

# Principles for Designing Intervention in Mathematics

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# Principles for Designing Intervention in Mathematics

## Purpose and Overview of Document

The purpose of this guide is to provide brief explanations of practices that can be implemented when working with students in need of intensive intervention in mathematics. Special education instructors, math interventionists, and others working with students who struggle with mathematics may find this guide helpful. Strategies presented in this guide should be used in conjunction with teaching guides developed for specific mathematical concepts. Specific topics covered include the following:

- Explicit, Systematic Instruction
- Effective Questioning
- Concrete, Representational/Visual/Pictorial, Abstract/Symbolic Models
- Teaching Mathematical Vocabulary and Symbols
- Fluency Building
- Error Analysis

## Explicit, Systematic Instruction

Explicit, systematic instruction in mathematics requires educators to clearly teach the steps involved in solving mathematical problems using a logical progression of skills (Hudson, Miller, & Butler, 2006; Montague & Dietz, 2009). Explicit instruction may take the form of teaching students how to use manipulatives, teaching specific algorithms for solving computational problems, or teaching strategies for solving more advanced mathematical concepts. Systematic instruction considers the scope and mathematical trajectories, such as the types of examples used for developing the foundational skills prior to introduction/re-teaching of grade-level material (Gersten et al., 2009; Kroesbergen & Van Luit, 2003; Maccini, Mulcahy, & Wilson, 2007). Regardless of the concept or skill being taught, explicit, systematic instruction should include the following components (Archer & Hughes, 2011; Hudson et al., 2006):

1. **Advance Organizer:** Providing students with an advance organizer allows them to know the specific objective of the lesson and its relevance to everyday life.
2. **Assessing Background Knowledge:** In assessing background knowledge, instructors determine whether students have mastered the prerequisite skills for successful problem solving in the new concept area. If the prerequisite skills were recently covered, assessment of background knowledge should be conducted quickly. If, however, those skills were taught several weeks ago, more time may be needed to refresh students' memories. Instructors can also determine whether students are able to generalize previously learned concepts to the new concept.

For example, if students have previously learned regrouping strategies in addition and subtraction, are they able to generalize these concepts to regrouping in multiplication and division? In addition, instructors should ask students questions about the new concept to assess their knowledge of the concept.

3. **Modeling:** During the modeling phase, instructors “think aloud” as they model the process of working through a computation problem; read, set up, and solve a word problem; use a strategy; or demonstrate a concept. During modeling, instructors should be clear and direct in their presentation; they also should be precise and mindful in using general and mathematical vocabulary as well as in selecting numbers or examples for use during instruction. During modeling, instructors should involve students in reading the problems and should ask questions to keep students engaged in the lesson.
4. **Guided Practice:** During guided practice, instructors engage all students by asking questions to guide learning and understanding as students actively participate in solving problems. During this phase, instructors prompt and scaffold student learning as necessary. Scaffolding is gradually eliminated as students demonstrate accuracy in using the material being taught. Positive and corrective feedback is provided during this phase, and instruction is adjusted to match student needs. Students should reach a high level of mastery (typically 85 percent accuracy or higher) before moving out of the guided practice phase.
5. **Independent Practice:** After achieving a high level of mastery, students move to the independent practice phase where they autonomously demonstrate their new knowledge and skills. During independent practice, the instructor closely monitors students and provides immediate feedback as necessary. Countless independent practice activities can be used with students, and the primary focus of the independent practice activity should be related to the content of the modeling and guided practice. If students demonstrate difficulty at this stage, instructors evaluate and adjust their instruction to re-teach concepts as needed.
6. **Maintenance:** Students with disabilities often have a difficult time maintaining what they have learned when the knowledge is not used on a regular basis. Students are given opportunities to independently practice these skills during the maintenance phase. During this phase, instructors use distributed practice to assess student maintenance at regularly scheduled intervals. Distributed practice is focused practice on a specific skill, strategy, or concept. The frequency of these practice assessments is determined by the difficulty level of the skill and according to individual student needs. Maintenance may also include cumulative practice.

Instructors often want to know how much time they should spend on each phase. Although there are no specific guidelines concerning how much time should be devoted to each phase, the bulk of the instruction should occur within the guided practice phase (National Center on Intensive Intervention, 2013).

## Concrete, Representational/Visual/Pictorial, and Abstract/Symbolic Models

Using multiple representations to teach mathematics allows students to understand mathematics conceptually, often as a result of developing or “seeing” an algorithm or strategy on their own. By building strong conceptual understanding, students are able to better generalize skills and understand algorithms (Gersten et al., 2009; Jones, Inglis, Gilmore, & Evans, 2013; Miller & Hudson, 2007). Moving through each phase is essential for every skill area, not just for early foundational skills (Jayanthi et al., 2008; National Mathematics Advisory Panel, 2008; Stein, Kinder, Silbert, & Carnine, 2005; Woodward, 2006). A description of the three phases follows.

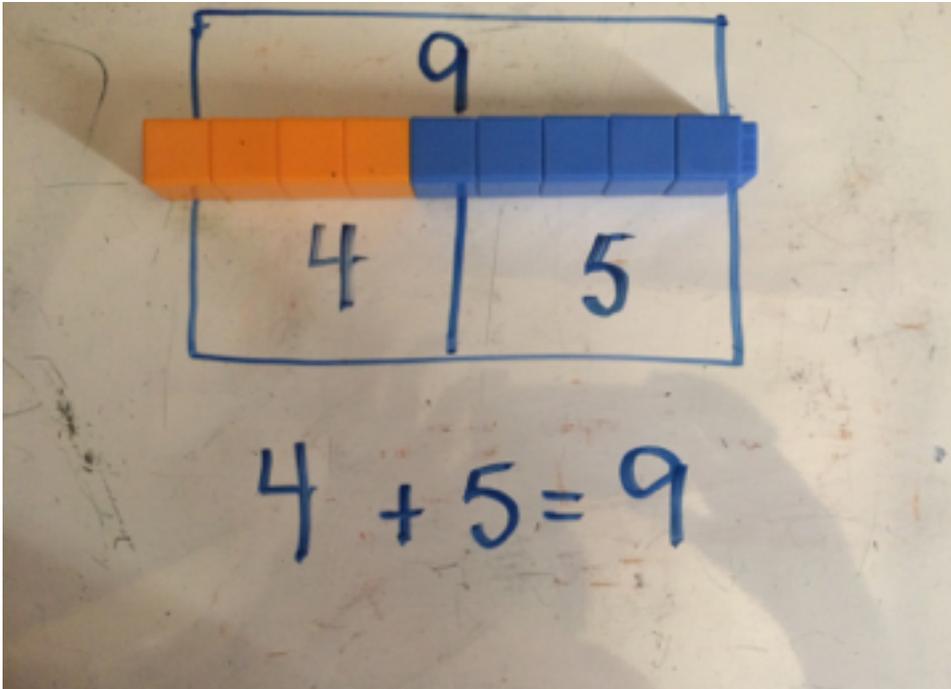
1. **Concrete:** In this phase, students use three-dimensional manipulatives to solve problems and to gain a better conceptual understanding of a concept. Examples of manipulatives include counting bears, snap cubes, base-10 blocks, real or plastic money, clocks, fraction tiles, geoboards, Algeblocks, algebra tiles, and others. It is helpful to use a variety of manipulatives (if possible) to teach concepts so that students can generalize the concept being taught. Using an assortment of manipulatives is not always possible, however; some concepts can only be taught using a specific manipulative.

It is important to note that although students may demonstrate proper use of a manipulative, this does not mean that they understand the concepts behind use of the manipulative. Explicit instruction and student verbalizations, such as explaining the concept or demonstrating use of the manipulative while they verbally describe the mathematical procedure, should accompany all manipulative use.

2. **Representational/Visual/Pictorial:** Students use two-dimensional pictures, drawings, or diagrams to solve problems. These pictures, drawings, or diagrams may be given to the students, or they may draw them when presented with a problem. These representations should be used to connect and solve the same concepts previously taught using concrete objects. Representational models also may be presented virtually through websites or tablet applications. With a virtual representation, students move the image with a mouse or with their hands.
3. **Abstract/Symbolic:** During this phase, students are expected to solve problems through the use of numbers and symbols rather than with the use of concrete objects or visual representations. Students are often expected to memorize facts and algorithms as well as to build fluency.

Following is an example that demonstrates use of the three phases to solve the problem  $4 + 5$ :

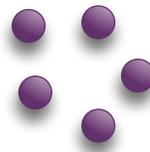
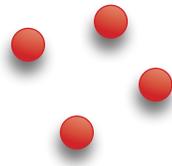
**Concrete:**



**Visual/Pictorial**



OR



**Symbolic:**

$$4 + 5 = 9$$

## Effective Questioning and Providing Feedback

Students who have difficulty with mathematics need many opportunities to respond to effective questions, explain their thinking, and receive feedback that allows them to improve their learning. To increase generalization of skills and flexibility in thinking mathematically, instructors need to ask questions that increase student engagement, that provide feedback, and that are linked to algebraic or higher level thinking and understanding (Cai & Knuth, 2005; Witzel, Mercer, & Miller, 2003). Specifically, beginning algebraic thinking, such as analyzing relationships, generalizing models, predicting, justifying, or noticing structure, can lead to greater gains in mathematics understanding in later years (Kieran, 2004).

Following are guidelines for asking questions that will move student learning forward, increase student engagement, and offer immediate feedback.

1. **Questioning:** The three main types of questions that should be used in mathematics are reversibility, flexibility, and generalization (Dougherty, Bryant, Bryant, Darrough, & Pfannenstiel, 2015).
  - **Reversibility** questions are those that change the direction of student thinking: for example, giving the student the answer and asking him or her to identify the correct equation. This type of question allows for multiple answers and gives students the opportunity to think about algorithms in different ways. Reversibility questions should be presented after the student has demonstrated mastery of a particular procedure or algorithm.
  - **Flexibility** questions support student understanding in finding multiple ways to solve a problem or in discerning relationships among problems. For example, the student might be asked to solve an addition problem using a specific strategy and then show or prove the answer using another method, such as a number line. Flexibility questions can be used during instruction to show relationships between similar problems or differences in models.
  - **Generalization** questions are those that ask students to create statements about patterns. In the past, instructors would explain algorithms or rules, and they did not afford students the opportunity to develop explanations on their own. To increase conceptual understanding, guided questions about patterns allow rules or generalizations to be “discovered” by the student. For instance, students are presented with a list of numbers multiplied by two and then asked to describe any patterns they notice (e.g., one factor is two, product is an even number, etc.). The use of generalization questions allows students to develop a deeper understanding of mathematics and to generalize their thinking to similar problems.

Regardless of the type of questions asked, instructors should use the questioning strategy to assess student understanding and then use the information obtained from the questioning to evaluate and adjust their instruction as necessary.

Instructors should involve all students in questioning. This involvement can be accomplished in several ways:

- **First**, instructors may invite all students to respond to questioning through **unison choral response**. Although this is an easy way to encourage students to respond, it is important to ensure that all students are responding to the questions at the same time.
- **Second**, instructors can use **equity sticks**. Instructors write each student's name on an ice pop stick and then draw a stick for each question they ask. The student whose name appears on the stick answers the question. All students have the same chance to be called upon.
- **Third**, instructors may use **response cards**. Write "A," "B," and "C" on separate cards. The instructor asks a question and presents three answer choices. Students select their choice and hold up the response card indicating their answer.
- **Fourth**, instructors may ask students to write their answers on **whiteboards**. Students hold up the answers so the instructor can check them for accuracy.
- **Fifth**, instructors may invite students to create a **model**. Students then pair-share their creations to identify differences and similarities among the models and answers to the mathematical questions.

Instructors may need to individualize their questions for students to gain a better understanding of a particular student's knowledge of the skill that is being taught.

2. **Feedback:** Providing students with both positive and corrective feedback is essential to their learning. It is important that students receive immediate feedback so that they do not continue to practice incorrectly. Students should also have an opportunity to practice/repeat the correct response after error correction has been provided (Archer & Hughes, 2011).

## Teaching Vocabulary and Symbols

Students with a strong mathematical vocabulary will have a better understanding of the concepts and skills being taught. Instructors, therefore, should use precise language when teaching mathematical concepts and skills. Explicit teaching of vocabulary and mathematical symbols is important in helping students understand mathematical concepts, and this explicit instruction should be integrated into all lessons. Encouraging student verbalization of mathematics vocabulary, paired with explicit instruction in identifying and using symbols, can increase overall mastery of symbolic representation (Driver & Powell, 2015; Frye, Barody, Burchinal, Carver, Jordan, & McDowell, 2013). Examples for teaching mathematics vocabulary and symbols are explained in this section.

1. **Word Walls/Word Banks:** Word walls and word banks can be used in two ways. First, instructors can create cards with vocabulary terms or symbols and the corresponding definitions. Second, students can create their own cards. Once cards have been created, they are placed on a word wall, or students can keep their own notebook containing the terms. When the word wall or student notebook is organized, the words should be placed in meaningful sections. For example, one section may be devoted to fractions, another section to basic operations, another to geometry terms, and so forth. This meaningful organization will help students when they are looking for the terms.

It is important to note that simply placing word cards on the wall or having students add them to their notebooks does not increase student understanding of the vocabulary or symbols. Instructors must teach these vocabulary terms and symbols and their definitions and then relate the terms and symbols to student learning. Instructors should use precise mathematical vocabulary in teaching and correcting, and they should also encourage students to use correct mathematical language in speech.



**parallelogram: a quadrilateral with both pairs of opposite sides parallel**



**difference: the result of subtracting one number from another**

$$15 - 9 = 6 \leftarrow 6 \text{ is the difference}$$

**Multiplication**

$$7 \times 8 = 56$$

factor (multiplier)      factor (multiplicand)      Product

$\sqrt{\dots}$  = **radical**

2. **Vocabulary Cards:** These cards may have the same vocabulary terms and symbols as those on the word wall or word bank. If instructors use vocabulary cards to teach definitions, they should write each term on the front of a card and its definition on the back. When the cards are used to teach symbols, the instructor should write each symbol on the front of a card and its name on the back. The cards are used as a practice activity in which students quickly say the word and state its definition or identify a symbol and cite its meaning. Through vocabulary card practice activities, students will learn to automatically recognize mathematical vocabulary terms and symbols.

<b>denominator</b>	<b>the number of parts into which one whole is divided</b>
$\infty$	<b>infinity</b>
$<$	<b>less than</b>

3. **Labeling:** Students are expected to label parts of a problem or figure in mathematics. Often, students need opportunities to identify vocabulary terms prior to solving problems. To increase overall mathematical vocabulary and flexibility, problems should be written in a variety of ways to show variability. Providing students with examples such as the ones below will enable instructors to assess students' understanding of terms prior to problem solving.

**Label and define the parts of this division problem.**

**Example 1:**      dividend      divisor      quotient

$$49 \div 7 = 7$$

**Example 2:**      dividend      divisor      quotient

$$\frac{49}{7} = 7$$

**Example 3:**      divisor      quotient      dividend

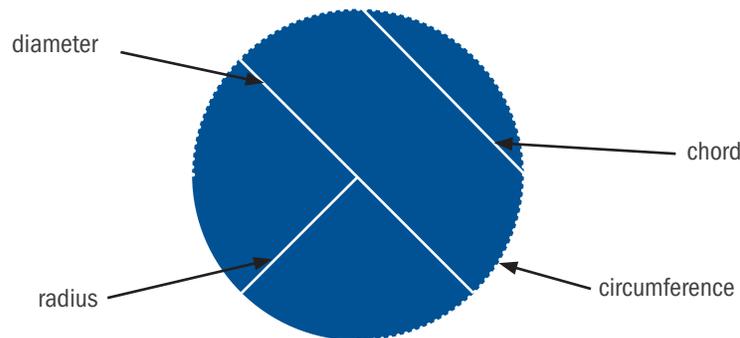
$$7 \overline{)49}$$

dividend = how many total

divisor = how many groups

quotient = how many in each group

**Label and define the parts of a circle.**



circumference = the distance around the outside of a circle

chord = line segment that joins two points of the circumference

diameter = the length of the line through the center of a circle

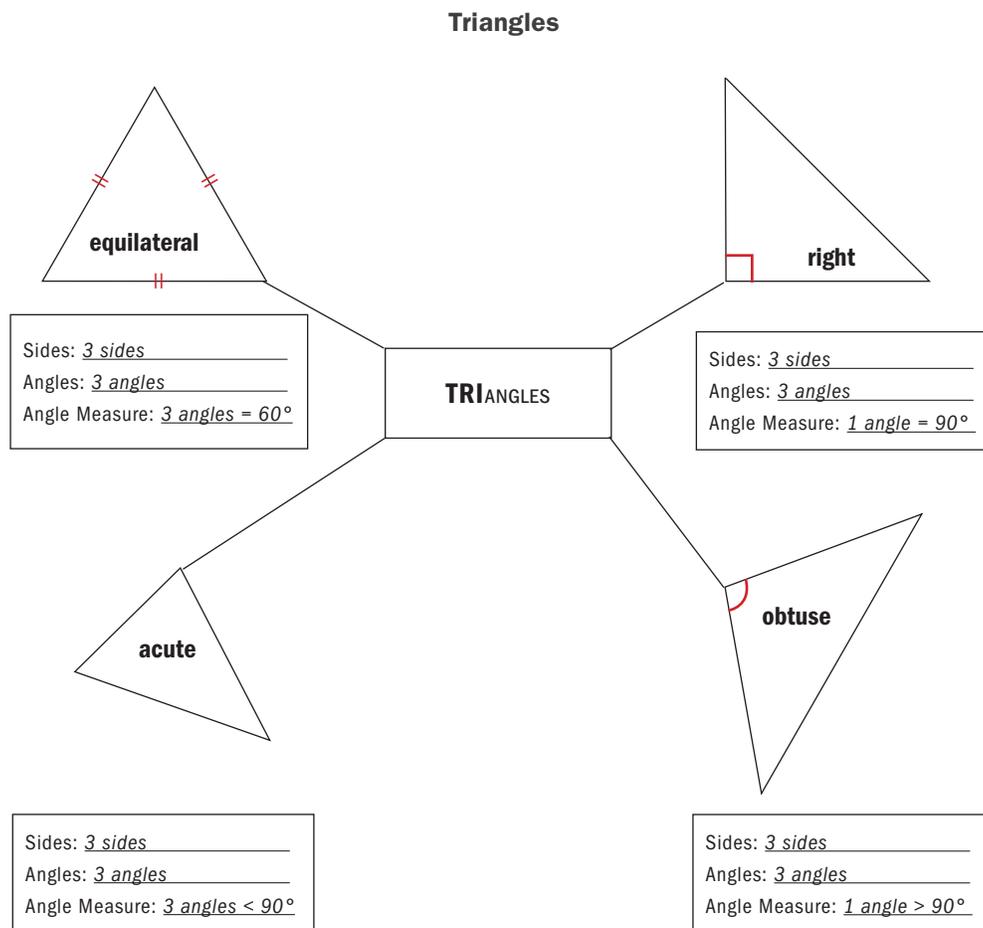
radius = line segment from the center of the circle to the perimeter

4. **Identifying Characteristics:** Some mathematical terms or concepts are more complicated than others and require further explanation as well as examples and non-examples. A characteristics table presents information in a manner that is easy for students to access. When a term or concept is introduced, instructors and students should complete a characteristics table together. In the first box, characteristics of the term/concept are listed. In the second box, examples of the term/concept are provided. In the third box, non-examples are listed. For some concepts or terms, it may be helpful to provide pictures of the examples and non-examples.

polygon: a simple, closed plane figure made up of three or more line segments		
Characteristics	Examples	Non-Examples
Closed Plane figure (two-dimensional) 3 or more sides No curved sides No intersecting lines	Triangle (3 sides) Quadrilateral (4 sides) Pentagon (5 sides) Octagon (8 sides) Dodecagon (12 sides) Icosagon (20 sides)	Circle Cube Sphere Cone Rectangular Pyramid Triangular Prism

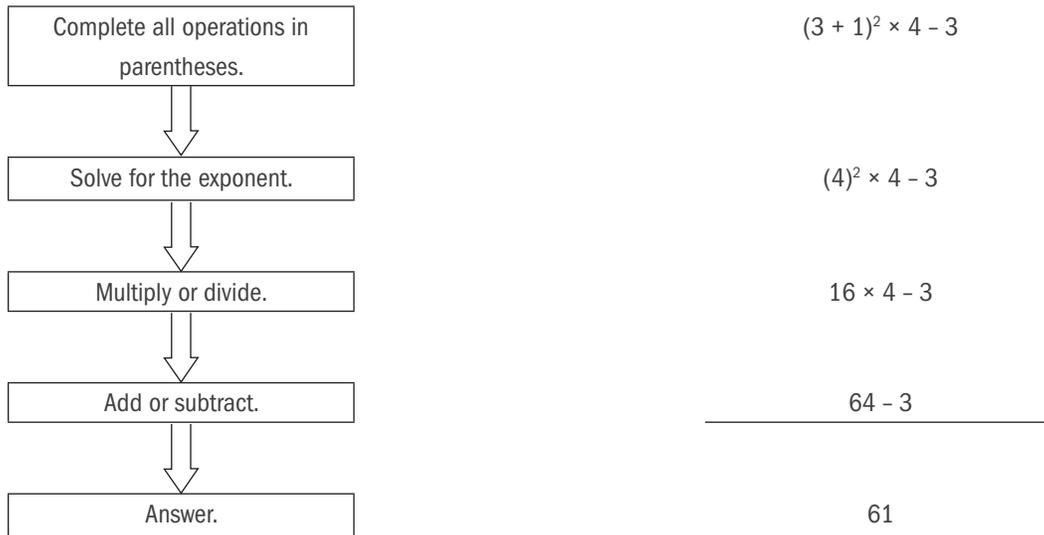
## Graphic Organizers

Graphic organizers are helpful mathematics tools; they allow a great deal of information to be logically organized in one place. Graphic organizers, such as the three variants of the Frayer Model that follow, are an efficient alternative to extensive note taking. Instructors explicitly teach students how to use the graphic organizer and the content provided. Graphic organizers can be used to illustrate most mathematical concepts.

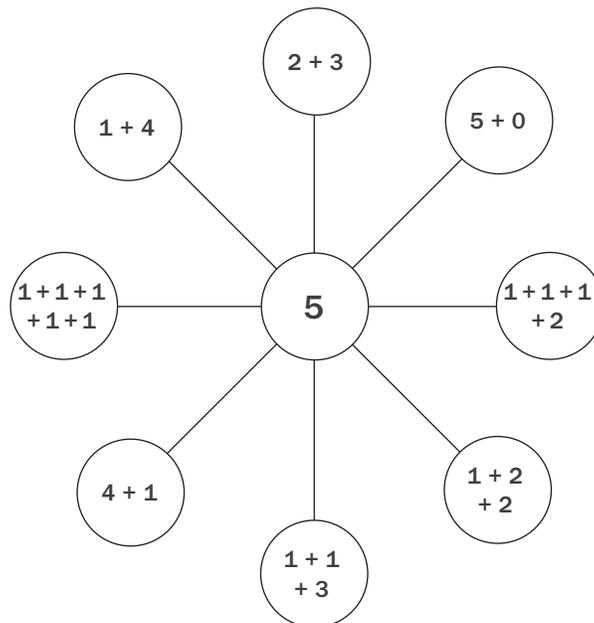


## Flow Map Order of Operations

Review order of operations: P (parentheses) E (exponents) D (division) M (multiplication)  
A (addition) S (subtraction)



### Different Ways to Decompose the Whole Number 5



## Fluency Building

In addition to conceptual knowledge, students need to develop procedural fluency in mathematics. Providing students with practice and timed activities that build fluency is essential. Opportunities to work through multi-step problems allow students to develop the higher-level thinking skills they need in order to progress to more complicated math concepts. Students need effective strategies and ample practice to increase their fluency in basic mathematics skills such as operational facts. The only way to truly increase fluency is to combine timed activities with additional practice opportunities (Raghubar et al., 2010; Woodward, 2006).

Another benefit of fluency can be enhanced motivation. When students become more fluent in mathematics skills, their motivation and confidence often increase. To heighten motivation, students should self-correct whenever possible for immediate feedback and then graph the results. Instructors also can integrate goal setting to further motivate and increase student self-regulation (Burns et al., 2010; Coddling et al., 2009; Montague, 2007; Rock, 2005). Following is a list of suggested activities that instructors may use for fluency-building practice. Many of these activities can be incorporated into peer tutoring activities.

1. **Timed Activities:** The use of timed activities to increase fluency in demonstrating knowledge of basic facts is a mainstay of mathematics education. The purpose of timed tests is to motivate students to increase their speed and to surpass their previous scores. Although timed activities are an effective tool for building fluency, they should not be the sole mode of instruction. Instructors should explicitly teach strategies that aid students in demonstrating their knowledge of mathematical facts. It is important to note that timed activities are not a motivator for all students; the focus, therefore, should be on answering correctly as well as quickly answering questions related to mathematical facts.
2. **Flash Cards:** Flash cards are often used to improve fluency in demonstrating knowledge of basic facts. They also can be used in activities such as identifying coins and their values, reading clocks, identifying fractions, and performing other mathematical tasks. Flash cards can be used with students and instructors or with peer tutors. Answers are provided on the backs of the cards so that the flash cards can be worked through quickly. Peer tutors should be taught how to correct answers so that neither peer is practicing the wrong answer during flash card activities. Students should record items scored as “incorrect” so that they can further practice the specific skills associated with these items. Students can graph the total number of flash cards answered correctly under timed conditions. This graphing can be done in tandem with goal setting to motivate the development of fact fluency.
3. **Computer Software:** Computer software activities, when paired with explicit teaching, can be highly engaging for students. Computer software programs provide the additional practice that struggling students need to increase

mathematics fluency and accuracy. Instructors should evaluate software programs to ensure that they meet the needs of students and that they require students to actively solve problems. Effective computer software will contain clear directions and will provide students with positive and corrective feedback immediately after they have answered questions (i.e., worked through problems). Computer programs should complement, rather than replace, instructor-led learning.

4. **Instructional Games:** Games provide students with fun, stimulating ways to practice skills that they have already been taught. Instructional games, including board games, have been found to increase skills in estimation, magnitude comparison, identification of numbers, and counting (Ramani, Hitti, & Siegler, 2012; Ramani & Siegler, 2008; Siegler & Ramani, 2008). The games should include mathematical components and foundational skills that correlate to the state standards. Following are some common games that can be adapted for teaching most mathematical concepts:
- **Bingo:** The instructor draws a card and reads the number, basic facts, fraction, or other item. Students mark the number or solution on their bingo cards. The first student who completes a row or column wins only if he or she can read all the numbers or answer all the problems in the row or column.
  - **Concentration/Memory:** Students play the game as they would with cards; however, before students can pick up a match, they must read the numbers or solve the problem.
  - **Dominoes:** Students play the game as they would regular dominoes by matching numbers with objects, math facts, fraction names with pictures of fractions, and so forth. Students must be able to answer the problem before they place their dominoes.
  - **Board games:** Using commercially produced board games can assist students in counting, estimation, and understanding real-world applications of money. Board games also tend to be linear and link to understanding of measurement and fractions in later grades.
  - **I have \_\_\_\_\_; who has \_\_\_\_\_?** This game can be used to practice a variety of mathematical skills. The sentence structure “I have \_\_\_\_\_; who has \_\_\_\_\_?” is written on each card. The cards are evenly distributed among students. One card has the word Start written on it. Examples are as follows:

“I have 5; who has 6 more?”

“I have 11; who has 2 less?”

“I have 9; who has its double?”

“I have 18; who has 7 less?”

The game continues until all cards have been used. This game can be used to practice knowledge of basic facts or more advanced skills such as adding and subtracting fractions with unlike denominators.

## Error Analysis

Error analysis is the process of analyzing student work to determine why students solved a problem incorrectly (Ashlock, 2010). Many errors can easily be detected—for example, regrouping ones instead of tens or adding denominators rather than finding common denominators. Other errors that are specific to an individual student’s understanding of a process are more difficult to identify. Even more confusing, some errors lead to the correct answer, and, in turn, students develop misconceptions. These errors require more careful examination, and often, students need to explain their thinking before the errors can be identified. Developing a step-by-step task analysis for some skills may help the instructor identify where in the process a student is having difficulty. Once errors have been identified, instructors should quickly address them so that the student does not continue to practice incorrectly, and educators should adjust their instruction to facilitate student understanding (Archer & Hughes, 2011; Stein et al., 2005).

## References

- Archer, A., & Hughes, C. (2011). *Explicit instruction: Effective and efficient teaching*. New York, NY: Guilford Publications.
- Ashlock, R. B. (2009). *Error patterns in computation: Using error patterns to help each student learn*. Upper Saddle River, NJ: Merrill.
- Burns, M. K., Coddling, R. S., Boice, C. H., & Lukito, G. (2010). Meta-analysis of acquisition and fluency math interventions with instructional and frustration level skills: Evidence for a skill-by-treatment interaction. *School Psychology Review*, 39(1), 69.
- Cai, J., & Knuth, E. J. (2005). The development of students’ algebraic thinking in earlier grades from curricular, instructional, and learning perspectives. *ZDM (formerly Zentralblatt für Didaktik der Mathematik)*, 37(1), 1–4.
- Coddling, R. S., Chan-Iannetta, L., Palmer, M., & Lukito, G. (2009). Examining a classwide application of cover-copy-compare with and without goal setting to enhance mathematics fluency. *School Psychology Quarterly*, 24(3), 173.
- Dougherty, B., Bryant, D. P., Bryant, B. R., Darrough, R. L., & Pfannenstiel, K. H. (2015). Developing concepts and generalizations to build algebraic thinking: The reversibility, flexibility, and generalization approach. *Intervention in School and Clinic*, 50(5), 273–281.
- Driver, M. K., & Powell, S. R. (2015). Symbolic and nonsymbolic equivalence tasks: The influence of symbols on students with mathematics difficulty. *Learning Disabilities Research & Practice*, 30(3), 127–134.
- Frye, D., Baroody, A. J., Burchinal, M., Carver, S. M., Jordan, N. C., & McDowell, J. (2013). *Teaching math to young children: A practice guide* (NCEE 2014–4005). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education.
- Gersten, R., Chard, D. J., Jayanthi, M., Baker, S. K., Morphy, P., & Flojo, J. (2009). Mathematics instruction for students with learning disabilities: A meta-analysis of instructional components. *Review of Educational Research*, 79(3), 1202–1242.
- Hudson, P., Miller, S. P., & Butler, F. (2006). Adapting and merging explicit instruction within reform based mathematics classrooms. *American Secondary Education*, 35, 19–32.

- Jayanthi, M., Gersten, R., & Baker, S. (2008). *Mathematics instruction for students with learning disabilities or difficulty learning mathematics: A guide for teachers*. Portsmouth, NH: RMC Research Corporation, Center on Instruction.
- Jones, I., Inglis, M., Gilmore, C., & Evans, R. (2013). Teaching the substitutive conception of the equals sign. *Research in Mathematics Education*, 15(1), 34–49.
- Kieran, C. (2004). Algebraic thinking in the early grades: What is it? *The Mathematics Educator*, 8(1), 139–151.
- Kroesbergen, E. H., & Van Luit, J. E. (2003). Mathematics interventions for children with special educational needs: A meta-analysis. *Remedial and Special Education*, 24(2), 97–114.
- Maccini, P., Mulcahy, C. A., & Wilson, M. G. (2007). A follow-up of mathematics interventions for secondary students with learning disabilities. *Learning Disabilities Research & Practice*, 22(1), 58–74.
- Montague, M. (2007). Self-regulation and mathematics instruction. *Learning Disabilities Research & Practice*, 22(1), 75–83.
- Montague, M., & Dietz, S. (2009). Evaluating the evidence base for cognitive strategy instruction and mathematical problem solving. *Exceptional Children*, 75(3), 285–302.
- Miller, S. P. and Hudson, P. J. (2007), Using evidence-based practices to build mathematics competence related to conceptual, procedural, and declarative Knowledge. *Learning Disabilities Research & Practice*, 22(1), 47–57
- National Center on Intensive Intervention. (2013). *Data-based individualization: A framework for intensive intervention*. Washington, DC: Office of Special Education, U.S. Department of Education.
- National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.
- Ramani, G. B., & Siegler, R. S. (2008). Promoting broad and stable improvements in low-income children's numerical knowledge through playing number board games. *Child development*, 79(2), 375–394.
- Ramani, G. B., Siegler, R. S., & Hitti, A. (2012). Taking it to the classroom: Number board games as a small group learning activity. *Journal of Educational Psychology*, 104(3), 661.
- Raghubar, K. P, Barnes, M. A., & Hecht, S. A. (2010). Working memory and mathematics: A review of developmental, individual difference, and cognitive approaches. *Learning and Individual Differences*, 20(2), 110–122.
- Rock, M. L. (2005). Use of strategic self-monitoring to enhance academic engagement, productivity, and accuracy of students with and without exceptionalities. *Journal of Positive Behavior Interventions*, 7(1), 3–17.
- Siegler, R., & Ramani, G. (2008). The development of mathematical cognition. *Developmental Science*, 11(5), 655–661.
- Stein, M., Kinder, D., Silbert, J., & Carnine, D. (2005). *Designing effective mathematics instruction: A direct instruction approach* (4th ed.). Upper Saddle River, NJ: Pearson.
- Witzel, B. S., Mercer, C. D., & Miller, M. D. (2003). Teaching algebra to students with learning difficulties: An investigation of an explicit instruction model. *Learning Disabilities Research & Practice*, 18(2), 121–131.
- Woodward, J. (2006). Developing automaticity in multiplication facts: Integrating strategy instruction with timed practice drills. *Learning Disability Quarterly*, 29(4), 269–289.