

Mathematical Engagement Skills

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In this paper, the mathematical engagement of Colin and Robyn is compared. Through this comparison, and informed by longitudinal research into the mathematical journeys of a group of students in New Zealand, a set of engagement skills emerged. Both students had high levels of engagement in mathematics. However, Colin was a thriving mathematics student with effective engagement skills, whereas Robyn had ineffective engagement skills and was, over time, vulnerable to disengagement, negative feelings and non-participation in the subject of mathematics.

Introduction

In a longitudinal study using methods and analysis informed by qualitative methodology, the author captured students' journeys through mathematics by investigating a class of adolescents in New Zealand (Ingram, 2011). Specifically, this research sought to discover the nature of students' relationships with mathematics, how these relationships were associated with mathematical learning, and how they changed over time to form students' mathematical journeys. Emerging from the analysis of the data, the elements of these relationships were the students' views of mathematics, their mathematical knowledge, feelings, identities and habits of engagement. Negotiating with these elements and the current context, the students engaged in each mathematical task in different ways and therefore had individual experiences and performance outcomes. The meaning the students derived from these experiences reinforced or altered elements of their relationship with mathematics, which over time formed their mathematical journeys.

It is one of these elements, engagement, which has been detailed in this paper. With its strong links to mathematical learning (Op 't Eynde, 2004), students' engagement in mathematics is vital to the acquisition of knowledge and strategies and for continued participation in the subject. Yet, students' low levels of engagement in mathematics are often lamented (Sullivan, McDonough and Harrison, 2004). *Engagement* is considered in this paper to be students' involvement in the mathematical activity of the classroom and their commitment to learning the mathematical content. The term 'engagement' is viewed as different to 'participation', which here describes students choosing to enrol in the subject of mathematics.

The comparison of two of the students, Colin and Robyn, was the catalyst for the notion of a student having a set of engagement skills. In the same mathematics class, surrounded by the same classmates, taught by the same teachers, and doing the same tasks, Colin and Robyn were both highly engaged in the mathematics, yet they had very different learning experiences and performance outcomes because of their unique relationships with the subject. Researchers tend to talk about the degree of a student's engagement (e.g., Williams & Ivey, 2001), but in this research, the level of engagement was inadequate in describing the quality of that engagement and it is this quality, in the form of engagement skills, that is focussed on.

In this paper, engagement literature in mathematics education has been reviewed. Then the methods and analysis used in the larger research project have been described with a focus on the emergence of engagement as a category. This category has been described at a class level. Colin and Robyn's engagement is then compared and discussed in terms of their engagement skills in mathematics.

Students' engagement

The construct of engagement has been researched increasingly in mathematics education, often in association with constructs such as affect and motivation. Sometimes called participation or effort (e.g., Sullivan, Tobias, & McDonough, 2006), engagement has been linked to various aspects of *affect* – the experience of feelings and emotions (McLeod, 1992). Greenwood's (1984) early research describes people's disengagement in mathematics as their avoidance of negative feelings. Williams and Ivey (2001) used the term *motivation for engagement* to describe the degree to which students choose to actively engage in the classroom activities available to them because of their affective assessment of mathematics. Affective researchers (e.g., Malmivuori, 2006) included habitualised behaviours and patterns of engagement in their conceptualisations of a students' mathematical core.

Researchers into motivation attempt to explain students' choice of tasks, their persistence and vigour when doing them, and their related performance in those tasks (Wigfield & Eccles, 2000). These aspects of students' learning are explained through research into intrinsic vs. extrinsic motivation, goal orientations, interest, and self-schema or a combination of these (Murphy & Alexander, 2000).

Sullivan et al. (2006) drew on Dweck's (1999) research to investigate students' perceptions of the extent to which their efforts contributed to their success in, and enjoyment of, mathematics. Sullivan et al. (2006) found that the students chose not to engage because of the classroom culture, rather than because they were unable to engage. "It seems to us that classroom culture may be a more important determinant of participation than the curriculum, methods of teaching, modes of assessment, teacher experience, level of resources, or anything else" (Sullivan et al., 2006, p. 97). These findings highlight the limitations of studies that view students' learning as a product of individual cognitive processes.

Learning is considered in this research to be a fundamentally social phenomenon. How a researcher understands and uses the term 'learning' depends on their theoretical approach. In the wider field of mathematics education, two major theories of intellectual development have been dominant: *constructivism*, which is seen as part of the cognitive perspective (Greeno, Collins, & Resnick, 1996) and, more recently, *socioculturalism*, which includes the situated perspective (Lerman, 1996). The main differences in these perspectives arise in the ways that the interaction between the individual and the social are theorised and the extent to which researchers take account of the context within which the learning process takes place. In much of the affective literature, theoretical perspectives on learning are often unexplored or a cognitive perspective implied.

Also viewing learning as social, Boaler and Greeno (2000) describe mathematical learning as the construction of meaning through participation in the practices of the mathematics classroom. Similarly, Op 't Eynde (2004) described a students' learning as taking place through engagement in the language, rules and practices that govern activities in the community of the mathematics classroom. Evident in this research is the notion that students' engagement in mathematics can be viewed as similar to learning.

Research that is not situated in the classroom can still provide a fine-grained detail of individual's cognitive processes when they are engaging in a task. DeBellis and Goldin (2006), who conducted clinical, task-based interviews with individual children across two years, characterised a person as having *affective competencies* – the individual's capabilities to encode their affect and to respond with action that enabled them to complete a problem. They introduced the new constructs of *mathematical intimacy*, which refers to "deep vulnerable emotional engagement an individual may have with mathematics" (p. 132) and *mathematical integrity*, which refers to the "individual's fundamental commitment to

mathematical truth, search for mathematical understanding, or moral character guiding mathematical study” (p. 132). These seem potentially useful in describing students’ engagement because they describe the quality, rather than the level of engagement.

While there has been much research surrounding the conditions necessary for engagement to occur (e.g., Sullivan & McDonough, 2007) and its association with affect, there has only been some research into the quality of engagement during individual tasks and this has not incorporated the social nature of learning. In this paper, the quality of a student’s engagement in mathematics as a subject and in individual mathematical tasks is explored.

Methodology

The 31 participants of this research attended a co-educational secondary school in New Zealand. At the beginning of the research, the students were in Year 10, aged around 14. The two-year research period allowed students’ relationships with mathematics to be explored through changes of teachers, classmates, physical locations and expectations in the years leading up to mathematics being no longer compulsory. The research had a qualitative framework. Ethnographic and case study techniques were used to fully describe the classroom and the sociocultural context of the students (Merriam, 1998). Caution was used in interpreting students’ behaviour. For example, if one student was observed to be working steadily, and another to be talking while they completed their work, it was possible that both of the students were engaged in the mathematics. It was more tenable to describe the students’ engagement from their perspective – it was the meanings students took from a situation or experience that were important, rather than the researcher’s interpretations.

As “no single method can grasp all the subtle variations in ongoing human experience” (Denzin & Lincoln, 2005, p. 21), multiple tools were needed to ensure data was from the students’ perspective, to make their relationships with mathematics visible in different ways, to confirm the emerging findings, and to make decisions about the research process. Observations were made, the students were interviewed, and they were asked to submit a variety of written responses. Various school documents were collected, teachers were interviewed and parents were asked to fill in a questionnaire. The interviews were semi-structured with both individuals and groups of students. The student questionnaires were informed by Kaasila, Hannula, Laine and Pehkonen’s (2007) use of autobiography, Chiu and Henry’s (1990) questionnaire about students’ feelings about mathematics and a mathematics attitude survey by Fennema and Sherman (1976).

The research was inductive (Merriam, 1998) because of the constant movement between data collection and the three stages of analysis. In stage one, initial interpretations were made, shaped by the researcher’s background and experiences, the associated literature, data already gathered in the research process, and the emerging findings from all of the stages of analysis.

The analysis at stage two sought to understand students’ relationships with mathematics and how these were associated with mathematical learning. Grounded theory procedures of analysis were used such as the constant comparison method, multiple stages of data collection and the seeking, refinement and interrelationship of explanatory categories (Strauss & Corbin, 1998). However, inconsistent with grounded theory, the data set was analysed keeping in mind potential aspects of students’ relationships with mathematics. NVivo (QSR International, 2006), a qualitative data analysis software package, was used to enable the large data set to be managed and to support the grounded theory approach by facilitating the coding of data according to developing categories.

With six other categories, engagement emerged as an important theme. The students never used the word ‘engagement’. They instead talked about aspects of their engagement,

for example “working” or the “doing” of mathematics. Sometimes they used these terms synonymously with “learning”. The students perhaps assumed that, if they engaged in mathematical activity, they would accumulate mathematical content knowledge, and learning would take place. Engagement was explored further to form sub-categories about students’ habits of engagement, pathways of engagement, the influence of the context of the moment and engagement skills.

In the third stage of analysis, case studies allowed the internal themes unique to each student to be considered in relation to other students and the class’s relationship with mathematics. Through constant comparison the analysis moved to and fro between the general and the specific (Strauss & Corbin, 1998); between gradually unfolding personal journeys of the students and the emerging tentative theorising of the students’ engagement in mathematics, and within this, their engagement skills.

Strategies have been used throughout this research to ensure its quality such as minimising researcher bias through intensive, long term involvement, validating responses through checking transcripts directly with the students, searching for discrepant data, contradictions, negative cases, and seeking alternatives (Merriam, 1998). All of the research methods were used to inform each other in a continual process of interaction and reanalysis, and the research methods were adapted to the context and circumstances. Credibility was mainly concerned with ensuring data was as close as possible to the students’ perspective and minimising researcher interpretation.

Students’ habits of engagement are described in the next section in terms of the whole class and this provides a context for the specific discussion of students’ engagement skills, which have been detailed in terms of individual case studies.

Students’ habits of engagement

The students talked about their mathematical engagement both as a subject overall and their engagement in individual tasks. There was evidence the students had *habits of engagement* in the subject of mathematics – their involvement and commitment to the subject overall. Three students avoided mathematics as much as possible. Two others completed assigned work as quickly as they could to get it over with. These habits, which formed part of the students’ relationships with mathematics, continuously developed throughout students’ mathematical journeys, and were modified or reinforced by their engagement in each mathematical task.

As part of these habits, in some detail, the students described the pathways they usually took when attempting a mathematical task – their *pathways of engagement* – a term adapted from Goldin’s (2004) use of the term ‘affective pathways’. The pathways the students described were typified as avoidance, superficial engagement, and full engagement. If a problem initially looked hard, some students avoided doing it altogether, disengaging from the mathematics. Other students only made a superficial attempt to solve the problem, giving up when they experienced difficulty. Only three of the students described full engagement in mathematical tasks, persevering, using a variety of strategies and discussing the mathematics with others.

When the students engaged in a mathematical task, they were situated in a unique *context of the moment*. The mathematics teacher, the specific mathematics activity and the current social norms formed part of the context of the moment. Aspects of the classroom’s physical environment such as heating and lighting and seating arrangement further contributed. Students were also individually processing family or relationship issues, or were experiencing illness. A student’s individual interpretation of the context of the moment, interacting with

the elements of their relationship with mathematics, led to different ways the students' engaged in the mathematics.

The interaction between engagement and other elements of the students' relationships with mathematics was complex. All 31 of the students directly associated their views and feelings with the way they engaged in mathematics. Students, who enjoyed mathematics and viewed it to be a useful and important school subject, were more likely to engage in it to a fuller extent. These are similar to Sullivan et al. (2006) findings, which described students' engagement and motivation as vulnerable when they were unconvinced about the value of mathematics in their current or future lives. In the current research, the students' generally viewed mathematics as a boring and difficult subject. Boredom made the students feel tired and unhappy. It reduced the engagement of the students in terms of the amount of work they did, the depth to which they did the work, their level of perseverance and the time spent socialising. When students perceived mathematics to be an important subject, engagement was necessary because of the nature and difficulty of the subject. For these students, this helped to balance their loss of motivation to engage that they experienced because of the boredom. These findings emphasise the importance of investigating engagement in the context of students' whole relationships with mathematics.

Students' engagement skills

In analysing the different pathways of engagement that individual students had, and then comparing those with the other students in the class, a set of *engagement skills* emerged. These are called engagement *skills* because a skill implies something that can be actively fostered rather than simply a habit that has developed over time. These skills are detailed in Table 1 below.

Table 1
Engagement skills

Engagement skill	Description
Perseverance	The skill of continuing to do a mathematical task, despite experiencing difficulty.
Integrity	The commitment to searching for mathematical truth and understanding – searching for more than the correct answer.
Intimacy	Deep emotional engagement with mathematics.
Independence	The skill of solving problems autonomously.
Concentration	The skill of remaining focussed on the mathematics, and continuing engagement despite disruption.
Utilisation of feelings	The skill of being resilient to negative micro-feelings, and instead using them as a signal to persevere or change strategy.
Cooperation	The skill of discussing mathematical with others, to solve the tasks cooperatively, and to ask for help as a strategy, rather than as a form of disengagement or dependence on others.
Reflection	Being self-aware. Reflecting on own and others' engagement.

Researchers in the affective domain have discussed similar concepts to various degrees. DeBellis and Goldin's (2006) terms 'mathematical intimacy' and 'mathematical integrity' have been adopted in this research. Hannula (2006) talked about students' need for autonomy and their self-regulation and how this could be conscious or unconscious. Sullivan and In V. Steinle, L. Ball, & C. Bardini (Eds.)(2013). *Mathematics Education: Yesterday, 406 today, and tomorrow* (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia. pp. 402-409). VIC: MERGA.

McDonough (2007) found the students' willingness to persist was one of the factors necessary for engagement to occur. Outside the domain, Yair (2000) talked about the students' need to focus on the instruction. Indeed, many of the terms in Table 1 are familiar. Underpinning the New Zealand Curriculum (Ministry of Education, 2007) are values such as inquiry, curiosity, participation and respect. A student's key competencies include thinking, managing self, relating to others and participating. Furthermore, learning to learn is an important principle in the New Zealand curriculum.

Colin and Robyn each had a unique relationship with mathematics. These are described and their engagement skills discussed in context of these relationships. The implications of their unique engagement skills are then described in terms of their mathematical journeys. Phrases written in quotation marks are direct quotations from Colin or Robyn, unless otherwise indicated.

Colin's relationship with mathematics was very positive. His view of mathematics was that it was a challenging, exciting and important part of life. He knew a lot of mathematics and enjoyed accumulating new knowledge. He loved the subject, was confident in his ability and had high expectations of success. Colin had highly effective engagement skills. He usually engaged fully because he saw each mathematical situation as an opportunity to increase his mathematical knowledge. He demonstrated mathematical integrity and mathematical intimacy. In other words, Colin was emotionally involved in the mathematics and he demonstrated vigour in his search for mathematical truth and understanding. Colin sought understanding by thinking about the task more "complicatedly" because of its potential to add to his knowledge, and because he enjoyed the experience. When Colin came across a difficult problem, he anticipated that he would experience feelings such as confusion, slight anxiety, as well as curiosity. He was not only resilient to these feelings, but he was able to utilise them. They were affordances, rather than constraints, and a signal to persevere or change strategy. He was also skilled at both working independently and cooperatively; often discussing the mathematics with the teacher and his classmates. Colin knew if there were difficult problems he would "learn how they work eventually". Furthermore, Colin thought about his processes of engagement. He actively fostered his engagement skills by reflecting on them. He described himself as good at what he called "meta-cognition". Colin was aware that the way he engaged in the mathematics differentiated him from other students and contributed to his greater level of mathematical knowledge. Colin had high self-awareness, control of his responses, and high personal agency. Consequently, he could be described as actively regulating his affective responses (Malmivuori, 2006).

Robyn's relationship with mathematics was very different. Robyn believed that mathematics was not her strength. To her, mathematics was a difficult, controlling and obscure subject, a part of her school life that she did not enjoy. To Robyn, mathematics was a set of procedures and her focus was on learning these "off by heart ... to grasp everything with certainty". She sought instrumental understanding, as defined by Skemp (1976). Understanding to Robyn meant knowing the "definite steps of how to do it" and knowing "the storyline" of mathematics. She did not construct these steps herself; rather she learnt how to do them and then practised them so she could "really understand". Robyn believed difficulties in mathematics could be overcome by hard work. She attempted to meet her own high expectations of success with intense engagement. She worked steadily throughout the lesson, discussed the mathematics with her classmates and teachers, asked for help, and completed her homework. She experienced intensely negative feelings during mathematics because she engaged highly in the mathematics but still did not achieve success.

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Robyn's engagement skills were not effective. Although Robyn did have some mathematical integrity, wanting to go over and over an answer until she knew the steps, unlike Colin, Robyn did not emotionally engage with the mathematics. Robyn rarely discussed the mathematics with her classmates and only engaged superficially. She did not persevere when she experienced confusion. Rather, she asked for help. Help-seeking was described by Yair (2000) as an indicator of engagement. However, Robyn was dependent on this help. Unlike Colin, she did not learn to think for herself. Any reflection Robyn did on her mathematical learning led her to feeling "inferior" and "stupid"

Colin was a thriving mathematics student. His strong need for enrichment and achievement and his very effective engagement skills continued to support him during his journey through mathematics – making him more resilient to a wide range of experiences in the mathematics classroom. Boaler, William and Brown's (2000) research found that girls in particular were affected by fast-paced lessons because they wanted an in-depth understanding of what they were doing. Although many of Robyn's skills were effective in the daily routine of answering questions individually from the textbook, they were ineffective as she moved into the senior school. The amount of content and the faster-paced lessons in the class meant that learning the procedures fully was difficult for her to accomplish. As Robyn continued with mathematics, her relationship with mathematics, and in particular, her ineffective engagement skills, made her vulnerable to negative feelings, disengagement, and non-participation as the number of procedures in mathematics that she needed to learn continued to accumulate.

Conclusions

Interacting with other elements of their relationship with mathematics, and depending on their interpretation of the context of the moment, the research students engaged in mathematics in unique ways, which in turn affected their experiences and success in mathematics. This paper has described the students' engagement in the subject of mathematics as a whole and their engagement in individual mathematics tasks. The students had habits of engagement and tended to disengage, engage fully or superficially engage in the mathematics. Colin thrived in mathematics and Robyn was vulnerable to non-participation because of their very different relationships with the subject and their unique set of mathematical engagement skills. These sets of skills have enabled Colin and Robyn to be compared according to the quality as well as the level of their engagement in mathematics. Rather than a student being described as engaged or disengaged, now students' engagement can be compared according to their perseverance, mathematical intimacy and integrity, the way they utilise their feelings, their concentration, independence, cooperation and reflection. Although this was a small-scale study, the set of engagement skills that emerged and the fine-grained understanding of the how students' engagement operates in relation to their wider relationship with mathematics, has potential for further research involving students' engagement.

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