, research on mathematics

teaching has included a focus on the knowledge teachers use and need for their craft of teaching (Grossman & McDonald, 2008).

Teacher knowledge has been described as complex and existing of many facets. Schulman's (1987) theoretical framework described seven categories which became the foundation for labelling the knowledge base for teaching. Content knowledge is a central feature of his framework and is referred to as the "...*amount and organisation of knowledge in the mind of the teacher...*" (Schulman, 1987, p. 9).

Many studies have further analysed this complex knowledge. Ma (1999) describes a teacher's deep knowledge of content as Profound Understanding of Fundamental Mathematics, demonstrating breadth, depth, connectedness and thoroughness. Schoenfeld and Kilpatrick (2008) depict teachers' knowledge as similar, with proficient teachers of mathematics knowing multiple methods as well as broad and deep knowledge of mathematics. Effective teachers can draw on a range of mathematical knowledge such as procedural knowledge, procedural fluency, conceptual knowledge and mathematical connections (Ball & Bass, 2003; National Curriculum Board, 2009). These descriptions are examples that describe the unique characteristics of teachers' MCK. Ball and colleagues (2008) refer to this knowledge as SCK which is unique to teaching.

Pre-service teachers bring to the course CCK. For example, CCK is simply when someone is able to calculate an answer, demonstrate competence with the content, or recognise an incorrect answer (Ball et al., 2008). CCK knowledge will vary amongst pre-service teachers depending on their experiences before entering teacher education.

Recent Australian studies have identified weaknesses in pre-service teachers' MCK with many relying on procedural methods (Goos, Smith, & Thornton, 2008). Mewborn's (2001) critique of research found no significant correlation between the number of mathematics courses taken by a teacher and their mathematical knowledge for teaching. Possibly these courses and pre-service teachers' secondary mathematics education focused on procedural or CCK rather than developing SCK. Future pre-service teachers may bring to their course a different breadth of foundation knowledge as Australian schools implement a national curriculum (ACARA, 2012). The introduction of the proficiency strands of Reasoning, Problem Solving, Understanding and Fluency within the Australian Curriculum may deliver more students with a more connected understanding of mathematics.

Framework for MCK

The 'Knowledge Quartet' framework was designed as a means for reflecting on the content of mathematics lessons, assisting teachers to focus on what they know and what they do when teaching (Rowland et al., 2009). It has four main categories describing types of MCK required to teach mathematics well: foundation, connection, transformation and contingency. In this paper the dimensions of foundation and connection were applied when coding interview transcripts of two second-year pre-service teachers:

- Foundation: the knowledge pre-service teachers would bring with them to teacher education acquired from their schooling
- Connection: making connections between procedures and concepts and the cognitive demands of mathematics

Methodology

Context and Participants

The two pre-service teachers selected for this study were completing the Bachelor of Education Course (P-12). At graduation they will have the qualifications to teach in primary schools as well as their discipline specialisations in secondary schools. Most pre-service teachers enrolling in the Bachelor of Education (P-12) course have completed two years of the Victorian Certificate of Education (VCE) as course entry requirements. When completing the final year (Year 12) of secondary school, students may choose to study mathematics: Further Mathematics Units 3 and 4, Mathematical Methods (CAS)¹ Units 3 and 4 or Specialist Mathematics Units 3 and 4 (VCAA, 2010).

The participants in this study, Con and Rose, were part of a larger longitudinal study of 17 pre-service teachers' primary MCK. Two main factors contributed to selection for this study: the pre-service teachers' level of mathematics completed during their secondary education and their school based experiences during the first two-years of their course.

Both pre-service teachers had entered the course with different experiences of mathematics during their secondary schooling. During Year 12, Con completed Specialist Mathematics, which requires students to demonstrate a strong knowledge of mathematics and enables them to take further studies in mathematics or related disciplines when entering university. Con was completing mathematics as a major as part of his course in order to qualify to teach secondary mathematics.

Rose selected Art and Drama as her major for secondary school preparation. She completed and passed VCE, but chose not to continue to study mathematics in Year 12. During Year 11, she passed Mathematics Methods (CAS) Units 1 and 2. This unit is a pre-request for the Mathematics Methods (CAS) Units 3 and 4 that provides students with skills to undertake further study in, for example, science, humanities, economics or medicine (VCAA, 2010).

During their course all pre-service teachers participate in school based experiences known as Project Partnership. They mostly attend Project Partnership each Tuesday and complete full week block experiences. The second criteria for selecting Con and Rose for this study was that their Project Partnership experiences were similar. During first-year Con was in a Year 1 classroom and Rose a Year 1/2 composite classroom. During second-year Con was in a Year 6 classroom and Rose was in a composite Year 5/6 classroom. They both experienced different schools.

Data Collection

During the middle of second-year the pre-service teachers met with the researcher at their placement school for Project Partnership. The researcher observed the pre-service teachers teaching a primary mathematics lesson. As a coincidence both pre-service teachers chose to teach the same topic angles. After their lesson the researcher interviewed the pre-service teachers. The interview questions related to MCK during Project Partnership and course experience.

At the time of their interviews both pre-service teachers had completed different course experiences at university. Rose was interviewed first in May and had completed four education units relating to teaching primary mathematics. During first-year she had

¹ Computer Algebra System

completed Inquiry for Understanding but thought she had only ‘learnt a little mathematics.’ Therefore, during summer school, (before second-year) she enrolled in an elective unit Numeracy and Mathematics (Victoria University, 2007). This unit was designed to assist pre-service teachers who required further understanding of MCK. If required, pre-service teachers usually enrolled in Numeracy and Mathematics during second-year. In particular this unit assisted pre-service teachers who were unable to demonstrate competency in the Mathematics Competency Skills and Knowledge Test for second-year education unit, Reasoning for Problem Solving (Victoria University, 2007).

Con was interviewed one month later but had only completed the first-year education unit relating to primary mathematics. Rose and Con studied their course at different campuses. Therefore, second-year units were completed during different semesters. Rose had completed two education units related to teaching primary mathematics. Con completed these units during second semester of second-year. Therefore, Con had not completed these units at the time of the lesson observation and interview. He had completed three elective units relating to mathematics as part of his course requirement for teaching mathematics in secondary schools. The first unit, Mathematical Foundations 1 was revision of VCE Mathematical Methods; the second two extended his mathematical knowledge.

Data Analysis

This study used qualitative methods to analyse a second-year lesson and interview data of two pre-service teachers’ MCK. Both lessons and interviews were audio taped; these were then transcribed and coded for analysis. Two dimensions of Rowland and colleagues’ (2009) ‘Knowledge Quartet’ framework foundation and connection were used for coding. Findings from lesson observations and interview data relating to MCK, primary mathematical education units of study and Project Partnership experiences during the first and second years of their course are reported below. A selection of reflections and comparison of the two pre-service teachers’ developing foundation and connected knowledge with reference to the ‘Knowledge Quartet’ (Rowland et al., 2009) are reported.

Results

Con’s Reflections on Study and Partnership

Remembering back to the first-year of the course Con was concerned that he did not have ‘*primary maths knowledge*’. He wanted to build on his ‘primary’ MCK. He did not think the first-year primary mathematics unit helped him develop his MCK. Con wanted to learn about what content he should be teaching and said he, “I just wanted help remembering it all.”

He also wanted to know how primary mathematics should be taught. For example, he was concerned about how to multiply. He had forgotten because he was used to using a calculator. To help him practice he chose to tutor a Year 6 and Year 7 student. He now remembers how to do multiplication and has developed a deeper understanding. He also mentioned how you might approach multiplication with larger digits. “A primary teacher should also be able to do bigger times tables, like 11 times 32 and [think] 10 times 32 plus 32 [equals 352].” Con is exploring his connected knowledge of mental strategies using partitioning (distributive law) to solve a multiplication problem.

Con rated his MCK by reflecting on his responses to a practice Mathematical Skills and Knowledge Test he completed during first-year. This test assessed pre-service teachers’

foundation knowledge and understanding of: Space, Number, Common and Decimal Fractions, Measurement, Chance and Data, Structure and Working Mathematically (Victoria University, 2006). When explaining his MCK he said, “I wouldn’t say I am weak with any of them [the topics in the test]. I got 89% or something like that for my practice test in first-year.” By correctly answering these test items, Con is providing evidence that he has brought to the course comprehensive foundation knowledge.

Before the interview, Con had taught a geometry lesson. He drew on his foundation knowledge throughout the lesson as he focused on the aim within his lesson plan: “Assist students to understand the different types of angles that two lines make and to measure angles”. When introducing the lesson he confidently provided definitions, discussing and drawing different angles onto the whiteboard for the students. For example; “If we have an angle that is smaller than a right angle... That is right an acute angle [drew an example onto the board]... Can anyone tell me what the ones are (sic) that are bigger than a 90 degree angle? Or right angle? [Student response: obtuse]... Anything between 90 degrees and 180 [degrees] is called obtuse...”

Con enjoyed teaching mathematics and could rely on his foundation knowledge “I enjoy primary maths [lessons] because it is not a stress to think about it and it is really easy.” His mentor teacher also consolidated his foundation knowledge when planning this lesson. For example, “There are angles, intervals and raise. An interval of a line is so much of a line. A continuous line, the interval would be from point A to point B. A raise is a point A and no end... My mentor told me this and it took a second to learn.”

During primary school Year 5 and Year 6 students would be expected to develop an understanding that angles can be static and dynamic (DEECD, 2006). Con thought primary teachers required more MCK knowledge than their students. He explained that learning about angles is important knowledge required for trigonometry. “I believe it is important for a primary teacher to know up to Year 9 mathematics... for example, like angles have connections to trigonometry, which have connections... and the connections go up...” Con is demonstrating his connected knowledge as he elaborates on how his lesson connects with later learning of mathematics. Secondary school students draw on knowledge of angles when studying trigonometry, for example the angles of triangles.

He also thought it was important to know mathematics because “If a student says, why do I need to learn this? You have the knowledge to explain...They [teachers] should also know all the basic formulas for area, perimeter, trigonometry...fractions, decimals, percentages are really big in primary maths from Year 3/4...I don’t think that teachers at primary school level should have to look it up the day before. They should know it...”

Rose’s Reflections on Study and Partnership

Rose remembered struggling at secondary school during mathematics classes. Mathematics’ textbooks and work sheets provided a negative experience and she was also taught rules, which she forgot. Since starting the course Rose believed her understanding of mathematics had changed dramatically. The four mathematics education units had assisted her to begin to develop connected knowledge. “I am starting to think now. Why do I want to do that? How is that related to anything I have done previously? Seeing the links...I need[ed] someone to explain it clearly to help me.” Rose is developing connections by thinking about breaking down mathematical concepts.

Rose believed doing the course and studying for the test had assisted her mathematical knowledge. “In first year we only learnt a little... I did the practice test...” Realising that she required extra support for developing her MCK, Rose enrolled in the elective unit,

Numeracy and Mathematics. Rose completed a lot of sample Mathematical Skills and Knowledge Tests. She would *'go back and look at how they got the answer'*. Rose sometimes asked her lecturer for help or researched on the internet and completed a mathematical learning log which was an assessment task for Numeracy and Mathematics. "It forced you study, but I would have studied anyway, that made me study more, it made it more clear it was a good assignment it made you learn." During first-year of her course Rose selected course opportunities that assisted her to build her foundation knowledge.

Rose highlighted the areas of mathematics in which she was most confident during second-year. "I feel like we focused more on number because there are so many different areas within number that I think we did... Space I remember as being one of the hardest things for me... I memorised the names of the shapes but now it is fading and I would have to research it again but at least I have all those notes now to go back to. There are so many things and definitions it is hard to remember..."

When planning the lesson Rose used websites and resources to check the names of different angles: acute, right, obtuse, straight, reflex and revolution. "I looked up the definitions and I used them in the lesson. I would have had a rough idea of the definitions for the lesson and now I think I know them off by heart..." In the introduction to her lesson with the students she implemented her revised foundation knowledge of the names of angles. She could correctly draw and label the different angles on the board as she explained these with the students.

Rose also made connections with her foundation knowledge as she began to think about activities, curriculum, knowing her students and recognising the importance of enhancing understanding by not drawing on rote methods or rules. "...the teacher needs to be able to not just use the rules but do hands-on activities... in the course you get to understand how the VELS Levels (DEECD, 2006) work and so you know what to teach certain kids and how to actually put them into groups and find out what they know..."

Discussion and Conclusion

Rowland and colleagues' (2009) 'Knowledge Quartet' framework was suitable for coding pre-service teachers' transcripts of lesson observations and interviews when classifying their foundation and connected knowledge. The findings of this small study suggest both pre-service teachers developed foundation and connected knowledge during the first two years of their course by doing primary mathematics units of study and participating in Project Partnership experiences. The pre-service teachers developed foundation knowledge in conjunction with their connected knowledge. For example, during Project Partnership when teaching students, talking with their mentor or planning lessons. Course experiences at university included: completing practice Mathematical Skills and Knowledge Tests, developing a mathematical learning log as well as completing primary mathematics education units.

At the beginning of their course Con had more advanced mathematics when compared to Rose. However, the practice Mathematical Skills and Knowledge Test during first-year provided an opportunity for both pre-service teacher to identify strengths and weakness relating to their CCK. In particular it assisted Rose in making the decision to enrol in an elective primary mathematics unit. Con identified some gaps in his MCK and chose to tutor mathematics students. These choices would have assisted development of foundation knowledge. A concern is that all pre-service teachers may not be as motivated as the participants in this study, failing to address and implement strategies to consolidate gaps in their MCK.

Con and Rose experienced upper levels of the primary school curriculum during second-year Project Partnership. Teaching older primary students could provide opportunities to practice the foundation knowledge they may have revised for their Mathematical Skills and Knowledge Test. Their foundation knowledge of a topic could be drawn on in the classroom as they made connections when planning, teaching or responding to students' mathematical problems.

Before their interviews both pre-service teachers had taught a similar lesson focusing on geometry and the understanding of the names of different angles. However they both modelled procedural knowledge when introducing their lessons and naming and labelling different angles for the students. Rowland and colleagues (2009) describe this in trainee teachers as an instrumental understanding, excessively relying on procedures. A different approach would have been to allow the students to construct their own angles and sort them into various groups, acute, obtuse etc. This would assist students with a conceptual understanding rather than a rote method and would demonstrate pre-service teachers' connections with knowing the sequence of the topic.

When comparing the two pre-service teachers in this study, knowing more advanced mathematics did not especially promote working towards demonstrating SCK. Mewborn's (2001) critique of research reported similar findings with teachers. During their interviews, both pre-service teachers explained their thinking and provided examples of how they were developing conceptual connections. However, when both pre-service teachers were observed teaching a primary mathematics lesson related to angles they lacked connected knowledge and relied on procedures when introducing the different angle names to their students. They lacked cognitive demands of mathematics for planning and sequencing the lesson (Rowland et al., 2009). Goos and colleagues (2008) also reported that other studies suggest pre-service teachers rely on procedural methods.

From this study, one can conclude that the practice Mathematical Skills and Knowledge Test provided an opportunity for pre-service teachers to assess and address needs relating to gaps in foundation knowledge. Providing opportunities to enrol in an elective numeracy unit assisted development of foundation knowledge. Both pre-service teachers demonstrated that they knew the foundational knowledge for the lesson they taught. A concern was that both pre-service teachers were teaching rote knowledge when introducing their lesson during the second-year of their course. They required further guidance and understanding when designing a lesson for demonstrating connected knowledge of the topic. Making connections is at the heart of teaching and teaching is more than telling students what they need to learn (Rowland et al., 2009). A teacher uses their SCK for identifying mathematical connections when working with students and planning lessons (Ball et al., 2008).

There are limitations to this study as data was only collected from two pre-service teachers. A larger in depth study of 17 pre-service teachers' MCK will use a cross case analysis of various course experiences to test the findings reported here. Further analysis of lessons including those in the final year of their course will provide further detail and will be reported in the larger longitudinal study of pre-service teachers' MCK.

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