Early mathematical competencies and later outcomes: Insights from the longitudinal study of Australian children

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International research suggests that early mathematical competences predicts later mathematical outcomes. In this paper, we build on our previous study of young children's mathematical competencies (MacDonald & Carmichael, 2015) to explore the relationship between mathematical competencies at 4-5 years, as measured by teacher ratings, and later results on Years 3, 5, 7 and 9 NAPLAN numeracy tests. Data from a nationally-representative sample of 2343 children participating in the Longitudinal Study of Australian Children (LSAC) are examined. In line with overseas studies, we report moderate correlations between pre-entry mathematics and later NAPLAN results. However, analysis of individual growth trajectories suggests that in fact early mathematics predicts the initial (Year 3) level, but not subsequent growth. This suggests that early mathematical competences are important for enhancing outcomes in early schooling, but that the quality of mathematics education provided in the schooling years is critical for future development.

Introduction

Both Australian and international research has established that young children engage with a range of mathematical concepts and processes prior to starting school (e.g. Gervasoni & Perry, 2015; Sarama & Clements, 2015). Recent research has also identified a link between children's early mathematical skills and their later mathematical outcomes (e.g. Watts, Duncan, Siegler, & Davis-Kean, 2014), with research noting, in particular, the predictive power of mathematical knowledge at school entry for later mathematical achievement (e.g. Duncan et al., 2007). An opportunity to explore the relationship between early mathematical competencies and later mathematical outcomes has been afforded through the Longitudinal Study of Australian Children (LSAC) (Sanson, Nicholson, Ungerer, Zubrick, Wilson et al., 2002). LSAC utilises a cross-sequential design to follow two cohorts of children: a Birth cohort of approximately 5000 children aged between 6 and 12 months; and a Kindergarten cohort of approximately 5000 children aged between 4 years 6 months and 5 years. This study focuses on children from the Kindergarten cohort of LSAC when they were aged four to five years and examines both their early mathematical competencies and their later mathematical outcomes, as measured by their results on the Years 3, 5, 7 and 9 *National Assessment Program – Literacy and Numeracy* (NAPLAN) numeracy tests. The overarching research question guiding this study is: To what extent, and how, are competencies at age 4/5 related to mathematical achievement later in school?

Background

In this section we provide a brief review of extant research pertaining to the importance of early childhood education, young children's mathematical competencies, and the relationship between early mathematical competencies and later outcomes.

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Importance of early childhood education

The importance of early childhood education is well established. Conclusive international evidence demonstrates that early childhood is a vital period in children's learning and development (Department of Education, Employment and Workplace Relations [DEEWR], 2009), and that what happens in early childhood affects later development (Council of Australian Governments [COAG], 2009). Australia's national early childhood curriculum framework *Belonging, Being and Becoming: The Early Years Learning Framework for Australia* (EYLF) (DEEWR, 2009) was established "to assist educators to provide young children with opportunities to maximise their potential and develop a foundation for future success in learning" (p. 5). Furthermore, the EYLF contributes to realising COAG's (2009) vision that by 2020 "all children have the best start in life to create a better future for themselves and for the nation" (p. 4). Participation in effective early childhood education has a number of significant benefits, including improved child development, improved school readiness and performance at school, and improved educational attainment (Robinson, Silburn & Arney, 2011).

Young children's mathematical competencies

Doig, McRae and Rowe (2003) have suggested that the increasing numbers of children participating in early childhood programs, and the growing recognition of the importance of mathematics in general, provide compelling reasons for understanding children's mathematical development in the early childhood years. Indeed, children begin developing mathematical skills from a very young age. International research has shown that babies and toddlers demonstrate competence in regards to a range of mathematical concepts and processes, including number and counting, geometry, dimensions and proportions, location, and problem solving (Björklund, 2008; Reikerås, Løge & Knivsberg, 2012). Furthermore, several Australian studies have found that much of the content that forms the mathematics curriculum for the first year of school is already understood clearly by many children on arrival at primary school (Clarke, Clarke, & Cheeseman, 2006; Gervasoni & Perry, 2015; MacDonald, 2010), a finding echoed in international studies (e.g. Aubrey, 1993; Wright, 1994).

Early mathematical competencies and later outcomes

In recent years, studies have emerged which indicate the predictive power of early mathematical knowledge for later outcomes, both in mathematics specifically, and in terms of more general academic outcomes. Watts et al. (2014) conducted a tracking study of 1364 children from 4.5 years to 15 years and found that there were statistically significant associations between preschool mathematical ability and adolescent mathematics achievement, even after accounting for early reading, cognitive skills, and family and child characteristics. Furthermore, gains in mathematical knowledge from preschool through first grade were even more predictive of mathematics achievement at age 15 than preschool knowledge. Geary, Hoard, Nugent, and Bailey (2013) conducted a longitudinal study of 180 children from Kindergarten through to age 13 and found that early number system knowledge predicted functional numeracy more than six years later, controlling for intelligence, working memory, in-class attentive behaviour, mathematical achievement, demographic and other factors. Duncan et al. (2007) examined six longitudinal data sets to identify links between school-entry academic, attention and socioemotional skills and later school reading and mathematics achievement. Across all six studies, the strongest predictors of later achievement were school entry mathematics, reading, and attention skills. A meta-analysis of results showed that early maths skills have the greatest predictive power. This finding was consistent for boys and girls, and for children from high and low socioeconomic backgrounds. Similarly, Claessens, Duncan, and Engel (2009) used data from a nationally-representative sample of children from the Early Childhood Longitudinal Study – Kindergarten Cohort (ECLS-K) to estimate the predictive power of school-entry academic, attention-related and socioemotional skills for reading and maths achievement in first, third and fifth grade. School entry maths skills were consistently predictive of fifth-grade achievement. Early maths skills were not only highly predictive of later maths achievement, but of later reading achievement as well. Maths skills were the single most important set of kindergarten entry skills emerging from the analyses.

Given the growing body of research which suggests a relationship between early mathematics and later school achievement (Levine et al., 2010), it is important to ascertain the extent to which children's early mathematical competencies of children predict later mathematical outcomes, taking into account growth over time. This study considers the research question: To what extent, and how, are competencies at age 4/5 related to mathematical achievement later in school?

Method

Sample

The study is based on those 2343 students from the K-cohort for whom mathematical outcomes were available at age 4/5 and for whom at least three NAPLAN numeracy results were available when these students were in Years 3, 5, 7 and 9. Details of the design and implementation of LSAC are available in (Sanson, Nicholson et al., 2002). The size of this sample is considerably smaller than the original 4983 and is due to a number of factors including: attrition over the nine year period, difficulties in gaining parents' permissions for the release of their child's NAPLAN results, and difficulties in gaining teacher data.

Variables

Outcome. The main outcome variable in the study are the scores in the NAPLAN numeracy tests held when the students were in Years 3, 5, 7 and 9. These tests contain a mix of multiple-choice and short response items that reflect the *Australian Curriculum – Mathematics* (ACARA, 2013). Means and standard deviations for the outcome are shown in Table 1, which also includes population figures.

Table 1
NAPLAN means and standard deviations for sample and population

	Year 3, 2008	Year 5, 2010	Year 7, 2012	Year 9, 2014
Sample	426 (73)	506 (71)	559 (74)	606 (73)
Sample size	*n* = 1769	*n* = 2292	*n* = 2272	*n* = 1863
Population	397 (70)	489 (70)	538 (74)	588 (71)

Explanatory. Teachers were asked to answer "Yes" or "No" to whether their student had the ability to achieve each of five early mathematical competencies: (C1) sort and classify; (C2) count objects; (C3) count to twenty; (C4) recognize numbers; and, (C5) do simple additions. The total number of competencies achieved by the student was the main explanatory variable in the study and ranged from zero to five (M = 3.7, SD = 1.2). The majority of students (62%) gained four or more of the five competencies. Hereafter these students are referred to as high-competency.

Demographic variables. A number of student and family variables were also considered. Of the 2343 students in the sample, one half were male and their ages[1] when their data were first collected ranged from 4.3 to 5.6 years (M = 4.8, SD = 0.20). Students undertook an initial test of ability, as measured by the *Peabody Picture Vocabulary Test, 3rd Edition* (PPVT-III) (Dunn, Dunn, & Dunn, 1997). The PPVT-III is a measure of children's receptive vocabulary and a shortened version was used in LSAC as a screening test of verbal ability (Rothman, 2005). PPVT-III scores for this sample ranged from 31 to 85 (M = 65.1, SD = 5.8). Finally the social economic position (SEP) of the child, a standardised index developed specifically for LSAC, was included. This index, described in Blakemore, Strazdins et al. (2009), ranged from -2.3 to 3.0 (M = 0.2, SD = 1.0).

Analysis

Initially we provide some simple bivariate analyses in the form of correlations between the outcome variables and the explanatory variable. We also test the impact of each of the competencies, and children attaining a high number of competencies on later mathematics performance. In order to control for the influence of demographic and other factors on children's later mathematics achievement, we use multilevel regression models. These allow us to model the individual growth trajectories of students, whilst also accommodating missing data on the outcome variables. Model estimates were obtained using R (R Development Core Team, 2011) and in particular the Multilevel package (Bliese, 2012) as described in (Faraway, 2006).

Results

Bivariate analysis

The number of competencies gained at age 4/5 was moderately correlated with NAPLAN numeracy results in Year $3(r = 0.30, p = 0.00, df = 1767)$, Year $5(r = 0.28, p = 0.00, df = 2290)$, Year $7(r = 0.29, p = 0.00, df = 2270)$, and Year $9(r = 0.27, p = 0.00, df = 1861)$. After controlling for Year 3 results, however, the correlation between the number of competencies and later NAPLAN results failed to be significant. The number of competencies was also correlated with the socio-economic position (SEP) of the students (r = 0.22).

Given that most students gained four or five of the early competencies, we tested the difference in means in NAPLAN numeracy tests between this group of high-competency and the remaining low competency students. High-competency students scored significantly higher than their peers in Year 3 ($\Delta M = 40, t = 11.0$), Year 5 ($\Delta M = 37, t = 12.0$), Year 7 ($\Delta M = 41, t = 13.0$), and in Year 9 ($\Delta M = 39, t = 12.0$). Thus, high-competency students at age 4/5 performed on average more than one half a standard deviation higher than their peers and appeared to maintain that advantage throughout their schooling years.

Each of the individual competencies were assessed against later NAPLAN performance. The 2281 students who were able to sort and classify (C1) at age 4/5, on average, scored

[1]This was the age when children's parents completed the survey. Teacher data were obtained once parental permission was obtained.

better than their peers in Year 3 ($\Delta M = 50, t = 4.2$), Year 5 ($\Delta M = 40, t = 3.5$), and Year 7 ($\Delta M = 45, t = 4.2$). Similarly the 2242 students who could count objects (C2), on average, scored better than their peers in Year 3 ($\Delta M = 53, t = 6.2$), Year 5 ($\Delta M = 31, t = 3.5$), and Year 7 ($\Delta M = 33, t = 4.1$). The 1520 students who could count to 20 (C3) at age 4/5 scored significantly higher than their peers in Year 3 NAPLAN numeracy ($\Delta M = 40, t = 11.3$) and broadly maintained this advantage on each of the subsequent NAPLAN tests. Similarly, the 1794 students who could recognise numbers (C4) and the 856 who could do simple additions (C5) at age 4/5 scored significantly higher in Year 3 Numeracy ($\Delta M = 39$ and $\Delta M = 30$ respectively) and again broadly maintained this advantage.

The bivariate analyses reported above clearly demonstrate the advantage that highly competent children at age 4/5 have over their peers in later NAPLAN numeracy results. These analyses, however, do not consider the age difference of children when the competencies were assessed nor the impact of family and individual factors. Consequently a regression analysis is required.

Longitudinal analysis

Broadly, the modelling process applies lines of best fit to the three or four NAPLAN outcomes for each of the 2343 students. The intercepts and gradients from each of these 2343 lines, in turn, form two response variables that are predicted by the explanatory and demographic variables. More formally, the model assumes that at the student level, growth in NAPLAN numeracy is linear and expressed as:

$$Y_{ti} = \beta_{0i} + \beta_{1i}t + \epsilon_{ti} \text{ -------------------- (1)}$$

where $Y_{ti}$is the NAPLAN score of the ith student at time t = 0 (2008) to 3 (2014), β_{0i} the random intercept, β_{1i} the random slope and ϵ_{ti} the residual error.The terms β_{0i} and β_{1i} then form the response variables for two simple linear regression equations, each with their own random component, which is explained using predictor variables.

Initially, no predictor variables were entered into the model and this is shown as Model A in Table 3, which is equivalent to Equation 1. Following this, the number of competencies (*Ncomp*) together with demographic variables were entered into the model as predictors of the intercept. Finally, these variables were included also as predictors of the slope and significant predictors are shown as Model B in Table 3.

As shown in the table, the socio-economic position (*SEP*) predicted both the initial score in Year 3 (intercept) as well as subsequent growth (Time). Female students, on average, scored 13 points lower in Year 3, but gender did not influence subsequent growth. Similarly, the PPVT-III score of students at age 4/5 predicted their initial score but not their growth. Age differences when data were first collected did not impact on later NAPLAN numeracy performance. After controlling for these demographic variables, the number of competencies at age 4/5 did predict the initial score but it did not predict subsequent growth. A dichotomous variable indicating those students with high-competency was entered instead of *Ncomp*. Though not reported in Table 3, this variable was a significant predictor of the intercept (b = 28) but not of the slope. Similarly, dichotomous variables representing each of the five competencies were entered simultaneously into the model instead of *Ncomp*. Only counting to 20 (b = 20), recognizing numbers (b = 10) and doing simple additions (b = 11) were significant predictors of Year 3 NAPLAN numeracy.

Table 3
Results of multilevel models

Variable	Model A Estimate	SE	Model B Estimate	SE
Fixed effects				
Intercept	437	1.5	238.0	14.4
Time	59	0.4	58.3	0.5
Ncomp			12.7	1.1
SEP			16.5	1.5
PPVT-III			2.2	0.2
Female			-12.7	1.5
Time*SEP			2.2	0.5
Random effects				
Between intercepts variance	4003		2877	
Between slopes variance	112		111	
Residual variance	1188		1174	

Discussion and Limitations

Our analysis clearly demonstrates the importance of mathematics in early childhood for later performance on the Year 3 NAPLAN numeracy test. Students gaining four or more of the five competencies, on average, gained Year 3 NAPLAN numeracy results that were 0.5 standard deviations higher than their peers. More significantly, this gap in NAPLAN numeracy results appeared to remain over their schooling years. On the other side of the coin, the results show the disadvantage experienced by children who did not possess these early mathematical competencies and who also were more likely to come from less wealthy families.

With regards to the actual competencies gained by these students at age 4/5, our findings were consistent with Geary et al. (2013) in that students who were able to count to twenty, recognise numbers, and do simple additions were more likely to score higher in Year 3 NAPLAN numeracy, even after controlling for individual and family factors. However, our analysis does not agree with that undertaken by Watts et al. (2014), in that there was no evidence to suggest that these early childhood competencies were predictive of mathematics performance in adolescence, except via their contribution to performance in early primary school. This finding could be due to the differences in measures of early competencies and later mathematics achievement, with Watts using standardised tests on both occasions. However, Watts et al. (2014) also used average growth models, with research suggesting that individual growth models are better able to model the variance/covariance structure in longitudinal studies (Kwok, West, & Green, 2007). The only factor that predicted growth was socio-economic position, which is of concern as it suggests that children from wealthier families have an initial advantage and that this advantage increases during the school years. This finding is consistent with Sirin’s (2005) meta-analytic review of research pertaining to socio-economic status (SES) and academic achievement, which found a medium to strong SES-achievement relation and concluded that school success is greatly influenced by students’ family SES.

Of course, it is important to note that our analysis has been undertaken within the limits of the LSAC study design, including its measures. A limitation is that the assessment of early mathematical competencies was based on educators' judgements only and indeed was restricted to children enrolled in formal early childhood programs. Furthermore, the early mathematical competencies scale used in LSAC is very limited in its consideration of mathematical skills, and there may be more appropriate measures elsewhere; however, the analysis could only include data from the existing LSAC study. Our study points to the need for a new, large-scale study that utilises a more comprehensive assessment of early mathematical ability to further interrogate the relationship between early competencies and later achievement. Moreover, such a study needs to explore the growth in these competencies in those crucial early years.

Conclusion

Our study identified a reasonable correlation between pre-school-entry mathematics and Year 9 NAPLAN results. However, looking at growth trajectories suggests that early mathematical competence predicts the initial (Year 3) level results, but not subsequent growth. In turn, Year 3 then predicts Year 5, Year 5 predicts Year 7, and so forth. This is important to note, because it emphasises that while early competency in mathematics is predictive of children's mathematics achievement at Year 3, it is what happens afterwards that contributes to growth. This reflects the discourse around "early childhood education getting children off to the best start", but also emphasises that a good start alone isn't enough – what happens in the schooling years is critical.

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